University of Minnesota: Twin Cities

CE 4352 Groundwater Modeling

Project 4: Line Sinks

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Contents

Introduction

Up until this point, the modeling of discharge through wells have been thoroughly discussed. In these models, wells are represented by a single point of some infinitesimal dimension (length, width, etc.). However, on a small enough scale, wells are poor models for cases in which water enters or exits an aquifer over a significant length. This is the case for such surface water features as canals, rivers, streams, etc. Line-sinks are used to apply the source/sink effect of a well over a finite, linear length. Their use is explored in this project.

As discussed in section 6.8 of *Applied Groundwater Mechanics* by Professor Otto D. Strack, the complex potential at complex location z in a field of uniform flow with a single line sink centered at $z = 0$ is:

$$\Omega_{ls} = \frac{\sigma L}{4\pi} \left\{ (Z+1)\ln(Z_j+1) - (Z-1)\ln(Z-1) + 2\ln\frac{L}{2} - 2 \right\} \tag{1}$$

Where:
σ is the strength of the line sink (discharge density per length);
L is the linear length of the line-sink;
Z is a dimensionless value acting as a function of complex location z, constructed from line-sink geometry, as shown in equation 2.

$$Z = \frac{z - \frac{1}{2}(\overset{2}{z} + \overset{1}{z})}{\frac{1}{2}(\overset{2}{z} - \overset{1}{z})} \tag{2}$$

Equation 2 uses symbols $\overset{1}{z}$ and $\overset{2}{z}$ to represent the beginning and end points of the line-sink, respectively.

If it's end points are known, the length L of a line-sink can be calculated using the Pythagorean Theorem.

$$L = \sqrt{\Re(\overset{2}{z} - \overset{1}{z})^2 + \Im(\overset{2}{z} - \overset{1}{z})^2} \tag{3}$$

Often, line-sink models provide more accurate approximations with increasing line-sink quantity. However, each line-sink requires a separate term for complex potential. Considering n line-sinks in a field of uniform flow of discharge Q_0 oriented at angle α, ccw from the x^+ axis, the expression for complex potential at point z becomes:

$$\Omega = -Q_0 z e^{-i\alpha} + \sum_{j=1}^{n} \frac{\sigma_j L_j}{4\pi} \left\{ (Z_j+1)\ln(Z_j+1) - (Z_j-1)\ln(Z_j-1) + 2\ln\frac{L_j}{2} - 2 \right\} + C \tag{4}$$

Where Z is solved for all n line-sinks at that point:

$$Z_j = \frac{z - \frac{1}{2}(\overset{2}{z}_j + \overset{1}{z}_j)}{\frac{1}{2}(\overset{2}{z}_j - \overset{1}{z}_j)} \tag{5}$$

And C is an arbitrary constant solved by using head or discharge data at a reference location z_0.

As the expressions for the effect of each line-sink on complex potential are quite lengthy, it becomes convenient to express all knowns as a separate function, Λ, with complex location z, and line-sink geometry $\overset{1}{z}$ and $\overset{2}{z}$ as arguments.

$$\Lambda_{ls}\left(z, \overset{1}{z}_j, \overset{2}{z}_j\right) = \frac{L_j}{4\pi} \Re \left\{ (Z_j+1)\ln(Z_j+1) - (Z_j-1)\ln(Z_j-1) + 2\ln\frac{L_j}{2} - 2 \right\} \tag{6}$$

With this newly defined Λ, equation 7 for the complex potential resulting from n line sinks becomes:

$$\Omega = -Q_0 z e^{-i\alpha} + \sum_{j=1}^{n} \sigma_j \Lambda \left(z, \overset{1}{z}_j, \overset{2}{z}_j \right) + C \tag{7}$$

For convenience, an expression can also be created for the effects on complex potential from given non-line-sink elements. This expression ($\Phi_g(z)$) can incorporate terms for given wells, uniform flow, etc.

