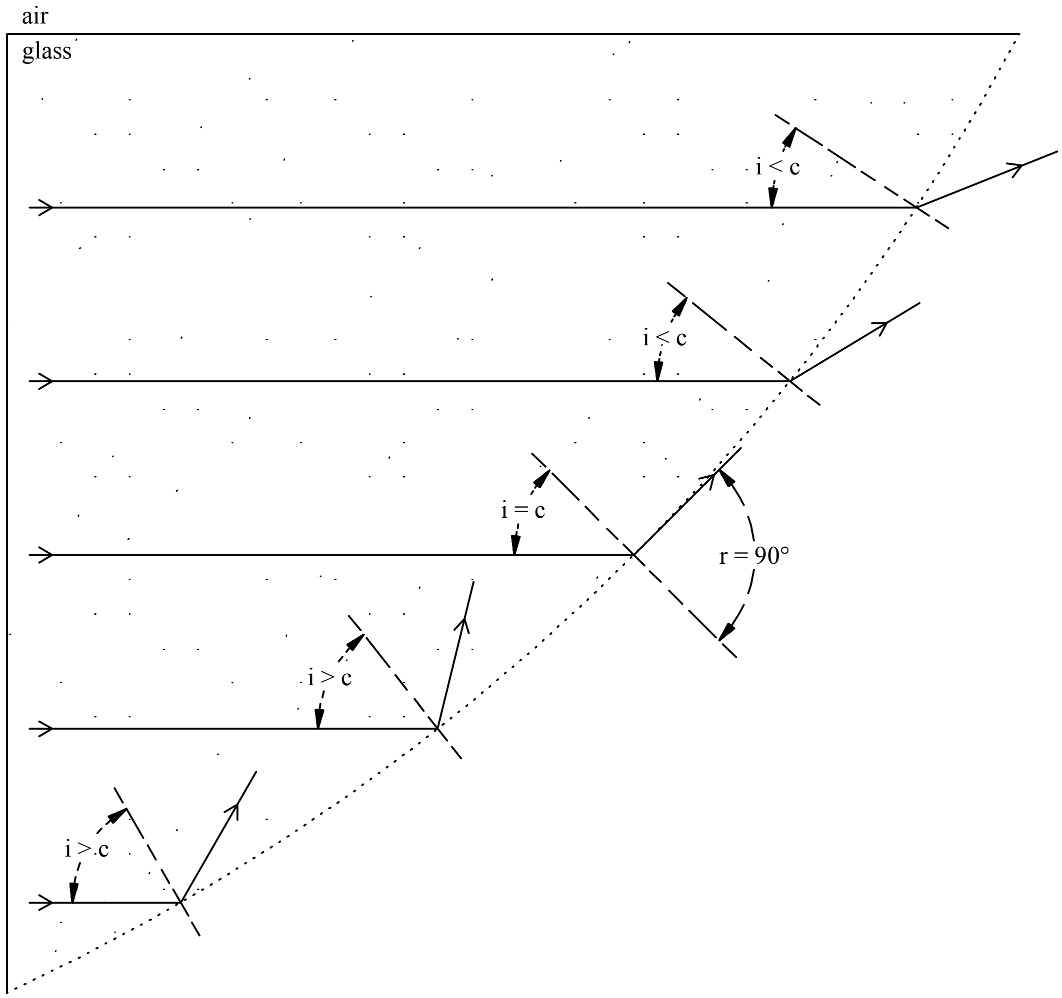
Intermediate Physics

Form 3



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GHS Ntumbaw

Contents

This document is intended to cover all of Form 3 Physics across 30 separate lessons. It has been customized for a teacher to cover all material within a typical 36-week school year with a 6-week allowance for delays and interruptions. Each lesson has been created according to the following needs, in order of priority.

1. Treat all objectives in the latest government-published scheme of work;
2. Minimize lesson length to allow for supplementary, student-centred learning activities;
3. Provide 10 multiple choice (GCE paper 1) questions immediately related to the material covered;
4. Provide at least 2 essay (GCE paper 2) questions and solutions immediately related to the material covered.

A considerable effort was made to minimize errors in this document, including those of grammar, syntax, concept, algebra, analysis and solution. However caution and healthy skepticism should be exercised when referring to its content. When technology permits, please send all questions and corrections as an email to the author at bulkl001@umn.edu.

3 Form 3

3.1 Introductory Material

3.1.1 Physical Quantities, Vectors and Scalars

Objectives

By the end of the lesson, students should be able to

1. define a physical quantity.
2. give examples and units of several physical quantities.
3. state measuring instruments of various physical quantities.
4. describe some basic physical quantities.
5. define and distinguish between vector and scalar quantities, giving examples.
6. convert units of various quantities, including distance, area and volume.
7. add and subtract vector quantities such as forces.

Physical Quantities

- A **physical quantity** is anything that can be measured.
- Base quantities are fundamental physical quantities measured in base units.
- Many quantities are measured in a system of units referred to as "Le Système International d'Unités".
- This system is abbreviated "SI".

base quantity	**quantity symbol**	**base unit name**	**base unit symbol**	**measuring device(s)**
distance	d or s	metre	m	metre rule, odometer
mass	m	kilogram	kg	beam balance, digital scale
time	t	second	s	stopwatch
electric current	I	Ampere	A	ammeter
temperature	θ	Kelvin	K	thermometer, thermocouple
particle amount	N	mole	mol	-

Table 3.1.1.1

- Secondary quantities are derived physical quantities measured in derived, or combined base units.

derived quantity	**quantity symbol**	**defining equation**	**unit derivation**	**unit name**	**unit symbol**	**measuring device(s)**
area	A	length×width	m^2	-	-	-
volume	V	base-area×height	m^3	-	-	beaker
velocity	$\vec{v}$	displacement/time	$m\ s^{-1}$	-	-	speedometer
acceleration	$\vec{a}$	velocity/time	$m\ s^{-2}$	-	-	accelerometer
force	F	mass×acceleration	$kg\ m\ s^{-2}$	Newton	N	force gauge
work	W	force×distance	$N\ m$	Joule	J	-
power	P	energy/time	$J\ s^{-1}$	Watt	W	meter
pressure	P	force/area	$N\ m^{-2}$	Pascal	Pa	manometer

Table 3.1.1.2

Unit Prefixes

- Prefixes can be added to SI base and derived units to make them larger or smaller.
- NB: **G**randma **M**ay **k**illed **h**er **d**ear **b**oyfriend **d**eci, **c**ouldn't **m**aster μicroscopic **n**ames.

prefix	**symbol**	**multiplication factor**		**common example**		
		sci. notation	**decimal**	**quantity**	**abbreviated**	**base equivalent**
giga-	G	10^9	1000000000	1 gigabyte	1 GB	1000000000 bytes
mega-	M	10^6	1000000	1 megabyte	1 MB	1000000 bytes
kilo-	k	10^3	1000	1 kilometre	1 km	1000 metres
heca-	h	10^2	100	-	-	-
deca-	d	10^1	10	-	-	-
(base)	-	10^0	1	1 metre	1 m	1 metre
deci-	d	10^{-1}	0.1	-	-	-
centi-	c	10^{-2}	0.01	1 centimetre	1 cm	0.01 metres
milli-	m	10^{-3}	0.001	1 millimetre	1 mm	0.001 m
micro-	μ	10^{-6}	0.000001	1 micrometre	1 μm	0.000001 m
nano-	n	10^{-9}	0.000000001	1 nanometre	1 nm	0.000000001 m

Table 3.1.1.3

Vector Quantities

- A **vector quantity** is a physical quantity which has both a magnitude (size) and a specified direction.
- Examples include
  - displacement
  - velocity
  - acceleration
  - momentum
  - force
  - impulse
  - torque
  - electric current

Scalar Quantities

- A **scalar quantity** is a physical quantity which has a magnitude but no specified direction.
- NB: A scalar quantity can never be negative, but a vector quantity can.
- Examples include
  - area
  - volume
  - time
  - mass
  - speed
  - density
  - frequency
  - machine efficiency

Common Quantities in Detail

- **Distance** is the length of a body's path from a reference point.
  - It is a scalar quantity with the SI unit metre, abbreviated m.
- **Time** is the duration of a physical event.
  - It is a scalar quantity with the SI unit second, abbreviated s.
- **Area** is the amount of a 2 dimensional (2D) region enclosed by a boundary or occupied by a shape.
  - It is a scalar quantity with the SI units square metres, abbreviated m^2.
- **Volume** is the amount of a 3 dimensional (3D) space enclosed in a container or occupied by a body.
  - It is a scalar quantity with the SI units cubic metres, abbreviated m^3.
- **Mass** is the amount of matter in a body.
  - It is a scalar quantity with the SI unit kilogram, abbreviated kg.
- **Force** is the pushing or pulling effect of one body on another.
  - It is a vector quantity with the SI unit Newton, abbreviated N.

