

General Newton - Raphson's Method

Given $f(x)$, one can find x where $f(x) = 0$ by reiterating:

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

until $f(x_{i+1}) < k$, where k is some "cutoff" minimum value, ie 0.00001

Applying General Newton - Raphson's Method to The Colebrook Equation

Original Colebrook equation:

$$\frac{1}{\sqrt{f}} = -2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right)$$

Set function to root-finding format:

$$g(f) = \frac{1}{\sqrt{f}} + 2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right) = 0$$

Take derivative with respect to independent variable (f):

$$g'(f) = -\frac{\sqrt{f} (\epsilon Re + 8.06659D) + 9.287D}{2f^{3/2} (\sqrt{f}\epsilon Re + 9.287D)}$$

Start with trial value of f and repeat

$$f_{i+1} = f_i - \frac{\left[\frac{1}{\sqrt{f}} + 2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f}} \right) \right]}{\left[-\frac{\sqrt{f} (\epsilon Re + 8.06659D) + 9.287D}{2f^{3/2} (\sqrt{f}\epsilon Re + 9.287D)} \right]}$$

Until

$$g(f_{i+1}) = \frac{1}{\sqrt{f_{i+1}}} + 2 \log \left(\frac{\epsilon/D}{3.7} + \frac{2.51}{Re\sqrt{f_{i+1}}} \right) < k$$

Where:

f_{i+1} = Solved friction factor (unitless)

Re = Reynold's number (unitless)

ϵ = Absolute roughness of pipe's interior surface (usually in feet)

D = Interior diameter of pipe (usually in feet)

k = Minimum "cutoff" value for solver, set to 0.00001 in this case.