Considering uniform flow to be the only non-line-sink term, equation 7 becomes:

$$\Omega = \sum_{j=1}^{n} \sigma_j \Lambda \left(z, \overset{1}{z}_j, \overset{2}{z}_j \right) + C + \Omega_g(z) \tag{8}$$

I Complex Potential For Center Heads and Reference Point

I.1 Complex Potential With σ and Z

In a field of uniform flow with one line sink of strength σ and no other elements (wells), the complex potential can be expressed as a function of Z. As shown in equation 2, Z has arguments $\overset{1}{z}$ and $\overset{2}{z}$, which are also used to calculate L in equation 3.

$$\Omega_{ls} = \frac{\sigma L}{4\pi} \left\{ (Z+1)\ln(Z_j+1) - (Z-1)\ln(Z-1) + 2\ln\frac{L}{2} - 2 \right\} \tag{9}$$

I.2 System Matrix

If more than one line sink is used, Λ is substituted for the Z term, and equation 7 becomes a set expressions of $\Omega(z)$ for n line-sinks. The $\Phi_g(z)$ is also used for given elements.

$$\Omega(z) = \begin{cases} n = 1 : \sigma\Lambda\left(z, \overset{1}{z}, \overset{2}{z}\right) + C + \Phi_g(z) \\ n = 2 : \sigma_1\Lambda\left(z, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(z, \overset{1}{z}_2, \overset{2}{z}_2\right) + C + \Phi_g(z) \\ n = 3 : \sigma_1\Lambda\left(z, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(z, \overset{1}{z}_2, \overset{2}{z}_2\right) + \sigma_3\Lambda\left(z, \overset{1}{z}_3, \overset{2}{z}_3\right) + C + \Phi_g(z) \\ n = n : \sum_{j=1}^{n} \sigma_j\Lambda\left(z, \overset{1}{z}_j, \overset{2}{z}_j\right) + C + \Omega_g(z) \end{cases} \tag{10}$$

In the event that the piezometric head at the center of each line-sink and at some reference point z_0 is known, it is possible to solve for the strengths of each line-sink using a system of equations (assuming constant strength across sink-length). A matrix of knowns (b) and unknowns (A) will be used to solve the strength of each line-sink as well as the arbitrary constant C. Given this single additional unknown element for C, the length of b will always be $m \times 1$ in size, where m is one plus the number of line sinks $m = n + 1$. This matrix of knowns is therefore symbolized b_m. As heads are the argument of complex potential, this Φ value is considered at each known line-sink center and the reference point to create b_m.

$$\Phi(z) = \Re\Omega(z) = \Re\left[\sum_{j=1}^{n}\sigma_j\Lambda\left(z, \overset{1}{z}_j, \overset{2}{z}_j\right)\right] + C + \Phi_g(z) \tag{11}$$

In order to create a matrix of unknowns A, b_m must incorporate all knowns. Therefore, the discharge potential resulting from given, non-line-sink elements is included.

$$b_m = \Phi(z_m) - \Phi_g(z_m) : m = 1, ..., n+1 \tag{12}$$

For b_m, elements 1 through n are calculated at each of the n line-sink centers. The final element $m = n+1$, is the discharge potential knowns at reference point z_0

$$b_m = \begin{cases} \Phi(\overset{c}{z}_m) - \Phi_g(\overset{c}{z}_m) : m = 1, ..., n \\ \\ \Phi(z_0) - \Phi_g(z_0) : m = n+1 \end{cases} \tag{13}$$

The matrix of unknowns A is then created in which each element of a given row are the coefficients of the unknowns to be solved using of m conditions by using the process shown in equation 10. The iterations for $n = 1, 2, 3$ show that the coefficient of element 1 through n of any given row is the Λ of the associated line-sink center. The last element of a given row is the coefficient of unknown constant C, which is 1 for all conditions.

$$A_{m,j} = \begin{cases} \Lambda_{ls}\left(\overset{c}{z}_m, \overset{1}{z}_j, \overset{2}{;}z_j\right) : j = 1, ..., n \\ \\ 1 : j = n+1 \end{cases} \tag{14}$$

The values of the unknowns are then solved and placed in column vector s.

