UNIVERSITY OF MINNESOTA: TWIN CITIES

CE 4511 HYDRAULIC STRUCTURES

HW 7: Scour, Rip Raps

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Contents

0) Preliminaries

0.1) Introduction

This assignment reviews the process of predicting the extent of scouring using formulae provided by Bormann and Julian (1991). The hydraulic system shown in figure 1 is given in which a rectangular drop structure discharges into a wide, trapezoidal channel of natural material.

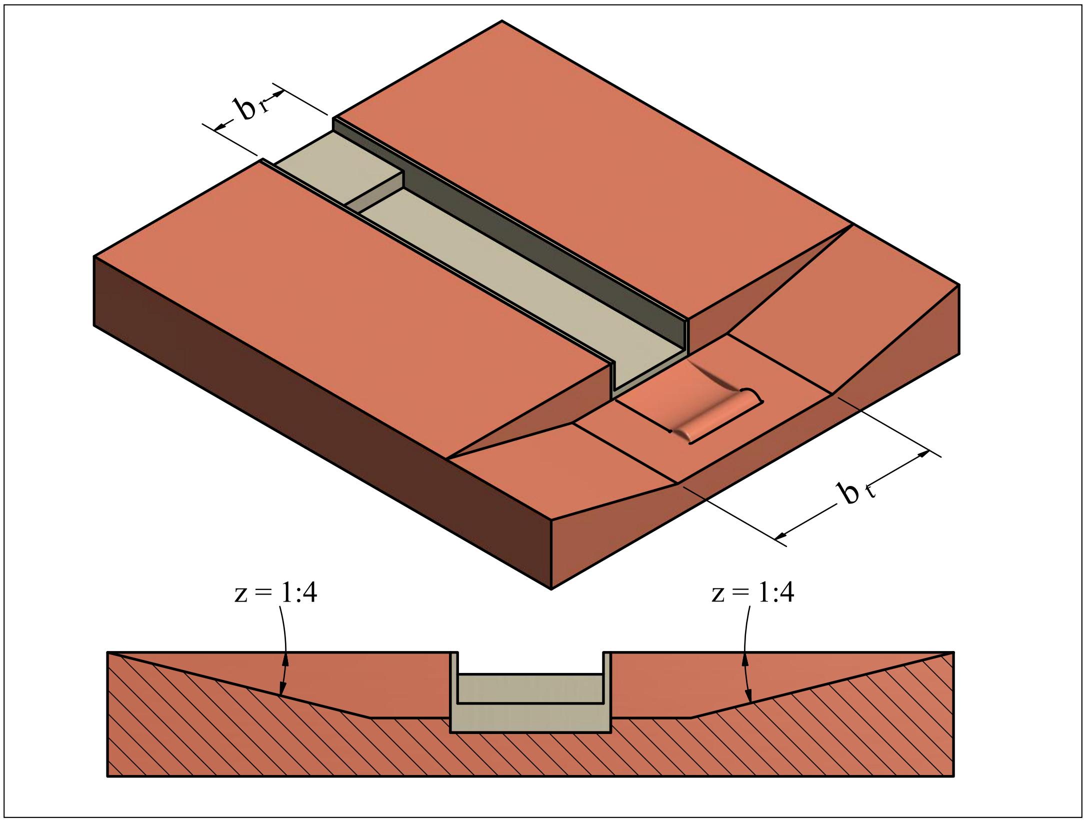

Figure 1: Given drop structure

In order to calculate the maximum scour depth as well as its horizontal location, a series of steps are taken to determine the hydraulic depth at the foot of the chute resulting from the concrete drop as well as the hydraulic depth at the end of the apron, just before the discharge enters the natural, downstream channel. With this apron-end depth known, Bormann and Julain's equations can be used to determine the location and extent (depth) of maximum scouring. As a final step, a rip rap is then designed to help reduce the affects of scouring.