Converting Linear Units

- Units that measure a particular quantity can be converted from one form to another.
- Given equivalences can be modified into **conversion factors**.
- Conversion factors have the quantity's given unit(s) on the bottom (denominator) and the given unit's equivalence of the desired unit on the top (numerator).
- If the equivalence between the units is maintained, the conversion factor is dimensionless and equal to 1.

$$\text{convserion factor} = \text{unit equivalence} \times \frac{\text{desired unit(s)}}{\text{given unit(s)}} = 1$$

$$\text{quantity in desired units} = \text{conversion factor} \times \text{quantity in given unit(s)}$$

Example: How many centimetres are in 3.5 metres?

$$\text{quantity in given unit: } d = 3.5\ m$$

$$\text{given unit: } m$$

$$\text{desired unit: } cm$$

$$\text{given equivalence: } 1\ cm = 0.01\ m$$

$$\text{turning equivalence into conversion factor: } \frac{1\ cm}{0.01\ m} = \frac{1}{0.01} \times \frac{cm}{m} = 1$$

$$\text{applying conversion factor: quantity in desired units} = \left(\frac{1}{0.01} \times \frac{cm}{m}\right)(3.5\ m)$$

$$\text{final answer: } \boxed{d = 350\ cm}$$

- Conversion factors can also be multiplied by each other in order to convert between several, separate units.

$$\text{compuound convserion factor} = \text{first conversion factor} \times \text{second conversion factor} = 1 \times 1$$

Example: How many seconds are in a 24 hour day?

$$\text{quantity in given unit: } t = 24\ hr$$

$$\text{given unit: } hr$$

$$\text{desired unit: } s$$

$$\text{given equivalences: } 1\ hr = 60\ min, \quad 1\ min = 60\ s$$

$$\text{turning first equivalence into first conversion factor: } \frac{60\ min}{1\ hr} = \frac{60}{1} \times \frac{min}{hr} = 1$$

$$\text{turning second equivalence into second conversion factor: } \frac{60\ s}{1\ min} = \frac{60}{1} \times \frac{s}{min} = 1$$

$$\text{applying conversion factors: } t = \left[\left(\frac{60}{1} \times \frac{min}{hr}\right)\left(\frac{60}{1} \times \frac{s}{min}\right)\right](24\ hr)$$

$$\text{final answer: } \boxed{t = 86400\ s}$$

Converting Units of Area by Converting Units of Distance

- Units of area can be expressed as the square of units of distance.

$$1\ m^2 = 1\ m \times 1\ m$$
$$1\ km^2 = 1\ km \times 1\ km$$
$$1\ cm^2 = 1\ cm \times 1\ cm$$
$$1\ mm^2 = 1\ mm \times 1\ mm$$

- When converting units of area by converting units of distance, the conversion factor is squared.

Example: 1 square centimetre is converted into square millimetres considering $1\ cm = 10\ mm$.

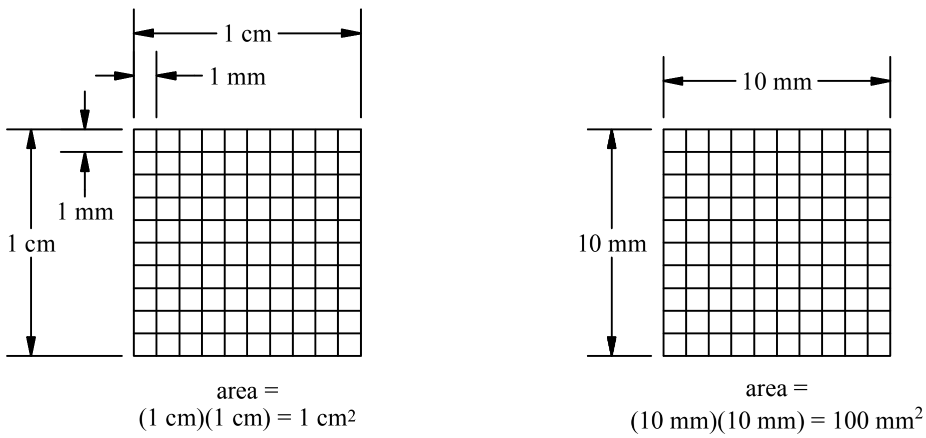

Figure 3.1.1.1

$$1\ cm^2 = (1\ cm)(1\ cm) = (1\ cm)\left(\frac{10\ mm}{1\ cm}\right)(1\ cm)\left(\frac{10\ mm}{1\ cm}\right) = (1\ cm)^2\left(\frac{10\ mm}{1\ cm}\right)^2 = 10^2\ mm^2$$

Example: 1 square metre is converted into square centimetres considering $1\ m = 10^2\ cm$.

$$1\ m^2 = (1\ m)(1\ m) = (1\ m)\left(\frac{10^2\ cm}{1\ m}\right)(1\ m)\left(\frac{10^2\ cm}{1\ m}\right) = (1\ m)^2\left(\frac{10^2\ cm}{1\ m}\right)^2 = 10^4\ cm^2$$

Example: 1 square kilometre is converted into square metres considering $1\ km = 10^3\ m$.

$$1\ km^2 = (1\ km)(1\ km) = (1\ km)\left(\frac{10^3\ m}{1\ km}\right)(1\ km)\left(\frac{10^3\ m}{1\ km}\right) = (1\ km)^2\left(\frac{10^3\ m}{1\ km}\right)^2 = 10^6\ m^2$$

Example: 1 square millimetre is converted into square metres considering $1\ mm = 10^{-3}\ m$.

$$1\ mm^2 = (1\ mm)(1\ mm) = (1\ mm)\left(\frac{10^{-3}\ m}{1\ mm}\right)(1\ mm)\left(\frac{10^{-3}\ m}{1\ mm}\right) = (1\ mm)^2\left(\frac{10^{-3}\ m}{1\ mm}\right)^2 = 10^{-6}\ m^2$$

Converting Units of Volume by Converting Units of Distance

- Units of volume can be expressed as the cube of units of distance.

$$1\ m^3 = 1\ m \times 1\ m \times 1\ m$$
$$1\ km^3 = 1\ km \times 1\ km \times 1\ km$$
$$1\ cm^3 = 1\ cm \times 1\ cm \times 1\ cm$$
$$1\ mm^3 = 1\ mm \times 1\ mm \times 1\ mm$$

- When converting units of volume by converting units of distance, the conversion factor is cubed.

Example: 1 cubic centimetre is converted cubic millimetres considering 1 $cm = 10\ mm$.

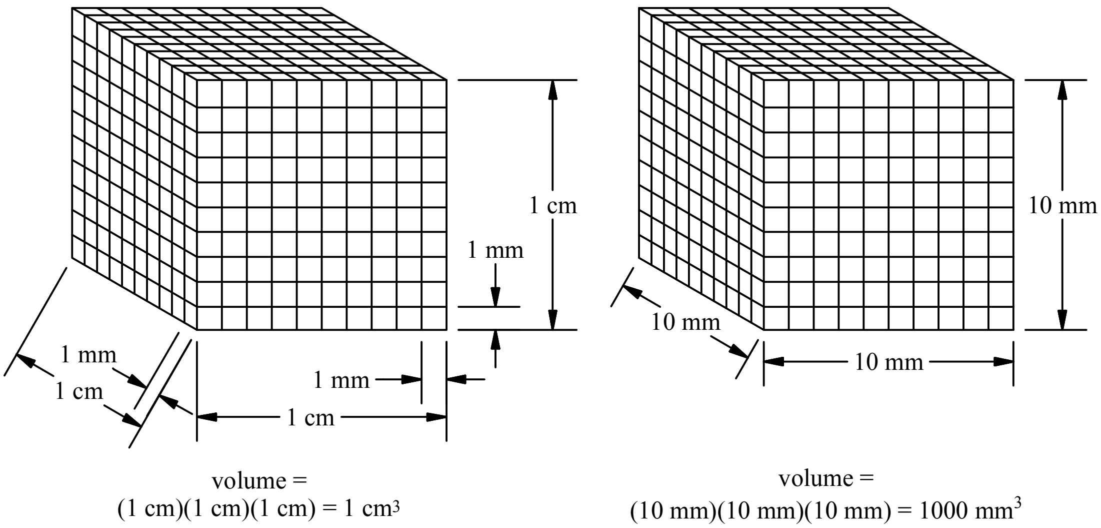

Figure 3.1.1.2

$$1\ cm^3 = (1\ cm)(1\ cm)(1\ cm) = (1\ cm)\left(\frac{10\ mm}{1\ cm}\right)(1\ cm)\left(\frac{10\ mm}{1\ cm}\right)(1\ cm)\left(\frac{10\ mm}{1\ cm}\right)$$
$$= (1\ cm)^3\left(\frac{10\ mm}{1\ cm}\right)^3 = 10^3\ mm^3$$

Example: 1 cubic kilometre is converted into cubic metres considering 1 $km = 10^3\ m$.

$$1\ km^3 = (1\ km)(1\ km)(1\ km) = (1\ km)\left(\frac{10^3\ m}{1\ km}\right)(1\ km)\left(\frac{10^3\ m}{1\ km}\right)(1\ km)\left(\frac{10^3\ m}{1\ km}\right)$$
$$= (1\ km)^3\left(\frac{10^3\ m}{1\ km}\right)^3 = 10^9\ m^3$$

Example: 1 cubic millimetre is converted into cubic metres considering 1 $mm = 10^{-3}\ m$.

$$1\ mm^3 = (1\ mm)(1\ mm)(1\ mm) = (1\ mm)\left(\frac{10^{-3}\ m}{1\ mm}\right)(1\ mm)\left(\frac{10^{-3}\ m}{1\ mm}\right)(1\ mm)\left(\frac{10^{-3}\ m}{1\ mm}\right)$$
$$= (1\ mm)^3\left(\frac{10^{-3}\ m}{1\ mm}\right)^3 = 10^{-9}\ m^3$$

Displacement, Velocity and Acceleration

- **Displacement**, or $\vec{\mathbf{s}}$, is the distance a body moves in a specific direction from a reference point.