$$s = A^{-1}b \tag{15}$$

Each of the first n single-element rows of s are the strengths of each line sink. The last single-element row provides the value of the constant C.

$$s_m = \begin{cases} \sigma_m : m = 1, ..., n \\ \\ C : m = n+1 \end{cases} \tag{16}$$

II Two Line Sinks, Well, Given Values

This approach to modeling with line-sinks is tested on an unconfined aquifer with the following elements and properties:

1 well with withdrawal of $Q = 800\frac{m^3}{day}$ located at $z_w = 100m + i100m$;

Two line-sinks spanning from $z_s = -200m$ to $z_e = 200m$, both with center heads of $\phi_c = 25m$;

At reference point of $z_0 = 1000m$, reference head is $\phi_0 = 28m$;

Uniform flow of $Q_0 = Q_{x0} = 0.4\frac{m^2}{day}$;

Hydraulic conductivity of water-bearing soil of $k = 10\frac{m}{day}$

II.1 Solving for Strengths and Constant

In order to solve for the strengths of both line-sinks as well as the arbitrary constant C, equation 11 is used to create three conditions.

$$\Phi(\overset{c}{z}_1) = \Re\left[\sigma_1\Lambda\left(\overset{c}{z}_1, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(\overset{c}{z}_1, \overset{1}{z}_2, \overset{2}{z}_2\right)\right] + C + \Phi_g(\overset{c}{z}_1)$$

$$\Phi(\overset{c}{z}_2) = \Re\left[\sigma_1\Lambda\left(\overset{c}{z}_2, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(\overset{c}{z}_2, \overset{1}{z}_2, \overset{2}{z}_2\right)\right] + C + \Phi_g(\overset{c}{z}_2)$$

$$\Phi(z_0) = \Re\left[\sigma_1\Lambda\left(z_0, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(z_0, \overset{1}{z}_2, \overset{2}{z}_2\right)\right] + C + \Phi_g(z_0)$$

The knowns of these conditions are clumped and used to create column-vector b.

$$b_1 = \Phi(\overset{c}{z}_1) - \Phi_g(\overset{c}{z}_1)$$

$$b_2 = \Phi(\overset{c}{z}_2) - \Phi_g(\overset{c}{z}_2)$$

$$b_3 = \Phi(z_0) - \Phi_g(z_0)$$

The discharge potential for the unconfined aquifer is function of piezometric head.

$$\Phi = \frac{1}{2}k\phi^2 \tag{17}$$

The complex potential resulting from uniform flow is a function of complex location z and uniform flow amount Q_{x0}.

$$\Omega_{UF} = -Q_{x0}z \tag{18}$$

The complex potential resulting from withdrawal at the well is a function of complex location z, well location z_w, and withdrawal discharge Q_w

$$\Omega_w = \frac{Q_w}{2\pi} \ln(z - z_w) \tag{19}$$

Equations 17, 18 and 19 are used to determine values of b.

$$b = \begin{pmatrix} \frac{1}{2}k{\phi_c}^2 - \Re\left[-Q_{x0}\overset{c}{z}_1 + \frac{Q_w}{2\pi}\ln(\overset{c}{z}_1 - z_w)\right] \\ \\ \frac{1}{2}k{\phi_c}^2 - \Re\left[-Q_{x0}\overset{c}{z}_2 + \frac{Q_w}{2\pi}\ln(\overset{c}{z}_2 - z_w)\right] \\ \\ \frac{1}{2}k{\phi_0}^2 - \Re\left[-Q_{x0}z_0 + \frac{Q_w}{2\pi}\ln(z_0 - z_w)\right] \end{pmatrix}$$

The matrix of unknowns, A is then populated according to equation 14.

$$A = \begin{pmatrix} \Lambda\left(\overset{c}{z}_1, \overset{1}{z}_1, \overset{2}{z}_1\right) & \Lambda\left(\overset{c}{z}_1, \overset{1}{z}_2, \overset{2}{z}_2\right) & 1 \\ \Lambda\left(\overset{c}{z}_2, \overset{1}{z}_1, \overset{2}{z}_1\right) & \Lambda\left(\overset{c}{z}_2, \overset{1}{z}_2, \overset{2}{z}_2\right) & 1 \\ \Lambda\left(z_0, \overset{1}{z}_1, \overset{2}{z}_1\right) & \Lambda\left(z_0, \overset{1}{z}_2, \overset{2}{z}_2\right) & 1 \end{pmatrix}$$