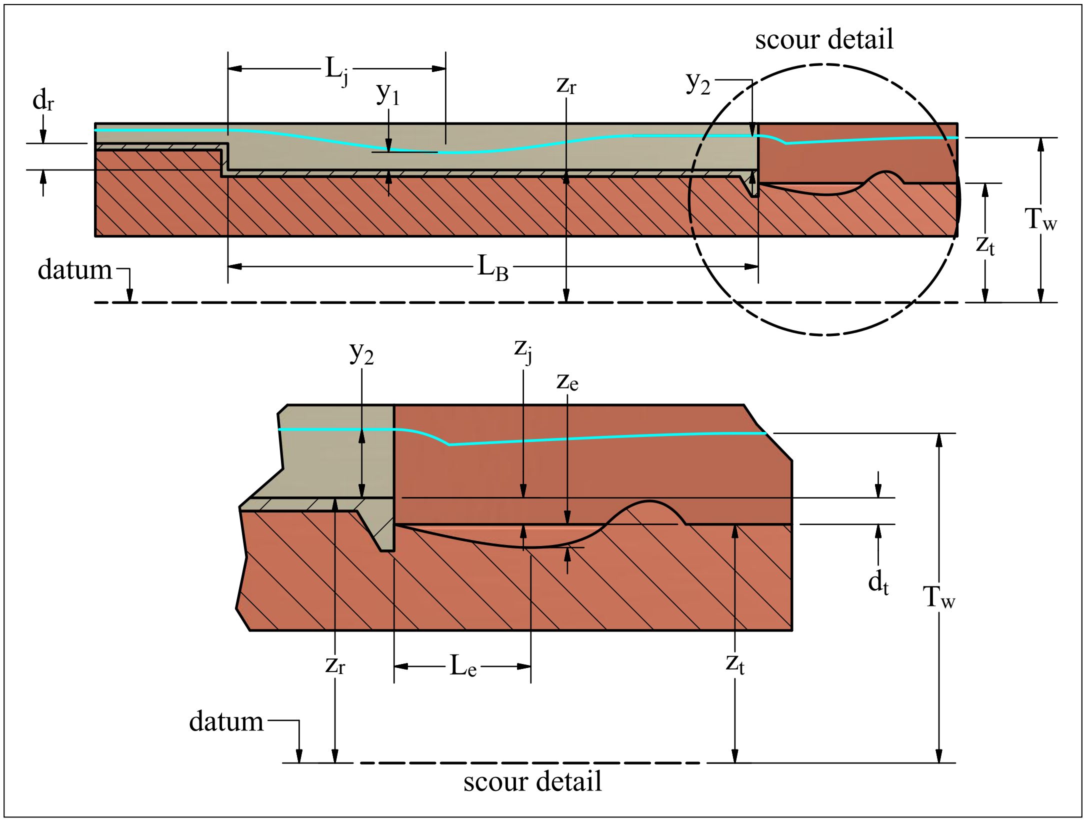

Figure 2: Given drop structure

As shown in figures 1 and 2, the following properties are given for this system:

- bed elevation of rectangular channel: $z_r = 821.1\ ft$;
- bed elevation of trapezoidal channel: $z_t = 820.1\ ft$
- cross sectional width of the rectangular drop structure: $b_r = 10\ ft$;
- base width of trapezoidal channel: $b_t = 22\ ft$;
- side slope of trapezoidal channel: $z = V : H = 1 : 4$;
- drop height of rectangular channel: $d_r = 2\ ft$;
- drop height into trapezoidal channel: $d_t = 1\ ft$;
- total drop structure length: $L_B = 40\ ft$

The following properties, not shown in either figure are also given/inferred:

- discharge per unit width in rectangular channel: $q_r = 20\ \frac{ft^2}{s}$
- rectangular channel bed is horizontal: $s_{0_r} = 0\%$;
- trapezoidal channel bed slope: $s_{0_t} = 0.56\%$;
- trapezoidal channel bed made of material with scouring property: $d_{90} = 0.5\ in$;
- Manning's n along concrete bed: $n = 0.030$;
- acceleration of gravity: $g = 32.2\ \frac{ft}{s^2}$

1) Calculating Tailwater Depth

Tailwater depth is crucial to understanding scouring effects. The tail-water depth is approximated to be the normal depth of flow in the trapezoidal channel and calculated as follows:

$$\text{using the formula for normal flow in a prismatic channel : } \quad Q = \left(\frac{k}{n}\right) A_n R_n^{\frac{2}{3}} \sqrt{s_{0_t}} \tag{1}$$

$$\text{where : } \quad k = 1.486 \text{ for English units} \tag{2}$$

$$\text{discharge assumed same from rectangular channel : } \quad Q = q_r b_r \tag{3}$$

$$\text{trapezoidal area : } \quad A_n = y_n(b_t + z y_n) \tag{4}$$

$$\text{trapezoidal wetted perimeter : } \quad P_n = b_t + 2y_n\sqrt{1+z^2} \tag{5}$$

$$\text{trapezoidal hydraulic radius : } \quad R_n = \frac{A_n}{P_n} \tag{6}$$

a value of y_n is solved for satisfying eq. 1 with known Q

This process, whose values are shown in table 1, yields:

$\boxed{y_{n_t} = 1.6013\ ft}$ $\boxed{T_w = y_{n_t} + z_t = 821.7013\ ft}$

$\mathbf{y_n}$ (ft)	$\mathbf{A_n}$ (ft^2)	$\mathbf{P_n}$ (ft)	$\mathbf{R_n}$ (ft)	$\mathbf{Q = q_r b_r}$ (ft^3/s)	$\mathbf{Q = \left(\frac{k}{n}\right) A_n R_n^{\frac{2}{3}} \sqrt{s_{0_t}}}$ (ft^3/s)
1.6013	45.4843	35.2044	1.292	200	200