- **Velocity**, or $\vec{\mathbf{v}}$, is the displacement a body achieves within a time interval.

$$\vec{v} = \frac{\vec{s}}{\Delta t}$$

  Where
  - $\vec{v}$ is the velocity of the body, in $m\ s^{-1}$;
  - $\vec{s}$ is the displacement of the body, in m;
  - Δt is the time interval of motion, in s.

- **Acceleration**, or $\vec{\mathbf{a}}$ is the change of a body's velocity within a time interval.

$$\vec{a} = \frac{\vec{v}}{\Delta t}$$

  Where
  - $\vec{a}$ is the acceleration of the body, in $m\ s^{-2}$;
  - $\vec{v}$ is the velocity of the body, in $m\ s^{-1}$;
  - Δt is the time interval of motion, in s.

- When a body is released from a particular height, it accelerates downwards towards the centre of the earth.
  - The surface of the earth (or ground) is often the obstacle that stops the body's downward acceleration.
  - The value of a body's downward acceleration is roughly consistent everywhere on the earth's surface.
  - **The acceleration of gravity**, or **g**, is the acceleration a body experiences when released from rest.
  - The value of this downward acceleration is roughly 9.81 $m\ s^{-2}$ and is often approximated as 10 $m\ s^{-2}$.
  - Figure 3.1.1.3 shows the displacement, velocity and acceleration of a body released from a height of 10 m.

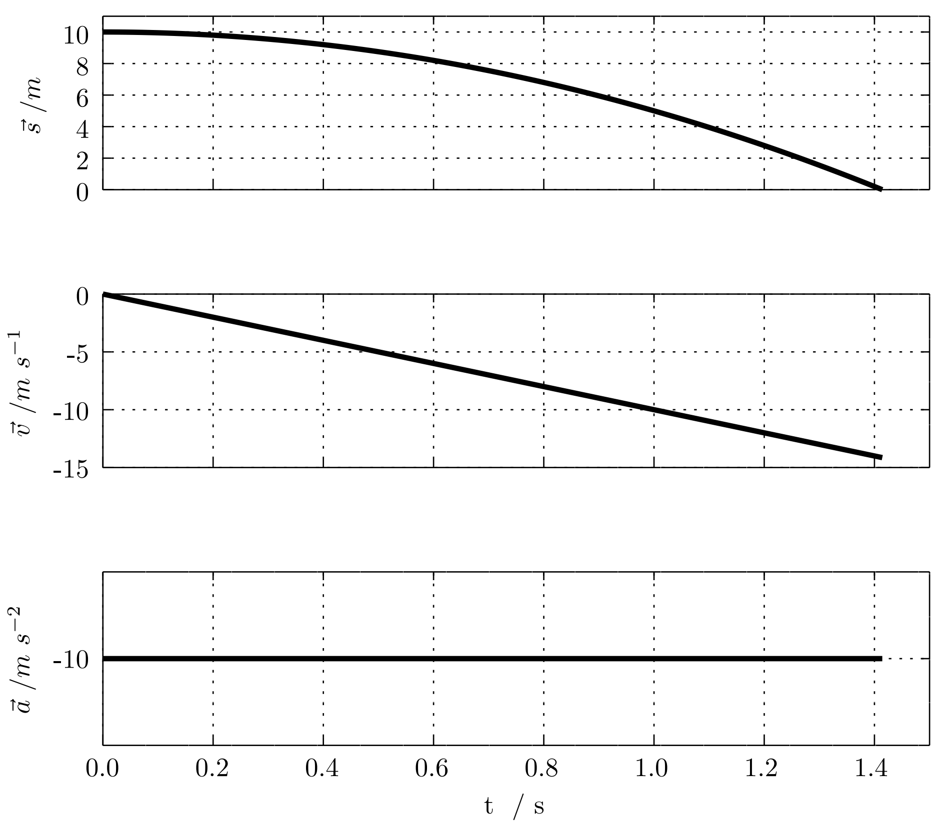

Figure 3.1.1.3

Forces as Vectors

- The acceleration of gravity causes a downward force on any body.
- The **weight**, or $\mathbf{F_w}$, of a body is the attractive force between its mass and that of another mass like the earth.
- A body's weight due to the earth is the product of its mass and the acceleration of gravity.

$$F_w = mg$$

Where
  - F_w is the body's force of weight, in N;
  - m is the mass of the body, in kg;
  - g is the acceleration of gravity, in $m\ s^{-2}$.

- Any body whose velocity is constant has a balance of forces.
  - Such a balance is referred to as an "equilibrium".
  - A body has a constant velocity if its velocity has no change in a particular time.
  - A body with no motion where $\vec{v} = 0\ m\ s^{-1}$ has a constant velocity.
  - A body moving at a constant rate of displacement per time where $\vec{v} = 5\ m\ s^{-1}$ also has a constant velocity.
- Vectors are often drawn and analysed with the following sign convention of
  - positive values for upward vectors
  - negative values for downward vectors
  - positive values for rightward vectors
  - negative values for leftward vectors
- A body suspended from a rope has a downward force of gravity in equilibrium with the upward force of tension.

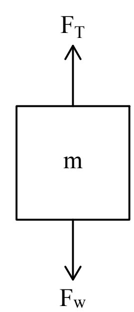

Figure 3.1.1.4

$$F_T = -F_w$$

- NB: The negative sign on F_w is consistent with its downward nature.
- A body suspended from multiple ropes is in a similar state of equilibrium.

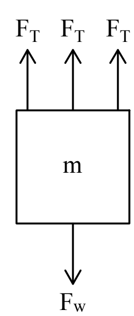

Figure 3.1.1.5

$$F_w = -3F_T$$

- All horizontal forces of a body in equilibrium sum to zero as do all vertical forces.

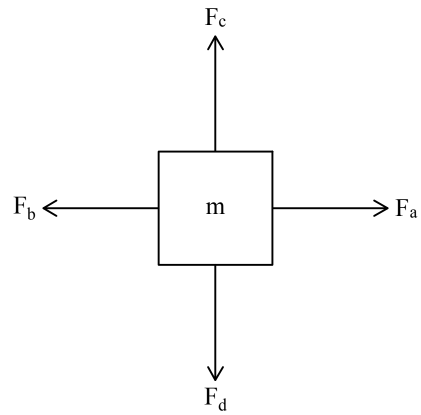

Figure 3.1.1.6

$$\text{summing horizontal forces:} \quad \sum F_x = 0 = F_a + F_b \rightarrow F_a = -F_b$$

$$\text{summing vertical forces:} \quad \sum F_y = 0 = F_c + F_d \rightarrow F_c = -F_d$$

- NB: The negative sign on F_b is consistent with its leftward nature.
- An angled force can be resolved into is horizontal and vertical components.

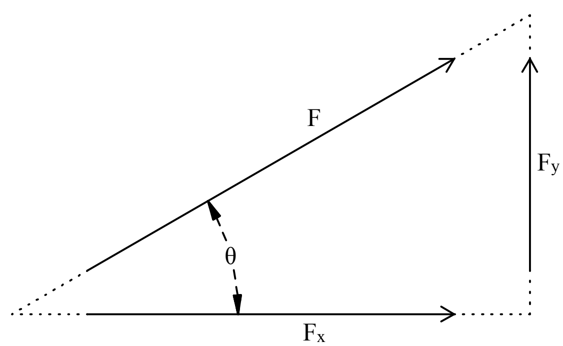

Figure 3.1.1.7

$$\text{considering sine:} \quad \sin(\theta) = \frac{F_y}{F}$$

$$\text{turning vertical component into subject:} \quad F_y = F\sin(\theta)$$

$$\text{considering cosine:} \quad \cos(\theta) = \frac{F_x}{F}$$

$$\text{turning horizontal component into subject:} \quad F_x = F\cos(\theta)$$

Where
- F_y is the vertical component of the force, in N;
- F_x is the horizontal component of the force, in N;
- F is the force, in N;
- θ is the angle of the the force's inclination, in degrees.

- Such a resolution can also be applied to a body in equilibrium on an inclined plane.