The column-vector of calculated values for unknowns s is then equated by multiplying the inverse of matrix A by b as shown in equation 15. Column-vector s is rendered as follows for two line sinks:

$$s = \begin{pmatrix} 7.8008 \\ 4.3231 \\ 776.9559 \end{pmatrix}$$

Interpreting these values for s yields the following results:

$$\sigma_1 = 7.8008\frac{m^2}{s}$$

$$\sigma_2 = 4.3231\frac{m^2}{s}$$

$$C = 776.9559\frac{m^3}{s}$$

II.2 Verification of Piezometric Heads

After solving for the sinks, it needs to be verified that the piezometric heads at the points of concern (link-sink centers and reference point) are indeed equal to given values.

By isolating ϕ in equation 17 for an unconfined aquifer, the piezometric head can be expressed as a function of discharge potential.

$$\phi = \sqrt{\frac{2\Phi}{k}} \tag{20}$$

Discharge potential can be expressed as a function of z by considering the real part of complex potential.

$$\Phi(z) = \Re\Omega(z) = \Re\left[\sigma_1\Lambda\left(z, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(z, \overset{1}{z}_2, \overset{2}{z}_2\right) + C - Q_{x0}z + \frac{Q_w}{2\pi}(z - z_w)\right] \tag{21}$$

Equations 20 and 21 can be combined to express piezometric head as a function of complex location.

$$\phi(z) = \sqrt{\frac{2\Re\left[\sigma_1\Lambda\left(z, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(z, \overset{1}{z}_2, \overset{2}{z}_2\right) + C - Q_{x0}z + \frac{Q_w}{2\pi}(z - z_w)\right]}{k}} \tag{22}$$

This can be used to verify piezometric heads at all points of concern.

$$\phi(\overset{c}{z}_1) = \sqrt{\frac{2\Re\left[\sigma_1\Lambda\left(\overset{c}{z}_1, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(\overset{c}{z}_1, \overset{1}{z}_2, \overset{2}{z}_2\right) + C - Q_{x0}\overset{c}{z}_1 + \frac{Q_w}{2\pi}(\overset{c}{z}_1 - z_w)\right]}{k}} = \phi_c \tag{23}$$

$$\phi(\overset{c}{z}_2) = \sqrt{\frac{2\Re\left[\sigma_1\Lambda\left(\overset{c}{z}_2, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(\overset{c}{z}_2, \overset{1}{z}_2, \overset{2}{z}_2\right) + C - Q_{x0}\overset{c}{z}_2 + \frac{Q_w}{2\pi}(\overset{c}{z}_2 - z_w)\right]}{k}} = \phi_c \tag{24}$$

$$\phi(z_0) = \sqrt{\frac{2\Re\left[\sigma_1\Lambda\left(z_0, \overset{1}{z}_1, \overset{2}{z}_1\right) + \sigma_2\Lambda\left(z_0, \overset{1}{z}_2, \overset{2}{z}_2\right) + C - Q_{x0}z_0 + \frac{Q_w}{2\pi}(z_0 - z_w)\right]}{k}} = \phi_0 \tag{25}$$

Using this method, the piezometric heads are indeed confirmed.

$$\phi(z) = \begin{cases} 25m : z = -200m + 0i \\ 25m : z = 200m + 0i \\ 28m : z = 1000m + 0i \end{cases} \tag{26}$$

II.3 Flow Through Canal

The volumetric discharge through a line-sink is the product of its strength and length.

$$Q_{ls} = \sigma L \tag{27}$$

In this project, for this step, a canal is modeled as two adjacent line sinks. The flows through each of these line sinks $Q_{1,ls}$ and $Q_{2,ls}$ are calculated considering their strengths calculated in section II.1 and the lengths they represent along the total canal.