Table 1: Process of determining y_{n_t}

2) Calculating Drop Structure Basin Depths

2.1) Drop Toe Depth

The with known discharge and drop height, the basin depth of the concrete, rectangular channel is fairly straight forward:

$$\text{toe depth is calculated}: \quad y_1 = d_r(0.54)Dr^{0.425} \tag{7}$$

$$\text{where}: \quad Dr = \frac{{q_r}^2}{g{d_r}^3} \tag{8}$$

This process, whose values are shown in table 2 yields:

$$\boxed{y_1 = 1.3021\ ft}$$

$\mathbf{d_r}$ (ft)	**Dr** -
2	1.5528

Table 2: Process of determining y_1

2.2) Conjugate Depth of Drop Toe Depth

In order to check if a hydraulic jump can occur, the conjugate elevation of y_1 is determined and compared to the elevation T_w. If it determined that the conjugate depth of y_1 is greater than the tail-water elevation, a hydraulic jump is not possible. Therefore, this conjugate depth is calculated as follows:

$$y_1 \text{ conjugate is calculated}: \quad \text{conj}\,(y_1) = \frac{y_1}{2}\left[1 + 8{Fr_1}^2\right] \tag{9}$$

$$\text{where}: \quad Fr_1 = \frac{V_1}{\sqrt{gy_1}} = \frac{\frac{q_r}{y_1}}{\sqrt{gy_1}} \tag{10}$$

This process, whose values are shown in table 3, yields $\boxed{\text{a conjugate depth of } y_1 \text{ of } 29.9579\ ft}$.

$\mathbf{V_1}$ $(^{ft}/_{s})$	$\mathbf{Fr_1}$ -
15.3597	2.3721

Table 3: Process of determining conjugate depth of y_1

In order to compare elevations, the post-concrete drop length is subtracted from T_w and compared:

$$T_w - d_t - z_t = 821.7013\ ft - 1\ ft - 820.1\ ft = 0.603\ ft << \text{conjugate depth of } y_1$$

Thus, it is determined that a hydraulic jump is not possible over the apron. Therefore, a water surface profile along the apron will developed in order to determine the apron-end depth y_2.

2.3) WSP Along Apron

2.3.1) Determing Drop Length

A water surface profile (WSP) is to be developed, starting at known depth y_1 and progressing downstream until the total apron length is reached. This requires a value of drop length L_d in order to determine the remaining horizontal distance to be covered. This is done using the following equation with values already known as shown in table 2:

$$L_d = d_r(4.3)Dr^{0.27} \tag{11}$$

This yields $\boxed{L_d = 9.865\ ft}$.

2.3.2) Determine Depth at Downstream End of Drop Structure

With this initial horizontal location along the apron known, the apron-end depth y_2 is calculated using the direct step method as follows:

a table is set up, the 1st column of which is the depth : y

(where the depth in the 1st row is set as the known drop depth)

the 2nd column is rectangular area : $$A = (y)b_r \tag{12}$$

the 3rd column is rectangular wetted perimeter : $$P = (2y) + b_r \tag{13}$$

the 4th column is hydraulic radius : $$R = \frac{A}{R} \tag{14}$$

the 5th column is average velocity : $$V = \frac{q_r}{y} \tag{15}$$

the 6th column is total energy : $$E = y + \frac{V^2}{2g} \tag{16}$$

the 7th column is friction slope : $$s_f = \left(\frac{nQ}{1.49AR^{\frac{2}{3}}}\right)^2 \tag{17}$$

(it is at this point that the 1st row is complete)

the 8th column is average friction slope between rows : $$\overline{s_f} = \frac{s_{f_i} + s_{f_{i-1}}}{2} \tag{18}$$

the 9th column is the corresponding horizontal progression : $$dx = \frac{E_i - E_{i-1}}{s_{0_r} - \overline{s_f}} = \frac{E_i - E_{i-1}}{-\overline{s_f}} \tag{19}$$

the 10th column is the cumulative sum of horizontal progression : $$x_i = \sum_{j=1}^{i} dx_j \tag{20}$$

process halted when horizontal progression reaches apron end : stop at $x_i > L_B$

Using a height step of $\Delta y = 0.001$, this process, a portion of the values of which are shown in table 4, yields:

$$\boxed{y_2 = 1.4181\ ft}$$

y (ft)	**A** (ft^2)	**P** (ft)	**R** (ft)	**V** (ft/s)	**E** (ft)	s_f -	$\overline{s_f}$ -	**dx** (ft)	**x** (ft)
1.3021	13.0210	12.6040	1.0331	15.3600	4.9655	0.0172	0.0000	0.0000	0.0000
1.3031	13.0310	12.6060	1.0337	15.3480	4.9608	0.0172	0.0172	0.2690	9.9540
1.3041	13.0410	12.6080	1.0343	15.3360	4.9562	0.0171	0.0171	0.2689	10.2230
1.3051	13.0510	12.6100	1.0350	15.3240	4.9516	0.0171	0.0171	0.2688	10.4920
1.3061	13.0610	12.6120	1.0356	15.3130	4.9471	0.0170	0.0171	0.2686	10.7600
1.3071	13.0710	12.6140	1.0362	15.3010	4.9425	0.0170	0.0170	0.2685	11.0290
1.3081	13.0810	12.6160	1.0368	15.2890	4.9379	0.0170	0.0170	0.2684	11.2970
1.3091	13.0910	12.6180	1.0375	15.2780	4.9334	0.0169	0.0169	0.2683	11.5650
1.3101	13.1010	12.6200	1.0381	15.2660	4.9289	0.0169	0.0169	0.2681	11.8340
1.3111	13.1110	12.6220	1.0387	15.2540	4.9243	0.0168	0.0169	0.2680	12.1020
1.3121	13.1210	12.6240	1.0394	15.2430	4.9198	0.0168	0.0168	0.2679	12.3690
1.3131	13.1310	12.6260	1.0400	15.2310	4.9153	0.0168	0.0168	0.2678	12.6370
1.3141	13.1410	12.6280	1.0406	15.2190	4.9109	0.0167	0.0167	0.2676	12.9050
1.3151	13.1510	12.6300	1.0412	15.2080	4.9064	0.0167	0.0167	0.2675	13.1720
1.3161	13.1610	12.6320	1.0419	15.1960	4.9019	0.0166	0.0167	0.2674	13.4400
1.3171	13.1710	12.6340	1.0425	15.1850	4.8975	0.0166	0.0166	0.2673	13.7070
1.3181	13.1810	12.6360	1.0431	15.1730	4.8931	0.0166	0.0166	0.2671	13.9740
1.3191	13.1910	12.6380	1.0437	15.1620	4.8887	0.0165	0.0165	0.2670	14.2410
1.3201	13.2010	12.6400	1.0444	15.1500	4.8842	0.0165	0.0165	0.2669	14.5080
1.3211	13.2110	12.6420	1.0450	15.1390	4.8799	0.0165	0.0165	0.2668	14.7750
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1.3991	13.9910	12.7980	1.0932	14.2950	4.5721	0.0138	0.0138	0.2561	35.1700
1.4001	14.0010	12.8000	1.0938	14.2850	4.5686	0.0138	0.0138	0.2559	35.4250
1.4011	14.0110	12.8020	1.0944	14.2740	4.5651	0.0138	0.0138	0.2558	35.6810
1.4021	14.0210	12.8040	1.0950	14.2640	4.5616	0.0137	0.0137	0.2556	35.9370
1.4031	14.0310	12.8060	1.0956	14.2540	4.5581	0.0137	0.0137	0.2555	36.1920
1.4041	14.0410	12.8080	1.0963	14.2440	4.5546	0.0137	0.0137	0.2553	36.4480
1.4051	14.0510	12.8100	1.0969	14.2340	4.5511	0.0136	0.0136	0.2552	36.7030
1.4061	14.0610	12.8120	1.0975	14.2240	4.5476	0.0136	0.0136	0.2550	36.9580
1.4071	14.0710	12.8140	1.0981	14.2140	4.5441	0.0136	0.0136	0.2549	37.2130
1.4081	14.0810	12.8160	1.0987	14.2030	4.5407	0.0135	0.0136	0.2547	37.4670
1.4091	14.0910	12.8180	1.0993	14.1930	4.5372	0.0135	0.0135	0.2546	37.7220
1.4101	14.1010	12.8200	1.0999	14.1830	4.5338	0.0135	0.0135	0.2544	37.9760
1.4111	14.1110	12.8220	1.1005	14.1730	4.5304	0.0135	0.0135	0.2543	38.2310
1.4121	14.1210	12.8240	1.1011	14.1630	4.5270	0.0134	0.0134	0.2541	38.4850
1.4131	14.1310	12.8260	1.1017	14.1530	4.5236	0.0134	0.0134	0.2540	38.7390
1.4141	14.1410	12.8280	1.1023	14.1430	4.5202	0.0134	0.0134	0.2538	38.9930
1.4151	14.1510	12.8300	1.1029	14.1330	4.5168	0.0133	0.0134	0.2537	39.2460
1.4161	14.1610	12.8320	1.1036	14.1230	4.5134	0.0133	0.0133	0.2535	39.5000
1.4171	14.1710	12.8340	1.1042	14.1130	4.5100	0.0133	0.0133	0.2534	39.7530
1.4181	14.1810	12.8360	1.1048	14.1030	4.5067	0.0133	0.0133	0.2532	40.0060

Table 4: Direct step table between y_1 and y_2

3) Scour Depth

The following equation, adopted into local notation, is given by the Bormann and Julian (1991) article for the maximum depth of scour after the apron:

$$\frac{z_e + z_j}{y_2} = 0.61 Fr_2{}^{1.6} \left(\frac{y_2}{d_{90}}\right)^{0.4} \frac{\sin(\alpha_j)}{[\sin(25 + \alpha_j)]^{0.8}} \tag{21}$$

Where, as shown in figure 3:

- z_j is the drop at the end of the apron into the trapezoidal channel (aka d_t);
- z_e is the maximum scour depth, referenced against the natural bed of the trapezoidal channel;
- α_j is the angle of the jet against the horizontal, at the point of contact, calculated according to the article as:

$$\alpha_j = 0.2\sin(\alpha_b) + 0.15\ln\left(1 + \frac{z_j}{y_2}\right) + 0.13\ln\left(\frac{T_w - z_t}{y_2}\right) - 0.05\ln(Fr_2) \tag{22}$$

- where α_b refers to the geometry of the downstream face of the drop structure, set as $\alpha_b = 90^\circ = \frac{\pi}{2}$ *rad*. for this vertical face.