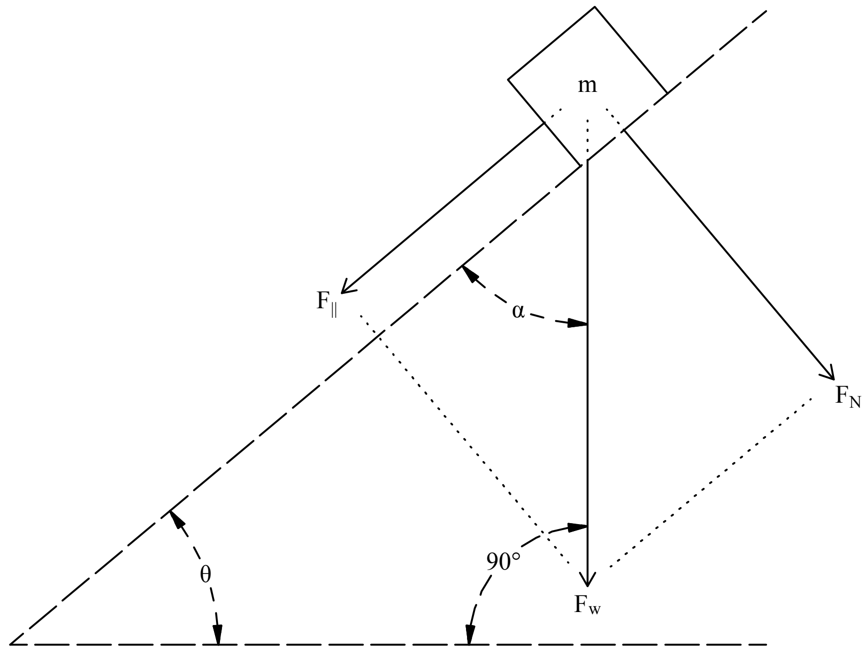

Figure 3.1.1.8

summing interior angles of triangle: $\theta + \alpha + 90^\circ = 180^\circ$

turning plane's angle against vertical into subject: $\alpha = 90^\circ - \theta$

considering components of force of weight: $\sin(\alpha) = \frac{F_N}{F_w}$

$$\cos(\alpha) = \frac{F_{\|}}{F_w}$$

turning components into subject: $F_N = F_w \sin(\alpha)$

$$F_{\|} = F_w \cos(\alpha)$$

substituting equation for plane's angle against vertical: $F_N = F_w \sin(90^\circ - \theta)$

$$F_{\|} = F_w \cos(90^\circ - \theta)$$

appling trigonometric equivalence: $F_N = F_w \cos(\theta)$

$$F_{\|} = F_w \sin(\theta)$$

Where

- F_N is the component of the body's force of weight normal to the inclined plane, in N;
- $F_{\|}$ is the component of the body's force of weight parallel to the inclined plane, in N;
- F_w is the body's force of weight, in N;
- θ is the plane's angle of inclination against horizontal, in degrees.

GCE Paper 1 Questions

1. Which of the following is a base unit?

   A Newton B second C Joule D Pascal

2. Which of the following is a scalar quantity?

   A momentum B force C torque D time

3. Which of the following quantities is a vector?

   A displacement B heat capacity C speed D time

4. A 10 N force angled 30° to the horizontal has a vertical component of

   A 5 N B 20 N C 8.66 N D 11.55 N

5. Which of the following pairs of physical quantities include two vectors?

   A mass and acceleration
   B momentum and time
   C velocity and momentum
   D volume and machine efficiency

6. A kilometre is equivalent to how many metres?

   A 0.001 B 1 C 1000 D 10^4

7. A carton having a mass of 1.2 kg is suspended by 4 equally-tensioned ropes. The force of tension in any one rope is

   A 48 N B 4.8 N C 0.3 N D 3 N

8. 0.3 cubic metres is equal to which of the following?

   A $3.0 \times 10^{-5}\ cm^3$ B $3.0 \times 10^{5}\ cm^3$ C $0.3 \times 10^{17}\ mm^3$ D $3.0 \times 10^{5}\ cm$

9. How many square metres are in $5 \times 10^4\ km^2$?

   A 5×10^{10} B 5×10^{13} C 5×10^{-10} D 5×10^{-13}

10. Given that a single hectare is equal to 10^4 square metres, 30 hectares is equal to which of the following?

    A $0.03\ km^2$ B $3 \times 10^4\ km^2$ C $30\ km^2$ D $0.3\ km^2$

GCE Paper 1 Solutions

1. B 2. D 3. A 4. A 5. C 6. C 7. D 8. B 9. A 10. D

GCE Paper 2 Questions

1. The distance between Ntumbaw and Ndu, when travelling through Wowo, is about 12 km.

   Providing the answer in both scientific and decimal notation, calculate
   (a) this distance in metres. **(3 mks)**
   (b) this distance in centimetres. **(3 mks)**

   ....

   Solution

   (a) *This conversion to a base unit uses the "kilo" row of table 3.1.1.3.*

   $$\text{quantity in given unit: } d = 12\ km$$

   $$\text{given equivalence: } 1\ km = 1000\ m$$

   $$\text{turning equivalence into conversion factor: } \frac{1000\ m}{1\ km} = \frac{1000}{1} \times \frac{m}{km} = 1$$

   $$\text{applying conversion factor: } d = \left(\frac{1000}{1} \times \frac{m}{km}\right)(12\ km)$$

   $$\text{final answer in decimal notation: } \boxed{d = 12000\ m}$$

   $$\text{converting to scientific notation: } \boxed{d = 1.2 \times 10^4\ m}$$

   (b) *This conversion uses the previously calculated base unit quantity as well as the "centi" row of table 3.1.1.3.*

   $$\text{calculated quantity in base unit: } d = 12000\ m$$

   $$\text{given equivalence: } 1\ cm = 0.01\ m$$

   $$\text{turning equivalence into conversion factor: } \frac{1\ cm}{0.01\ m} = \frac{1}{0.01} \times \frac{cm}{m} = 1$$

   $$\text{applying conversion factor: } d = \left(\frac{1}{0.01} \times \frac{cm}{m}\right)(12000\ m)$$

   $$\text{final answer in decimal notation: } \boxed{d = 1200000\ cm}$$

   $$\text{converting to scientific notation: } \boxed{d = 1.2 \times 10^6\ cm}$$

2. A year is a single unit of time. It is equivalent to approximately 365 days.

   (a) Calculate the duration of a day, expressing the answer in base units and in scientific notion. **(4 mks)**
   (b) Calculate the duration of a year, expressing the answer in base units and in scientific notion. **(4 mks)**
   (c) Explain why base units are not always used to measure physical quantities. **(2 mks)**

. . . .

Solution

(a) *There are approximately 24 hours in a day. This quantity of time must first be converted to minutes and then to seconds, the desired base unit.*

$$\text{quantity in given unit: } t_{\text{day}} = 24\ hr$$

$$\text{given equivalences: } 1\ hr = 60\ min, \quad 1\ min = 60\ s$$

$$\text{turning first equivalence into first conversion factor: } \frac{60\ min}{1\ hr} = \frac{60}{1} \times \frac{min}{hr} = 1$$

$$\text{turning second equivalence into second conversion factor: } \frac{60\ s}{1\ min} = \frac{60}{1} \times \frac{s}{min} = 1$$

$$\text{applying conversion factors: } t_{\text{day}} = \left[\left(\frac{60}{1} \times \frac{min}{hr}\right)\left(\frac{60}{1} \times \frac{s}{min}\right)\right](24\ hr)$$

$$\text{final answer: } \boxed{t_{\text{day}} = 8.64 \times 10^4\ s}$$

(b) *This conversion is done between the previosly calculated amount of base units (seconds) in a day as well as the given amount of days in a year.*

$$\text{quantity in given unit: } t_{\text{year}} = 365\ day$$

$$\text{previously calculated equivalence: } 1\ day = 8.64 \times 10^4\ s$$

$$\text{turning equivalence into conversion factor: } \frac{8.64 \times 10^4\ s}{1\ day} = \frac{8.64 \times 10^4}{1} \times \frac{s}{day} = 1$$

$$\text{applying conversion factor: } t_{\text{year}} = \left(\frac{8.64 \times 10^4}{1} \times \frac{s}{day}\right)(365\ day)$$

$$\text{final answer: } \boxed{t_{\text{year}} = 3.1536 \times 10^7\ s}$$

(c) Some quantities, such as the amount of time in a year, are so large that it is inconvenient to express them in significantly smaller base units. A similar problem is found when trying to express very small quantities, such as some amount of milligrams, in their significantly larger base quantities, like kilograms.

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3. A single cube of sugar has a volume of $7.29 \times 10^{-7}\ m^3$.

   Assuming its a perfect cube, and all sides are equal,
   (a) Calculate its volume in cubic centimetres. **(3 mks)**
   (b) Calculate the length of one of its sides, in metres. **(3 mks)**
   (c) Calculate the length of one of its sides, in centimetres. **(2 mks)**
   (d) Calculate the area of one of its faces, in square centimetres. **(2 mks)**

. . . .

Solution

(a) *The volume is converted considering* $1\ m = 10^2\ cm$.

$$\text{given quantity: } V = 7.29 \times 10^{-7}\ m^3$$

$$\text{applying conversion factor: } V = \left(7.29 \times 10^{-7}\ m^3\right)\left(\frac{10^2\ cm}{m}\right)^3 = \left(7.29 \times 10^{-7}\ m^3\right)\left(\frac{10^6\ cm^3}{m^3}\right)$$

$$\text{final answer: } \boxed{V = 7.29 \times 10^{-1}\ cm^3}$$

(b) *The length of one side is calculated as the cube root of the volume.*

$$\text{equation for volume of a cube: } V = l^3$$

$$\text{turning length into subject: } l = \sqrt[3]{V}$$

$$\text{substituting known values: } l = \sqrt[3]{7.29 \times 10^{-7}\ m^3}$$

$$\text{final answer: } \boxed{l = 9.0 \times 10^{-3}\ m}$$

(c) *The length of one side is already known in metres.*

$$\text{previously calculated quantity: } l = 9.0 \times 10^{-3}\ m$$

$$\text{applying conversion factor: } l = \left(9.0 \times 10^{-3}\ m\right)\left(\frac{10^2\ cm}{1\ m}\right)$$

$$\text{final answer: } \boxed{l = 9.0 \times 10^{-1}\ cm}$$

(d) *The area of one face is simply the square of the length of one of its sides.*

$$\text{equation for area of a square: } A = l^2$$

$$\text{substituting known values: } A = (9.0 \times 10^{-1}\ cm)^2$$

$$\text{final answer: } \boxed{A = 8.1 \times 10^{-1}\ cm^2}$$

3.2 Density

3.2.1 Calculating and Measuring Density

Objectives

By the end of the lesson, students should be able to

1. define density and state its units.
2. be able to solve problems relating a body's density to its mass and volume.
3. briefly explain the effect of a body's density on its buoyancy in water.
4. define relative density.
5. solve problems involving a body's density and its relative density.