$$Q_{1,ls} = \sigma_1 L_1 = 7.8008\frac{m^2}{day}(-200m) = 1560.16\frac{m^3}{day}$$

$$Q_{2,ls} = \sigma_2 L_2 = 4.3231\frac{m^2}{day}(-200m) = 864.62\frac{m^3}{day}$$

$$Q_{total} = Q_{1,ls} + Q_{2,ls} = 1560.16\frac{m^3}{day} + 864.62\frac{m^3}{day} = 2424.78\frac{m^3}{day}$$

The positive sign of both σ_1 and σ_2 indicates that flow is occurring out of the canal. As L cannot logically be negative, this sign convention also applies to discharge Q_{ls}. That is, in this case, the canal is withdrawing water from the aquifer.

II.4 Flow Net and Piezometric Head Contours

The complex potential is calculated for a set of points composing a grid across the aquifer within the region of interest. Figure 1 shows a *Flow Net* of this region, in which contours of constant Potential Discharge $\Phi = \Re\Omega$ are plotted in red, and contours of constant Streamlines $\Psi = \Im\Omega$ are plotted in blue.

In figure 2, just the potential discharge at each point of the region is considered and converted to piezometric head using equation 20. Contours of constant head are then plotted using a color-coded gradient.

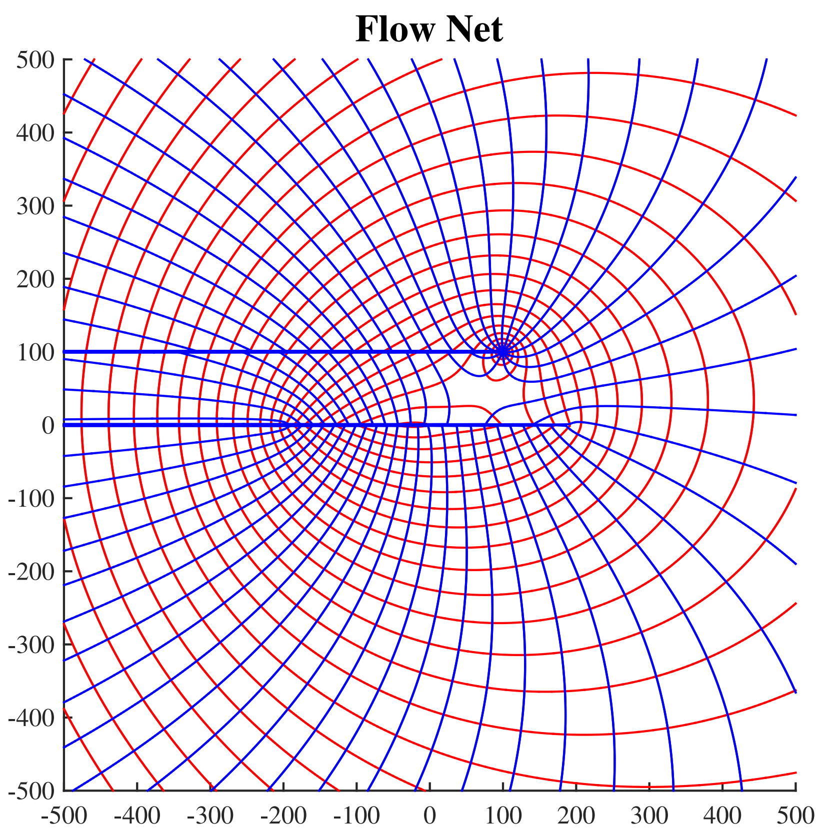

Figure 1: Discharge Potential and Streamline Contours

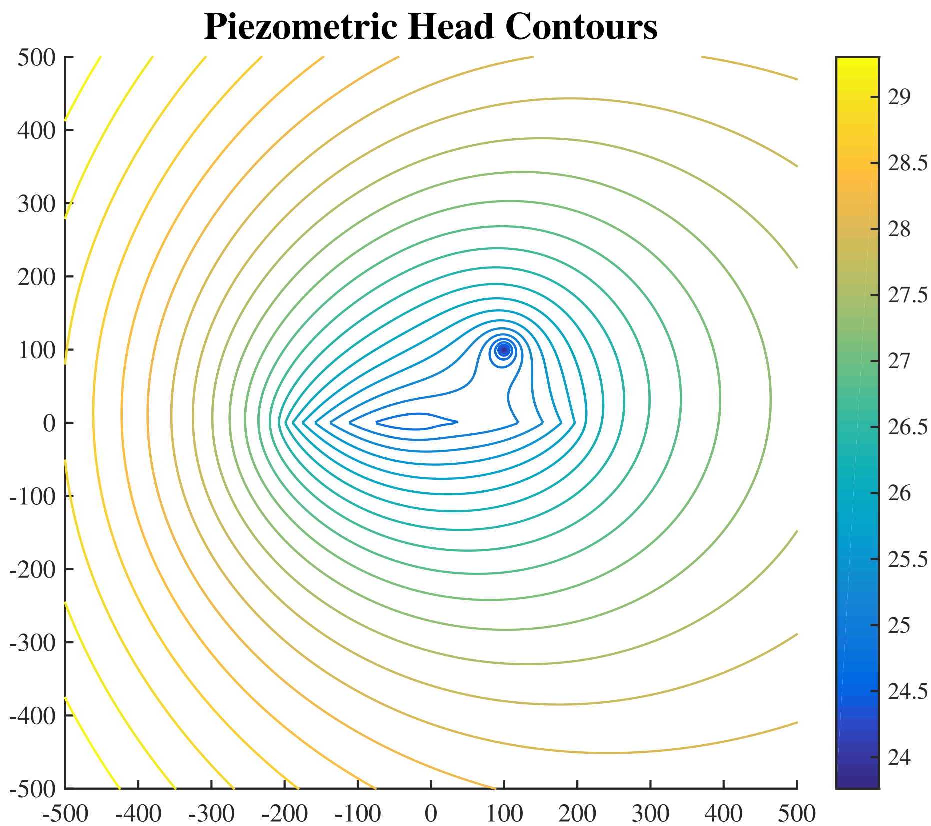

Figure 2: Contours of Piezometric Head

II.5 Flow Distributions

The sources of flow into or out of the aquifer are the canal, the well and uniform flow from infinity. The sum of these flows can be expressed by obeying continuity.

$$Q_{\infty} = Q_w + \sum_{j=1}^{n} Q_{ls,n} \tag{28}$$

Given that Q_{∞} is the only source of flow *into* the aquifer, both the well and line-sinks draw 100% of their discharge from infinity.

III Line Sink Quantity and Accuracy

As seen in table 1, the value for discharge from the aquifer converges to approximately 2650 $\frac{m^3}{day}$ with increasing line-sinks used in the model. This is the increasingly precise value calculated for Q_{ls}

Quantity	$Q_{ls}(\frac{m^3}{day})$
1	2312.7
2	2424.8
3	2501.3
4	2538.7
5	2561.2
10	2605.9
20	2628.2
30	2635.6
40	2639.3
50	2641.6
100	2646.0
500	2649.6
1000	2650.0
2000	2650.2
3000	2650.3