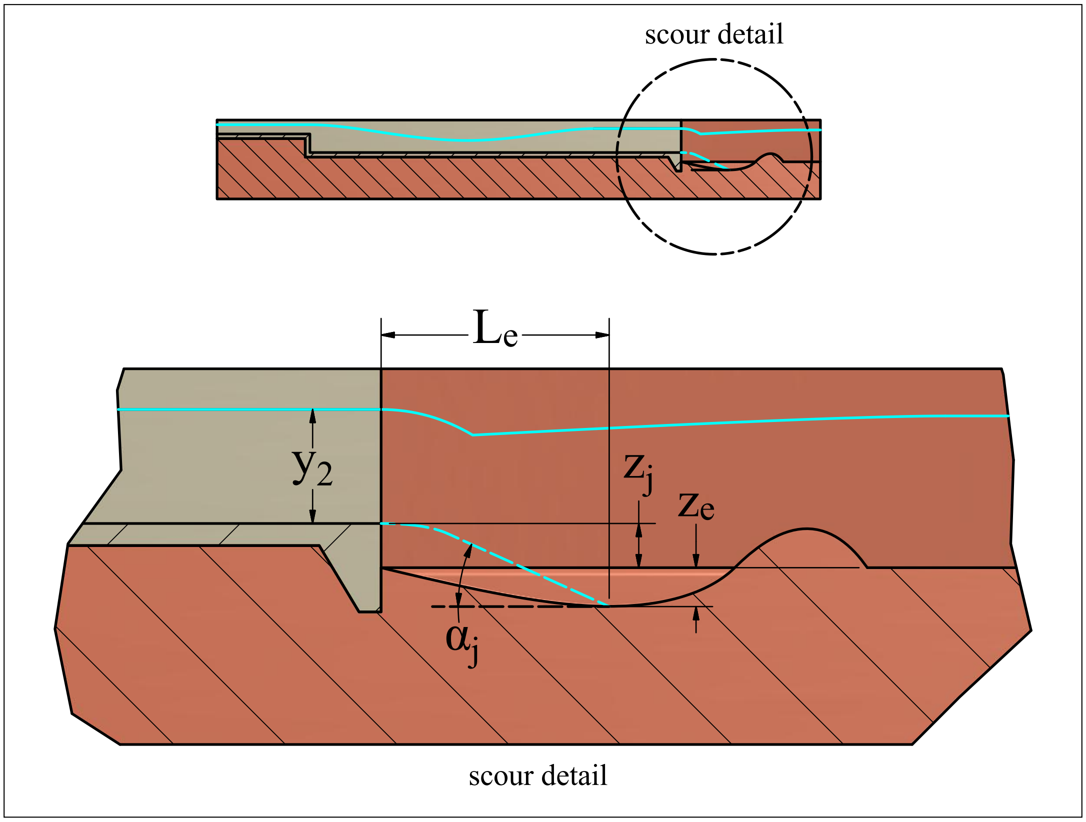

Figure 3: Scour calculation detail

Therefore, eq. 21 can be modified to include the distance to the location of maximum scour (L_e) as follows:

$$\frac{L_e}{y_2} = 0.61 \left[\frac{Fr_2{}^2}{\sin\left(25^\circ \frac{2\pi}{360^\circ} + \alpha_j\right)} \right]^{0.8} \left(\frac{y_2}{d_{90}} \right)^{0.4} \tag{23}$$

With known y_2, z_j, Fr_2 and d_{90}, eqs. 21, 22 and 23 are used to yield the following, with associated values shown in table 5:

$\boxed{z_e = 4.4906\ ft}$

$\boxed{L_e = 14.8377\ ft}$

$\mathbf{V_2}$ (ft/s)	$\mathbf{Fr_2}$ -	$\alpha_\mathbf{j}$ (rad)
14.1033	2.0871	0.3791

Table 5: Values involved in determining scour

4) Rip Rap Sizing

In order to reduce scouring in the bed of the natural, trapezoidal channel a rip rap composed of stones can be installed just at the foot of the apron end. The properties of this rip rap are determined using figure 4, which is borrowed from plate 30 of USACE Engineer Manual No. 1110-2-1601.