Density

- **Density**, or ρ, is the mass per unit volume of a substance.
- It is a scalar.
- Its SI units are kilograms per metres cubed, abbreviated $kg\ m^3$.

$$\rho = \frac{m}{V} \tag{3.2.1.1}$$

Where
- ρ is the body's density, in $kg\ m^3$;
- m is the body's mass, in kg;
- V is the body's volume, in m^3.

Calculations of Density

- Problems involving density can require that one of the following three properties be calculated while the other two are given - mass, volume and density.

Example: A 0.3 cubic-metre container of petrol has a mass of 240 kilograms. What is the petrol's density? The mass of the container itself is negligible.

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{substituting known values: } \rho = \frac{240\ kg}{0.3\ m^3}$$

$$\text{final answer: } \boxed{\rho = 800\ kg\ m^{-3}}$$

Example: A bar of iron has a density of 7900 $kg\ m^{-3}$ and a mass of 20 kg. Calculate its volume.

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{turning volume into subject: } V = \frac{m}{\rho}$$

$$\text{substituting known values: } V = \frac{20\ kg}{7900\ kg\ m^{-3}}$$

$$\text{final answer: } \boxed{V \approx 2.532 \times 10^{-3}\ m^3}$$

Example: A plastic container encloses a quantity of air having a density of 1.3 $kg\ m^{-3}$. If the container's volume is 0.1 cubic metres, calculate the mass of the air inside in both kilograms and grams.

given equation for density: $\rho = \dfrac{m}{V}$

turning mass into subject: $m = \rho V$

substituting known values: $m = \left(1.3\ kg\ m^{-3}\right)\left(0.1\ m^3\right)$

final answer in kilograms: $\boxed{m = 0.13\ kg}$

converting to grams: $(0.13\ kg)\left(\dfrac{1000\ g}{1\ kg}\right)$

final answer in grams: $\boxed{m = 130\ g}$

Relative Density

- If a solid body is placed in a liquid body, it will either sink or float.

$$\text{the solid body floats if } \rho_{solid} < \rho_{liquid}$$

$$\text{the solid sinks if } \rho_{solid} > \rho_{liquid}$$

- If a solid body has a density greater than that of water, it will sink in the water.
- If a solid body has a density less than that of water, it will float on the water's surface.
- A body is **buoyant** if it floats in water.
- **Relative density**, or $\boldsymbol{\rho_{\text{rel}}}$, is the ratio of a body's density to that of water at room temperature ($25°C$).
- It is a scalar.
- It has no units given that it is the value of one density divided by another.

$$\rho_{rel} = \frac{\rho_b}{\rho_w} \tag{3.2.1.2}$$

Where
  - ρ_{rel} is the body's relative density (unit-less);
  - ρ_b is the body's density, in $kg\ m^{-3}$;
  - ρ_w is the density of water, equal to approximately 1000 $kg\ m^3$.

- If a body's relative density is less than 1, its density is less than that of water, and it will float.
- If a body's relative density is greater than 1, its density is greater than that of water, and it will sink.

$$\text{body sinks:} \quad \rho_b > \rho_w \rightarrow \rho_{rel} > 1$$
$$\text{body floats:} \quad \rho_b < \rho_w \rightarrow \rho_{rel} < 1$$

- NB: A body's relative density can <u>never</u> be negative.

$$\rho_{rel} > 0,\ \text{always}$$

GCE Paper 1 Questions

1. The density of a fixed mass of gas depends on its

   A volume B color C pressure D surface area

2. Which of the following is true regarding the density and volume of a body with a fixed mass?

   A $\rho \propto V$ B $\rho \propto \frac{1}{V^2}$ C $\rho \propto \frac{1}{V}$ D $\rho \propto V^2$

3. Which of the following is true regarding the density and mass of a body with a fixed volume?

   A $\rho \propto m$ B $\rho \propto \frac{1}{m^2}$ C $\rho \propto \frac{1}{m}$ D $\rho \propto m^2$

4. Given that the density of aluminium is 2700 $kg\ m^{-3}$, its relative density is

   A 0.37 B 0.27 C 27 D 2.7

5. Given that 1000 litres is equal to 1 cubic metre, a 1.5 litre container holds what mass of water?

   A 0.15 kg B 15 kg C 1.5 kg D 6.7 kg

6. Given $\rho_{\text{gold}} = 19300\ kg\ m^{-3}$, the volume of a single gram of gold is approximately

   A $1.58 \times 10^{-7}\ m^3$ B $5.18 \times 10^{8}\ m^3$ C $5.18 \times 10^{-8}\ m^3$ D $8.15 \times 10^{-6}\ m^3$

7. A 596.3 gram sample of copper occupies a volume of 6.7×10^{-5} cubic metres. Its density is

   A 8.900 $kg\ m^{-3}$ B 8900 $kg\ m^{-3}$ C $8.9 \times 10^{6}\ kg\ m^{-3}$ D 8900 $kg\ m^{3}$

8. If a solid body is known to float in pure water, which of the following is true of its relative density?

   A $\rho_{rel} = 0$ B $\rho_{rel} > 1$ C $\rho_{rel} < 1$ D $\rho_{rel} < 0$

9. If a solid body is known to sink in pure water, which of the following is true of its relative density?

   A $\rho_{rel} = 0$ B $\rho_{rel} > 1$ C $\rho_{rel} < 1$ D $\rho_{rel} < 0$

10. The density of methylated spirit is 800.0 $kg\ m^{-3}$. A volume of 0.00003 m^3, has a mass of

    A 0.0024 kg B 0.024 kg C 0.24 kg D 2.4 kg

GCE Paper 1 Solutions

1. A 2. C 3. A 4. D 5. C 6. C 7. B 8. C 9. B 10. B

GCE Paper 2 Questions

1. Brine is a solution of salt in water. 56 g of salt is added to 1000 cm^3 of water.
   (a) Calculate the density of the brine produced if the density of the water is 1 $g\ cm^3$. **(3 mks)**
   (b) State the assumption made in the calculation for subsection 1 (a). **(1 mk)**

....

Solution

(a) *The density of the brine solution is the mass of both the water and the salt per the volume of the water. It is assumed that the volume added by the salt is negligible.*

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{considering both the salt and the water: } \rho_{\text{brine}} = \frac{m_{\text{salt}} + m_{\text{water}}}{V_{\text{salt}} + V_{\text{water}}}$$

$$\text{assuming the volume of the salt is negligible: } \rho_{\text{brine}} = \frac{m_{\text{salt}} + m_{\text{water}}}{V_{\text{water}}}$$

$$\text{calculating the water's mass given its volume and density: } \rho_{\text{brine}} = \frac{m_{\text{salt}} + \rho_{\text{water}} V_{\text{water}}}{V_{\text{water}}}$$

$$\text{substituting known values: } \rho_{\text{brine}} = \frac{56\ g + \left(1\ g\ cm^{-3}\right)\left(1000\ cm^3\right)}{1000\ cm^3}$$

$$\text{converting to SI units: } \rho_{\text{brine}} = 1.056\ \frac{g}{cm^3}\left(\frac{1\ kg}{1000\ g}\right)\left(\frac{100\ cm}{1\ m}\right)^3$$

$$\text{final answer: } \boxed{\rho_{\text{brine}} = 1056\ kg\ m^{-3}}$$

(b) This calculation is based on the assumption that when the salt is added to the water, there is no significant volume added. That is, the only volume to be considered in calculating the brine's density is the original volume of the water.

---

2. Mercury is a metal that is often liquid at room temperature.
   (a) Explain the meaning of the statement "the density of mercury is $13,600\ kg\ m^{-3}$". **(2 mks)**
   (b) If the mercury in a thermometer occupies a volume of $1.5 \times 10^{-6}\ m^3$. Calculate its mass. **(3 mks)**

....

Solution

(a) This means that a body of mercury having a volume of 1 m^3 has a mass of $13,600\ kg$.

(b) *The mass to be calculated is assumed to be that of the mercury column.*

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{turning mass into subject: } m = V\rho$$

$$\text{substituting known values: } m = \left(1.5 \times 10^{-6}\ m^3\right)\left(13600\ kg\ m^{-3}\right)$$

$$\text{final answer: } \boxed{m = 0.0204\ kg}$$

---

3. A plastic cube has a volume of 1 cm^3 and a mass of 0.6 grams. **(3 mks)**

   (a) Calculate its density in $g\ cm^{-3}$. **(3 mks)**
   (b) Calculate its density in $g\ m^{-3}$. **(2 mks)**
   (c) Calculate its density in $kg\ m^{-3}$. **(2 mks)**
   (d) Explain whether or not the cube would float if placed in water. **(2 mks)**

. . . .