Table 1: Line-sink quantity and increasingly precise discharge value

IV Code

IV.1 Master Script

```
%%%%%%%%%% CE 4352 Groundwater Modeling  %%%
%%%%%%%%%%%% Project 4 Master Script %%%%%%%

clear all
close all
clc

dispphi = 0; %display phi values? [1 = yes, 0 = no]
flownet = 0; %display flownet? [1 = yes, 0 = no]
sinkflow = 1; %display sink inflow values? [1 = yes, 0 = no]
zees = 0; %display z values? [1 = yes, 0 = no]
show_s = 0; %show s values? [1 = yes, 0 = no]

%%% provided data %%%
Qw = 800; %well discharge [m^3/day]
d = 100; %well location value [m]
zw = complex(d, d); %well location
zs = complex(-2*d, 0); %canal extends from
ze = complex(2*d, 0); %canal extends to
zref = complex(1000, 0); %reference head location
phi0 = 25; %piezomtric head at both line-sink centers [m]
phiref = 28; %reference piezometric head [m]
Qx0 = 0.4; %uniform flow rate [m/day]
k = 10; %hydraulic conductivity [m/day]

%%% line-sink locations %%%
n = 3000; %number of line sinks
L = sqrt((real(ze)-real(zs))^2 + (imag(ze) - imag(zs))^2);
zone = zeros(1, n);
ztwo = zeros(1, n);
zm = zeros(1, n+1);
loc = linspace(zs, ze, n+1);
for j = 1:n
zone(j) = loc(j);
ztwo(j) = loc(j+1);
zm(j) = (loc(j) + loc(j+1))/2;
end
zm(n+1) = zref;
if zees == 1
disp('=======');
disp('start points');
disp(zone);
disp('=======');
disp('end points');
disp(ztwo);
disp('=======');
disp('centers');
disp(zm);
disp('=======');
```

```
end
A = zeros(n+1, n+1);
for m =1:n+1
for j = 1:n
A(m,j) = Lambda(zm(m), zone(j), ztwo(j));
end
A(m,n+1) = 1;
end
b = zeros(n+1, 1);
for m = 1:n
b(m,:) = 0.5*k*phi0^2 + real(Qx0*zm(m))-real((Qw/(2*3.14))*log(zm(m)-zw));
end
b(n+1,:) = 0.5*k*phiref^2 + real(Qx0*zref)-real((Qw/(2*3.14))*log(zref-zw));
s = A\b;
if show_s == 1;
    disp(s);
end
if dispphi == 1
disp('=====');
for j = 1:n
disp('head of sink');
disp(j)
disp(phi(zm(j),s,zone,ztwo,Qx0, Qw, zw));
disp('=====');
end
disp('head of reference');
disp(phi(zm(n+1),s,zone,ztwo,Qx0, Qw, zw));
end
if sinkflow == 1
dispsig = 0;
disp('===========');
for j = 1:n
disp('inflow for line-sink');
disp(j)
disp(s(j)*L/n);
disp('===========');
dispsig = s(j) + dispsig;
end
disp('combined inflow for all line-sinks');
disp(dispsig*L/n);
end

%%%% Flow Net %%%%%%%%
window = 500;
xfrom = -window;
xto = window;
yfrom = -window;
yto = window;
Nx = 400;
Ny = 400;
nint = 50;
if flownet == 1
P4Q1ContourMe_flow_net(xfrom, xto, Nx, yfrom, yto, Ny,
@(z)Omega(z,s,zone,ztwo,Qx0, Qw, zw),nint);
```

```
axis([-window window -window window]);
set(gca,'FontSize',12, 'fontname','times');
title('Flow Net','FontSize', 18);
ax = gca;
ax.XTick = [-500,-400,-300,-200,-100,0,100,200,300,400,500];
ax.YTick = [-500,-400,-300,-200,-100,0,100,200,300,400,500];
hold off
P4Q2ContourMe_flow_net(xfrom, xto, Nx, yfrom, yto, Ny,
@(z)Omega(z,s,zone,ztwo,Qx0, Qw, zw),nint);
set(gca,'FontSize',12, 'fontname','times');
title('Piezometric Head Contours','FontSize', 18);
ax = gca;
ax.XTick = [-500,-400,-300,-200,-100,0,100,200,300,400,500];
ax.YTick = [-500,-400,-300,-200,-100,0,100,200,300,400,500];
colorbar;
hold off
end
```

IV.2 Λ Function

```
%%%%%%% Lambda Function for Project 4 %%%%%%%
function [ Lambda ] = Lambda(zm, zonej, ztwoj)
Lj = sqrt((real(ztwoj)-real(zonej))^2 + (imag(ztwoj)-imag(zonej))^2);
Zj = (zm - 0.5*(ztwoj+zonej))/(0.5*(ztwoj - zonej));
Lambda = (Lj/(4*3.14))*real((Zj + 1)*log(Zj + 1) -
(Zj - 1)*log(Zj- 1)+2*log(Lj/2) - 2);
end
```