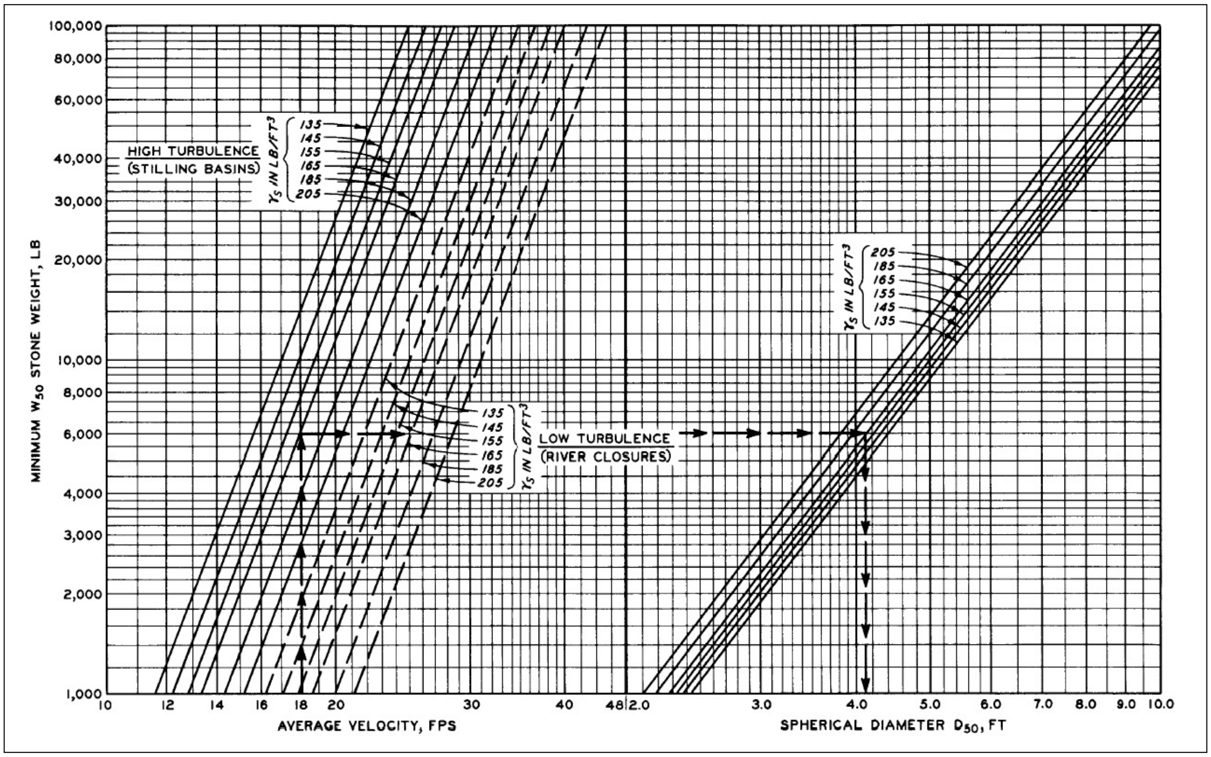

Figure 4: Velocity vs. stone weight, diameter, *Hydraulic Design Chart 712-1*

This figure provides the following:

weight of typical rip rap stone :	W_{50}
spherical diameter of typical rip rap stone :	D_{50}
recommended rip rap depth :	$2D_{50}$

With the significance of these properties known, figure 4 is read as follows:

- The velocity abscissa of the chart is found on the left-hand chart, set to V_2.
- Focus is placed on the $V_2 - W_{50}$ curves provided for high turbulence, stilling basins.
- In order to develop a most-accommodating estimate of rip rap depth (one with abundant depth), a design specific stone weight of $\gamma_s = 135\ ^{lbs}/_{ft^3}$ is chosen.
- The V_2 abscissa is followed upward until the appropriate γ_s curve is encountered.
- The ordinate of this location on the $V_2 - W_{50}$ curve is the read W_{50} value.
- This ordinate is then followed rightward until the $V_2 - D_{50}$ curve of the same γ_s is encountered.
- The abscissa of this second encounter is the read D_{50} value.

As shown in figure 5, this process yields the following values:

weight of typical rip rap stone : $W_{50} = 3,250\ lbs$

spherical diameter of typical rip rap stone : $D_{50} = 3.6\ ft$

rip rap depth : $\text{depth}_{\text{rip rap}} = 2D_{50} = 7.2\ ft$

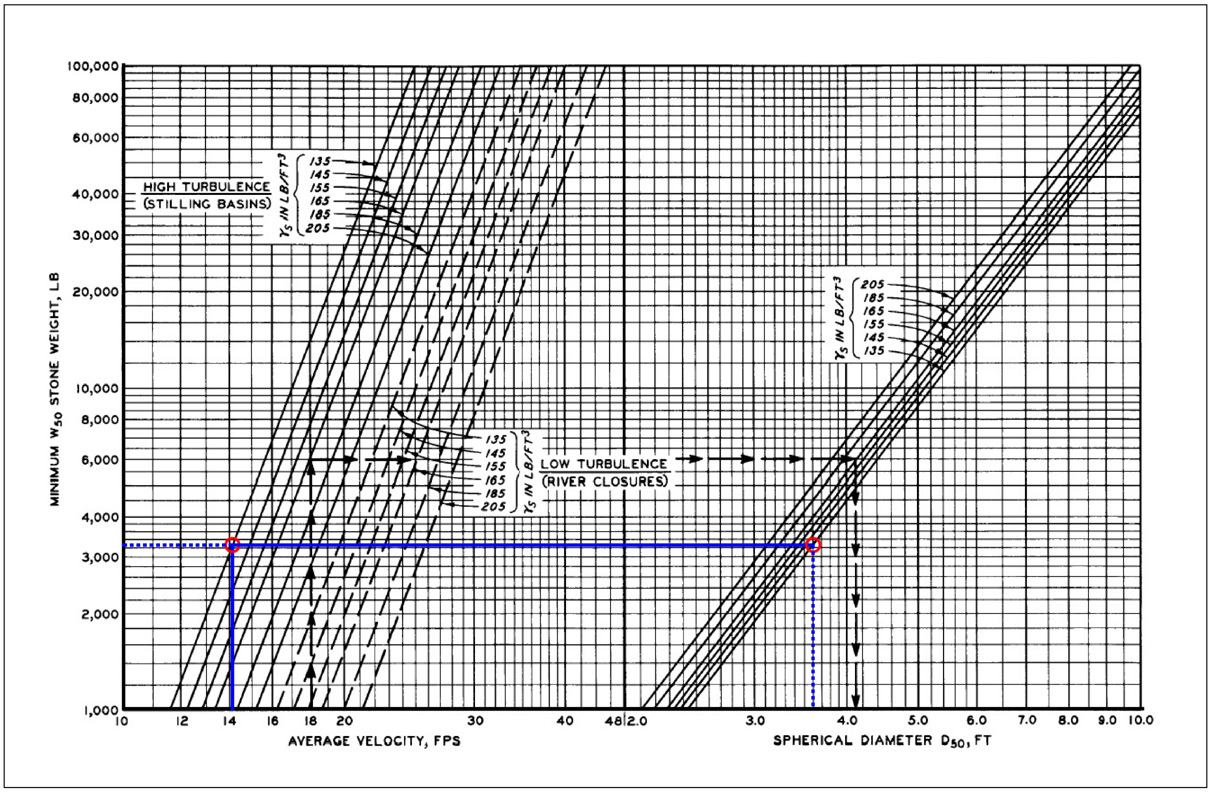

Figure 5: Reading of Hydraulic Design Chart 712-1

The placement of these values are shown, for clarity, in figure 6. Also, as shown, the rip rap horizontal length is set to be double L_e, in order to develop a conservative system.