Solution

(a) *It is requested that the density be provided in the given units.*

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{substituting known values: } \rho = \frac{0.6\ g}{1\ cm^3}$$

$$\text{final answer: } \boxed{\rho = 6 \times 10^{-1}\ g\ cm^{-3}}$$

(b) *It is requested that only the units of volume be converted.*

$$\text{previously calculated quantity: } \rho = 6 \times 10^{-1}\ g\ cm^{-3}$$

$$\text{applying conversion factor: } \rho = \left(6 \times 10^{-1}\ g\ cm^{-3}\right)\left(\frac{10^2\ cm}{1\ m}\right)^3$$

$$\text{final answer: } \boxed{\rho = 6 \times 10^{5}\ g\ m^{-3}}$$

(c) *Its now requested to convert the unit of mass.*

$$\text{previously calculated quantity: } \rho = 6 \times 10^{5}\ g\ m^{-3}$$

$$\text{applying conversion factor: } \rho = \left(6 \times 10^{5}\ g\ m^{-3}\right)\left(\frac{1\ kg}{1000\ g}\right)$$

$$\text{final answer: } \boxed{\rho = 6 \times 10^{2}\ kg\ m^{-3}}$$

(d) *The cube's buoyancy depends on its relative density.*

$$\text{given equation for relative density: } \rho_{rel} = \frac{\rho_{\text{cube}}}{\rho_{\text{water}}}$$

$$\text{substituting known values: } \rho_{rel} = \frac{6 \times 10^2\ kg\ m^{-3}}{10^3\ kg\ m^{-3}}$$

$$\text{calculated relative density: } \rho_{rel} = 0.6$$

The cube will float. This is because its density is less than that of water. This is indicated by its relative density having a value of 0.6, which is less than 1.

3.2.2 Applications of Density

Objectives

By the end of the lesson, students should be able to

1. describe an experiment to measure the density of a regularly shaped solid.
2. describe an experiment to measure the density of an irregularly shaped solid.
3. discuss the abnormal expansion of water while freezing.
4. explain the advantages and disadvantages of water's abnormal expansion while freezing.
5. state the densities of some common materials.

Experiment to Measure the Density of a Regular Solid

I Procedure

1 The mass of the solid, m_s, is measured using a scale balance.
2 The solid's length, l_s, width, w_s and height, h_s of the solid are measured using a metre rule.

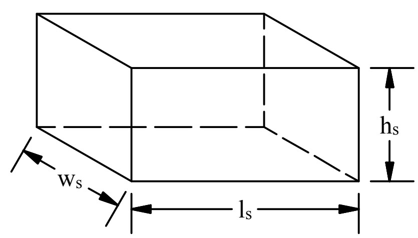

Figure 3.2.2.1

II Calculations

$$\text{The solid's volume is calculated as } V_s = (l_s)\,(w_s)\,(h_s)$$

$$\text{The density of the solid is then calculated as } \rho_s = \frac{m_s}{V_s}$$

III Precautions

- The experiment is repeated several times to get multiple values of ρ_s. The final value is then taken as the mean of these values in order to minimize error.

Experiment to Measure the Density of an Irregular Solid

I Procedure

1 The mass of the solid, m_s, is measured using a scale balance.
2 A measuring cylinder is filled with water.
3 The water's initial volume, V_i, is noted.
4 The object is attached to a string and slowly dropped into the water until its completely submerged.
5 The new volume, V_f, is then noted.

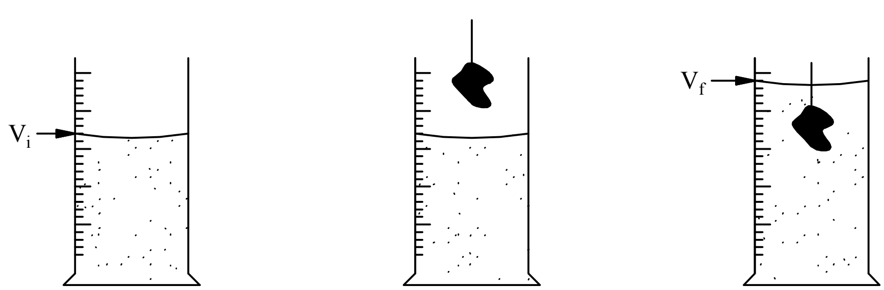

Figure 3.2.2.2

II Calculations

$$\text{The solid's volume is then calculated as } V_s = V_f - V_i$$

$$\text{The solid's density is then calculated as } \rho_s = \frac{m_s}{V_s}$$

III Precautions

- The solid is completely submerged in the water so that its volume is equal to that of the displaced water.
- The solid is lowered gently to avoid water splashing out of the cylinder.
- The experiment is repeated several times to get multiple values of ρ_s. The final value is then taken as the mean of these values in order to minimize error.

Abnormal Expansion of Water While Freezing

- Consider the volume and density of 1 kg of water across a range of temperatures, as shown in figure 3.2.2.3.
- Given a constant mass, $\rho \propto \frac{1}{V}$.
- As the liquid water cools, moving leftward from $100°C$ to $4°C$, its volume decreases and its density increases, which is typical.
- Just before it freezes, moving leftward from $4°C$ to $0°C$, its volume sharply increases and its density sharply decreases, which is abnormal.
- After the water freezes at $0°C$, it becomes solid ice with a density less than when it was a liquid.
- Therefore, $\rho_{rel,ice} < 1$, and ice floats.
- As the solid ice cools further, moving leftward from $0°C$, the volume decreases and the density increases, which is again typical.

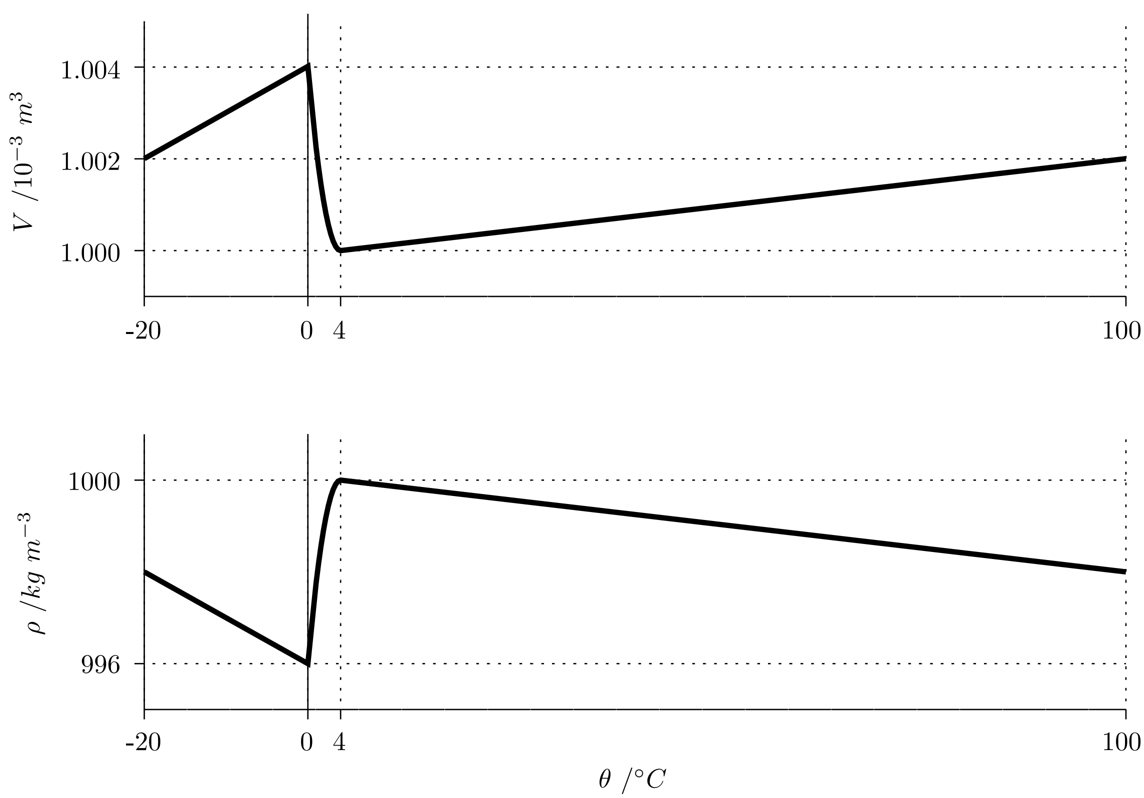

Figure 3.2.2.3

Advantages of Water's Abnormal Expansion

- Aquatic life can survive in water maintained at a warm $4°C$ in extremely cold regions.
- Expanding ice causes fracturing in rocks. Over time, this causes nutrient-rich debris to accumulate in nearby soil, which promotes regions of good agricultural land.
- Ice caps in polar regions float partially above the surface of the water, displacing less liquid water than if they floated entirely submerged. This reduces the average sea level globally, permitting coastal land to be utilized.

Disadvantages of Water's Abnormal Expansion

- Water cannot be used as a thermometric liquid given its irregular expansion.
- Water can break underground pipes when it expands while freezing.
- A liquid beverage left in refrigerator will break out of its container if it freezes.
- Floating ice caps in polar regions melt as the average global temperature increases, which causes the average sea level to increase. This causes coastal regions to become submerged.

