



$$R^2 = x^2 + y^2$$

therefore, $x = \sqrt{R^2 - y^2}$

$$dV = A = (2x)^2$$

therefore, $dV = A = (2\sqrt{R^2 - y^2})^2$

therefore, $dV = A = 4(R^2 - y^2)$

$$\frac{1}{2}V = \int_0^R dV$$

therefore, $\frac{1}{2}V = \int_0^R [4(R^2 - y^2)] dy$

therefore, $V = 8 \int_0^R (R^2 - y^2) dy$

therefore, $V = 8 \left[R^2 y - \frac{1}{3} y^3 \right]_0^R$

therefore, $V = 8 \left[R^2 R - \frac{1}{3} R^3 \right]$

therefore, $V = 8 \left[R^3 - \frac{1}{3} R^3 \right]$

therefore, $V = 8 \left[\frac{2}{3} R^3 \right]$

therefore, $V = \frac{16}{3} R^3$