IV.3 ϕ Function

```
%%%%%%% Phi Function for Project 4 %%%%%%%
function [ phi ] = phi(z,s,zone,ztwo,Qx0, Qw, zw)
n = length(s)-1;
Lambda = zeros(1, n);
for j=1:n
Lj = sqrt((real(ztwo(j))-real(zone(j)))^2 + (imag(ztwo(j))-imag(zone(j)))^2);
Zj = (z - 0.5*(ztwo(j) + zone(j)))/(0.5*(ztwo(j) - zone(j)));
Lambda(j) = (Lj/(4*3.14))*real((Zj + 1)*log(Zj + 1) -
(Zj - 1)*log(Zj- 1)+2*log(Lj/2) - 2);
end
Phi = s(length(s)) - Qx0*z + (Qw/(2*3.14))*log((z - zw));
for j = 1:n
Phi = real(Phi + Lambda(j)*s(j));
end
phi = sqrt(2*Phi/10);
```

IV.4 Ω Function

```
%%%%%%% Complex Potential Function for Project 4 %%%%%%%
function [ Omega ] = Omega(z,s,zone,ztwo,Qx0, Qw, zw)
n = length(s)-1;
Lambda = zeros(1, n);
for j=1:n
Lj = sqrt((real(ztwo(j))-real(zone(j)))^2 + (imag(ztwo(j))-imag(zone(j)))^2);
```

```
Zj = (z - 0.5*(ztwo(j) + zone(j)))/(0.5*(ztwo(j) - zone(j)));
Lambda(j) = (Lj/(4*3.14))*((Zj + 1)*log(Zj + 1) -
(Zj - 1)*log(Zj- 1)+2*log(Lj/2) - 2);
end
Omega = s(length(s)) - Qx0*z + (Qw/(2*3.14))*log(z-zw);
for j = 1:n
Omega = Omega + Lambda(j)*s(j);
end
end
end
```

IV.5 Flow Net Routine Function

```
function [Grid] = P4Q1ContourMe_flow_net(xfrom, xto, Nx, yfrom, yto, Ny, Omega,nint)
Grid = zeros(Ny,Nx);
X = linspace(xfrom, xto, Nx);
Y = linspace(yfrom, yto, Ny);
for row = 1:Ny
    for col = 1:Nx
        Grid(row,col) = Omega( complex( X(col), Y(row) ) );
    end
end
Bmax=max(imag(Grid));
Bmin=min(imag(Grid));
Cmax=max(Bmax);
Cmin=min(Bmin);
D=Cmax-Cmin;
del=D/nint;
Bmax=max(real(Grid));
Bmin=min(real(Grid));
Cmax=max(Bmax);
Cmin=min(Bmin);
D=Cmax-Cmin;
nintr=round(D/del);
figure;
hold on
contour(X, Y,real(Grid),nintr,'r');
contour(X, Y,imag(Grid),nint,'b');
axis square
axis equal
```

IV.6 Piezometric Head Contour Routine Function

```
function [Grid] = P4Q2ContourMe_flow_net(xfrom, xto, Nx, yfrom, yto, Ny, Omega,nint)
Grid = zeros(Ny,Nx);
X = linspace(xfrom, xto, Nx);
Y = linspace(yfrom, yto, Ny);
for row = 1:Ny
    for col = 1:Nx
        Grid(row,col) = Omega( complex( X(col), Y(row) ) );
    end
end
Bmax=max(imag(Grid));
Bmin=min(imag(Grid));
Cmax=max(Bmax);
```

```
Cmin=min(Bmin);
D=Cmax-Cmin;
del=D/nint;
Bmax=max(real(Grid));
Bmin=min(real(Grid));
Cmax=max(Bmax);
Cmin=min(Bmin);
D=Cmax-Cmin;
nintr=round(D/del);
figure;
hold on
contour(X, Y,sqrt(2*real(Grid)/10),nintr);
axis square
axis equal
```