Typical Densities

- Density varies largely for typical gases, liquids and solids.

material	**density** $\left(\mathbf{kg\ m^{-3}}\right)$
air (at sea level)	1.3
cork	250
bamboo	400 hspace*1cm
eucalyptus wood	750
petrol	800
paraffin wax	900
ice at $0°C$	920
water at $4°C$	1000
glass	$\sim$ 2500
sand	$\sim$ 2600
aluminium	2700
steel	7800
lead	11400
mercury	13600
gold	19300

Table 3.2.2.1

GCE Paper 1 Questions

1. A stone of mass 250 g displaces water in a measuring cylinder from the 35 cm^3 mark to the 90 cm^3 mark. The density of the stone, in $g\ cm^{-3}$ is

   A 72.86 B 2.77 C 2.27 D 4.55

2. A given mass of liquid water reaches its maximum volume at what temperature?

   A $-20°C$ B $0°C$ C $4°C$ D $100°C$

3. A given mass of liquid water reaches its minimum volume at what temperature?

   A $-20°C$ B $0°C$ C $4°C$ D $100°C$

4. A given mass of liquid water reaches its maximum density at what temperature?

   A $-20°C$ B $0°C$ C $4°C$ D $100°C$

5. A given mass of liquid water reaches its minimum density at what temperature?

   A $-20°C$ B $0°C$ C $4°C$ D $100°C$

6. Which of the following materials has a density greater than that of steel?

   A bamboo B paraffin wax C gold D aluminium

7. Which of the following materials would float in a pool of petrol?

   A paraffin wax B gold C glass D bamboo

8. Which of the following materials would float in pure, liquid water?

   A lead B ice C sand D glass

9. Which of the following materials would sink in pure, liquid water?

   A bamboo B ice C paraffin wax D glass

10. Does pure ice at $4°C$ float in pure water?

   A yes C no
   B yes, but only in Kumbo D yes, but only on country Sunday

GCE Paper 1 Solutions

1. D 2. B 3. C 4. C 5. B 6. C 7. D 8. B 9. D 10. A

GCE Paper 2 Questions

1. During cooling, water has a minimum volume at $4°C$. Below $4°C$, water expands until it freezes at $0°C$. The process of freezing itself involves a very sharp increase in volume.

   (a) Explain how this behaviour provides a natural advantage for aquatic life during winter in some regions of the world where ambient temperatures are as low as $-10°C$. **(3 mks)**
   (b) At what temperature could water be at its maximum density? Explain. **(1 mk)**

....

Solution

(a) As water on the surface cools from warmer temperatures down to $4°C$, its density increases, and it sinks. The colder, less dense water then rises to the top, where it becomes so cold that it freezes at $0°C$. This forms a solid, thermal cover, through which the heat of the warmer, $4°C$ and above water cannot easily escape. This creates a region hospital to aquatic life.

(b) The density of a fixed mass of pure water is inversely proportional to its volume. That is, $\rho \propto \frac{1}{V}$. Therefore, the temperature of its maximum density is the same as the temperature of its minimum volume, which is given to be $4°C$.

---

2. A cubic metre is equal to 1000 litres.

   (a) Calculate the mass, in kg, of a 1.5 L of paraffin wax. **(3 mks)**
   (b) Calculate the volume, in mL, 50 g of gold. **(5 mks)**

....

Solution

(a) *The density of paraffin wax can be read from table 3.2.2.1.*

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{turning mass into subject: } m = V\rho$$

$$\text{subtituting known values: } m = (1.5\ L)\left(900\ kg\ m^{-3}\right)$$

$$\text{applying concversion factor: } m = (1.5\ L)\left(\frac{1\ m^3}{1000\ L}\right)\left(900\ kg\ m^{-3}\right)$$

$$\text{final answer: } \boxed{1.35\ kg}$$

(b) *The density of gold can also be read from table 3.2.2.1.*

$$\text{given equation for density: } \rho = \frac{m}{V}$$

$$\text{turning volume into subject: } V = \frac{m}{\rho}$$

$$\text{subtituting known values: } V = \frac{50\ g}{19300\ kg\ m^{-3}}$$

$$\text{applying conversion factors: } V = \frac{50\ g}{19300\ kg\ m^{-3}}\left(\frac{1\ kg}{1000\ g}\right)\left(\frac{1000\ L}{1\ m^3}\right)\left(\frac{1000\ mL}{1\ L}\right)$$

$$\text{final answer: } \boxed{V \approx 2.59\ mL}$$

---

3. A student is given two miscible liquids A and B in separate jars as well as a cylinder/beaker of unknown volume V_x and a mass balance. She is asked to confirm that the density of liquid A is greater than that of liquid B.

   (a) List the measurements she should make in order to make this confirmation. **(2 mks)**
   (b) Using these measurements along with the unknown volume V_x, write down the expressions for the densities of the liquids, showing $\rho_A > \rho_B$. **(3 mks)**
   (c) State the conclusion that allows the student to confirm that $\rho_A > \rho_B$. **(2 mks)**

. . . .

Solution

(a) The student should measure:
   - The mass of the empty beaker, m_0;
   - The mass of the beaker filled with liquid A to its brim, m_α;
   - The mass of the beaker filled with liquid B to its brim, m_β;

(b) *Although the volume V_x is unknown, it is the same for the measurement of both liquid A and B.*

the mass of liquid A is calculated: $m_A = m_\alpha - m_0$

the mass of liquid B is calculated: $m_B = m_\beta - m_0$

the density of liquid A is calculated: $\rho_A = \dfrac{m_A}{V_x} = \dfrac{m_\alpha - m_0}{V_x}$

the density of liquid B is calculated: $\rho_B = \dfrac{m_B}{V_x} = \dfrac{m_\beta - m_0}{V_x}$

show $\rho_A > \rho_B$: $\dfrac{m_\alpha - m_0}{V_x} > \dfrac{m_\beta - m_0}{V_x}$

multiply both sides by unknown volume: $m_\alpha - m_0 > m_\beta - m_0$

add mass of beaker to both sides: $m_\alpha > m_\beta$

(c) If the mass of the volume of liquid A needed to fill the beaker is greater than that of the volume of liquid B needed to fill the same volume, the density of liquid A is greater than that of liquid B.

3.3 Pressure

3.3.1 Calculating Pressure

Objectives

By the end of the lesson, students should be able to

1. define pressure and state its unit.
2. state the factors that affect pressure and explain the effects of force and area on pressure.
3. solve problems relating pressure to force and area.

Pressure

- **Pressure**, or **P**, is the magnitude of a force acting perpendicularly per unit surface area.
- Absolute pressure is a scalar.
- Its SI is the Pascal, abbreviated Pa.

$$P = \frac{F}{A} \tag{3.3.1.1}$$

  Where
  - P is the pressure applied by a force, in Pa;
  - F is the magnitude of the force, in N;
  - A is the area on which the force is applied perpendicularly, in m^2.

- The SI unit of pressure, the Pascal, is equivalent to one Newton per square metre.

$$Pa = N\ m^{-2}$$

- NB: Pressure is the quotient of a force acting perpendicularly to the area of a surface.

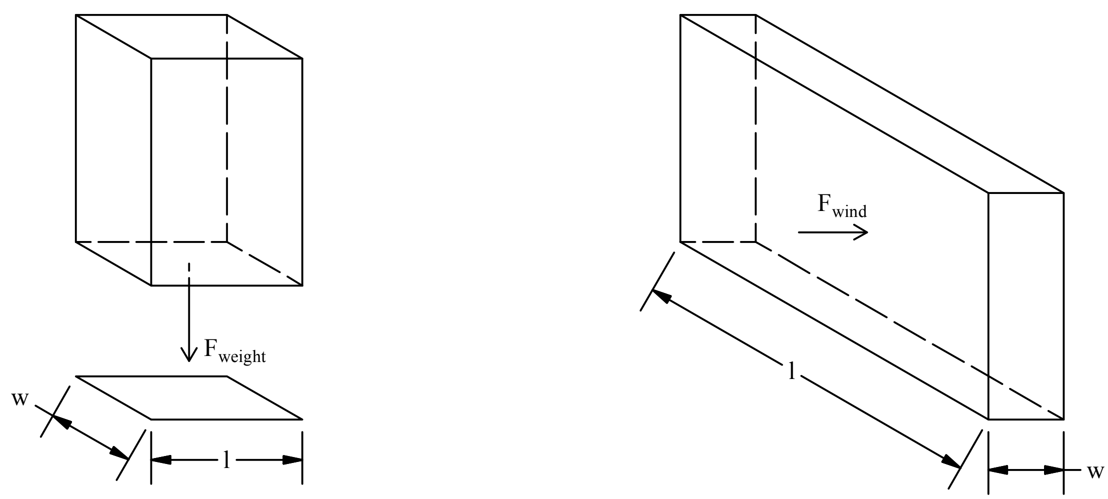

Figure 3.3.1.1

  - As shown on the left of figure 3.3.1.1, the force of the block's weight is applied over the surface of contact between the block and the ground.

$$\text{that is, } P_{\text{weight}} = \frac{F_{\text{weight}}}{A_{\text{ground}}} = \frac{F_{\text{weight}}}{l \times w}$$

  - As shown on the right of figure 3.3.1.1, the force of wind is applied over the surface area of a wall.

$$\text{that is, } P_{\text{wind}} = \frac{F_{\text{wind}}}{A_{\text{wall}}} = \frac{F_{\text{weight}}}{l \times w}$$

Factors of Pressure

- With a constant force, the pressure exerted on a surface is inversely proportional to the area of contact.

$$\text{with constant F, } P \propto \frac{1}{A}$$

  - Knife's are sharp to minimize contact area, causing higher pressure while cutting.
  - Heavy trucks have wide tires to maximize contact area, causing lower pressure on softer ground.
  - Thin-heeled shoes cause more pain when stepping on someone because their smaller contact areas cause larger pressure on a person's foot.
  - A boy exerts less pressure on the ground if he stands on two feet than on one because he increases the area of contact.
  - An snow-skier uses skies that have a large surface area so she does not cut too deep into the snow with a high pressure.
  - A nail is easier to hammer into wood than a bolt because the tip of the nail has a very small contact area.