rip rap horizontal length : $L_{\text{rip rap}} = 2L_e = 29.6755\ ft$

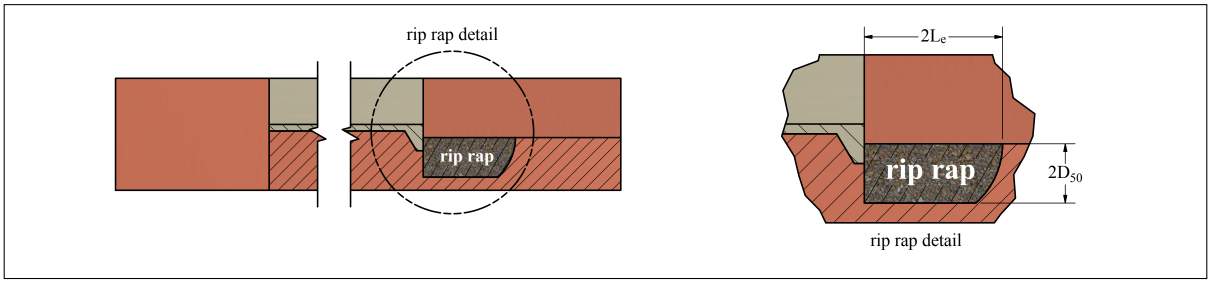

Figure 6: Rip rap sizing detail

A Matlab® Scripts

A.1 Master Script

```matlab
clc; close all; clear all

% --- given properties ---------------------------------------------------
qr = 20;                                   %discharge per unit width [f^2/s]
br = 10;                           %rectuangular drop structure width [ft]
drect = 2;                                   %rectangular drop height [ft]
Lr = 40;                            %rectangular drop structure length [ft]
zr = 821.1;                  %rectangular drop strucutre bed elevation [ft]

bt = 22;                                        %trapezoidal base width [ft]
zzt = 4;                                          %trapezoidal side slope
d90 = 0.5;                                  %trapezoidal bed material [in]
st = 0.56/100;                                   %trapezoidal bed slope [%]
zt = 820.1;                                 %trapezoidal bed elevation [ft]

nt = 0.03;                                                    %Manning's n
nr = 0.013;                                      %Manning's n for concrete
kn = 1.486;                       %normal depth unit conversion with english
g = 32.2;                                  %acceleration of gravity [ft/s^2]

% --- preliminary calculations -------------------------------------------
Q = qr*br; %total discharge [cfs]

% --- solver parameters --------------------------------------------------
tol = 1e-4;

% --- estimation of Tw ---------------------------------------------------
ynt = 0; err = 1e6;
while err > tol
Ant = ynt*(bt+zzt*ynt);                           %trapezoidal area [ft^2]
Pnt = bt + 2*ynt*sqrt(1 + zzt^2);        %trapezoidal wetted perimeter [ft]
Rnt = Ant/Pnt;                           %trapezoidal hydraulic radius [ft]
err = abs((kn/nt)*Ant*Rnt^(2/3)*sqrt(st) - Q);
ynt = ynt + err*tol;
end
Tw = ynt + zt;                            % determined tailwater elevation [ft]

% --- determining y1 -----------------------------------------------------
Dr = (qr^2)/(g*drect^3);                     %drop number into contrete channel
y1 = drect*0.54*Dr^0.425;  %depth at toe of fall into concrete channel [ft]

% --- testing for hydraulic jump -----------------------------------------
V1 = Q/(br*y1);                       %velocity at toe in concrete chan. [ft/s]
Fr1 = V1/sqrt(g*y1);
y1_conj = 0.5*y1*(1 + 8*Fr1^2);

% --- WSP along apron ----------------------------------------------------
ystep = 0.001;
Ld = drect*4.3*Dr^0.27;                  %length from concrete drop to toe [ft]

WSPtable = zeros(117, 10);
WSPtable(1,1) = y1;                                                      %y1
WSPtable(1,2) = WSPtable(1,1)*br;                                        %A1
WSPtable(1,3) = 2*WSPtable(1,1) + br;                                    %P1
WSPtable(1,4) = WSPtable(1,2)/WSPtable(1,3);                             %R1
WSPtable(1,5) = qr / WSPtable(1,1);                                      %V1
WSPtable(1,6) = WSPtable(1,1) + WSPtable(1,5)^2/(2*g);                   %E1
WSPtable(1,7) = ((nr*Q)/(1.49*WSPtable(1,2)*WSPtable(1,4)^(2/3)))^2;     %sf

for ii = 2:length(WSPtable)
WSPtable(ii,1) = y1+ystep*(ii-1);                                        %y1
WSPtable(ii,2) = WSPtable(ii,1)*br;                                      %A1
WSPtable(ii,3) = 2*WSPtable(ii,1) + br;                                  %P1
WSPtable(ii,4) = WSPtable(ii,2)/WSPtable(ii,3);                          %R1
WSPtable(ii,5) = qr / WSPtable(ii,1);                                    %V1
WSPtable(ii,6) = WSPtable(ii,1) + WSPtable(ii,5)^2/(2*g);                %E1
WSPtable(ii,7) = ((nr*Q)/(1.49*WSPtable(ii,2)*WSPtable(ii,4)^(2/3)))^2; %sf
WSPtable(ii,8) = 0.5*(WSPtable(ii,7)+WSPtable(ii-1,7));               %sfbar
WSPtable(ii,9) = (WSPtable(ii,6)-WSPtable(ii-1,6))/(-WSPtable(ii,8)); %delx
WSPtable(ii,10) = Ld + sum(WSPtable(:,9));
end
y2 = max(WSPtable(:,1));

% --- solve for ze and and Le --------------------------------------------
alphab = 90/180*pi;
zj = zr - zt;
V2 = Q/(br*y2);
Fr2 = V2/sqrt(g*y2);
alphaj = 0.32*sin(alphab)+0.15*log(1+zj/y2)+0.13*log(ynt/y2)-0.05*log(Fr2);
ze=y2*(0.61*(Fr2^1.6)*((y2/(d90/12))^0.4)*sin(alphaj)/...
                                 (sin(25*pi/180 + alphaj))^0.8) - zj;
Le = y2*(0.61*(((Fr2^2)/(sin(25*pi/180+alphaj)))^0.8)*((y2/(d90/12))^0.4));

% --- rip rap sizing -----------------------------------------------------
D50 = 3.6; rrdepth = 2*D50; rrlength = 2*Le;

ori = [102  729]; maxlim = [1234  13] ;
point1 = [223 547]; point2  = [868 point1(2)];

figure;
I = imread('chart01.jpg'); B = imrotate(I,0); imshow(B); hold on;
plot([ori(1) maxlim(1)],[ori(2) ori(2)],'-k','linewidth',1.5);
plot([ori(1) ori(1)],[maxlim(2) ori(2)],'-k','linewidth',1.5);
plot([point1(1) point1(1)],[ori(2) point1(2)],'-b','linewidth',1.5);
plot(point1(1),point1(2),'ro','linewidth',1.5);
plot([ori(1) point1(1)],[point1(2) point1(2)],':b','linewidth',1.5);
plot([point1(1) point2(1)],[point1(2) point2(2)],'-b','linewidth',1.5);
plot(point2(1),point2(2),'ro','linewidth',1.5);
plot([point2(1) point2(1)],[point2(2) ori(2)],':b','linewidth',1.5);
```