- With a constant area, the pressure exerted on a surface is proportional the force applied.

$$\text{with constant A, } P \propto F$$

  - A man will exert more pressure on the ground if he picks up a baby because he increases the force he applies downward.
  - A loaded truck exerts more pressure on the ground than when it is empty.

Calculations of Pressure

- Problems involving pressure can require that one of the following three properties be calculated while the other two are given - pressure, force and area.

  Example: A boy has a weight-force of 750 N. Each of his feet has a ground-contact area of $3.1 \times 10^{-2}\ m^2$. Calculate the pressure he exerts on the ground if he stands on one foot. Hence calculate the pressure he exerts as he stands on both feet.

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{substituting known values for one foot: } P = \frac{750\ N}{3.1 \times 10^{-2}\ m^2}$$

$$\text{final answer: } \boxed{P \approx 24131.3\ Pa}$$

$$\text{substituting known values for both feet: } P = \frac{750\ N}{2 \times (3.1 \times 10^{-2}\ m^2)}$$

$$\text{final answer: } \boxed{P \approx 12096.8\ Pa}$$

  Example: A truck having a weight force of 20000 N exerts a pressure of 40000 Pa on the ground. Calculate the area of contact between the truck's tires and the ground.

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{turning area into subject: } A = \frac{F}{P}$$

$$\text{substituting known values: } A = \frac{20000\ N}{40000\ Pa}$$

$$\text{final answer: } \boxed{A = 0.5\ m^2}$$

Example: A nail will only pierce a plank of wood if it exerts a pressure of 3.5 GPa. If the area of contact between the nail and the wood is 1 mm^2, calculate the force, in both N and kN, that a hammer must exert on the nail in order for it to enter.

given equation for pressure: $P = \frac{F}{A}$

turning force into subject: $F = PA$

substituting known values: $F = (3.5\ GPa)\left(1\ mm^2\right)$

applying conversion factors: $F = (3.5\ GPa)\left(1\ mm^2\right)\left(\frac{1 \times 10^9\ Pa}{1\ GPa}\right)\left(\frac{1\ m}{1000\ mm}\right)^2$

final answer in Newtons: $\boxed{F = 3500\ N}$

applying conversion factor: $F = (3500\ N)\left(\frac{1\ kN}{1000\ N}\right)$

final answer in kiloNewtons: $\boxed{F = 3.5\ kN}$

Common Units of Pressure

unit	**unit abbreviation**	**equivalence in base**
kiloPascal	kPa	$10^3\ Pa$
MegaPascal	MPa	$10^6\ Pa$
GigaPascal	GPa	$10^9\ Pa$
atmosphere	atm	$1.013 \times 10^5\ Pa$
pound per square inch	PSI	$6.895 \times 10^3\ Pa$
millimetres of mercury	$mmHg$	$133.289\ Pa$
inches of water	inH_2O	$257.302\ Pa$
bar	bar	$10^5\ Pa$

Table 3.3.1.1

GCE Paper 1 Questions

1. A concrete block of dimensions 0.1 m by 0.8 m by 1 m weighs 1000 N. The maximum pressure the block can exert on the ground in $N\ m^{-2}$ is

   A 12500 B 1250 C 10000 D 1000

2. The mass of a student standing on a beam balance with both feet is observed to be 60 kg. When he now stands on one foot, his mass will be

   A 30 kg B 45 kg C 600 N D 60 kg

3. If a force of 200 N acts on an area of 4 m^2, the pressure exerted is:

   A $\frac{1}{50}\ N\ m^{-2}$ B 204 $N\ m^{-2}$ C 50 $N\ m^{-2}$ D 800 $N\ m^{-2}$

4. A sharp knife cuts better than a blunt one because

   A it has a large surface area of contact.
   B it applies a small force.
   C it applies very little pressure.
   D it applies a large pressure on a small contact area.

5. The circular cross sectional area of the piston in a car engine is 0.006 m^2. When the pressure in the cylinder is 3000000 Pa, the force on the piston, in Newtons, is

   A 5.0×10^8 B 1.8×10^4 C 1.8×10^5 D 1.8×10^7

6. Heavy duty vehicles have broad tyres that are at times doubled so as to be able to

   A withstand the weight of the load they carry.
   B last longer.
   C reduce pressure.
   D reduce friction due to rough roads.

7. Which of the following values represents the greatest quantity of pressure?

   A 1 kPa B 1 Pa C 1 atm D 1 PSI

8. If the area over which a force is applied is halved, its pressure

   A doubles B halves C stays the same D becomes zero

9. If the area over which a force is applied is doubled, its pressure

   A doubles B halves C stays the same D becomes zero

10. Which of the following is <u>not</u> a unit of pressure?

    A kPa B GPa C PSI D N

GCE Paper 1 Solutions

1. A 2. D 3. C 4. D 5. B 6. C 7. C 8. A 9. B 10. D

GCE Paper 2 Questions

1. What is the minimum pressure that a rectangular block can exert on the ground if it has dimensions of $2\ m \times 4\ m \times 5\ m$ and a mass of $300\ kg$? Assume $g = 10\ m\ s^{-2}$. **(3 mks)**

....

Solution

The minimum pressure occurs when the force of the block's weight is applied over its largest face.

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{substituting force of weight: } P = \frac{F_{\text{weight}}}{A}$$

$$\text{substituting equation for force of weight: } P = \frac{mg}{A}$$

$$\text{substituting known values, using largest area: } P = \frac{(300\ kg)\left(10\ m\ s^{-2}\right)}{4\ m \times 5\ m}$$

$$\text{final answer: } \boxed{P = 150\ Pa}$$

---

2. The pressure acting on a surface is $10\ Pa$. What does this mean? **(3 mks)**

....

Solution

This means that a pressure equal to the force of $10\ N$ applied over $1\ m^2$ is occurring.

---

3. From your understanding of pressure, explain the following observations.

   (a) In order to avoid sinking, soldiers sometimes crawl on their bellies across swampy ground instead of walking. Explain why. **(2 mks)**

   (b) A woman wearing sharp-heeled shoes causes more pain if she steps on someone's foot than if she were wearing flat-heeled shoes. **(2 mks)**

....

Solution

(a) When soldiers crawl on their bellies, they distribute the force of their weight over a larger distance than if they walked on their feet. This reduces the pressure they exert on the ground which helps reduce the change of sinking if the surface is wet and soft like in a swamp.

(b) The woman with the sharp-heeled shoes will apply the force of her weight over a smaller area than she would with flat-heeled shoes. Given that pressure increases with decreasing area over which the is force applied, the sharp-heeled shoes will cause greater pressure, and therefore greater pain if she accidentally steps on someone's foot than would the larger-area, flat-heeled shoes.

---

4. Walking on pebbles with bare feet is more painful than walking on sand. Explain why. **(3 mks)**

. . . .

Solution

This is because the fewer, larger pebbles have a smaller surface of contact with someone's foot than do the many, smaller particles of sand. This smaller surface of contact creates a larger pressure on their foot, causing more pain.

---

5. A pressure of 1 kPa acts on an area of 2000 cm^2.

   (a) Calculate the force applied, in Newtons. **(3 mks)**
   (b) With this force, calculate the pressure, in kPa, if the area is doubled. **(3 mks)**

. . . .

Solution

(a) *The quantities in their given units must be converted into SI units.*

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{turning force into subject: } F = AP$$

$$\text{substituting known values: } F = \left(2000\ cm^2\right)(1\ kPa)$$

$$\text{applying conversion factors: } F = \left(2000\ cm^2\right)\left(\frac{1\ m}{100\ cm}\right)^2 (1\ kPa)\left(\frac{1000\ Pa}{1\ kPa}\right)$$

$$\text{final answer: } \boxed{F = 200\ N}$$

(b) *Only the area is changed in this calculation.*

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{doubling area: } P = \frac{F}{2A}$$

$$\text{substituting known values: } P = \frac{200\ N}{2\left(2000\ cm^2\right)}$$

$$\text{applying conversion factor: } P = \frac{200\ N}{2\left(2000\ cm^2\right)}\left(\frac{100\ cm}{1\ m}\right)^2$$

$$\text{answer in Pa: } P = 500\ Pa$$

$$\text{applying conversion factor: } P = (500\ Pa)\left(\frac{1\ kPa}{1000\ Pa}\right)$$

$$\text{final answer: } \boxed{P = 0.5\ kPa}$$

3.3.2 Boyle's Law and Charles' Law

Objectives

By the end of the lesson, students should be able to

1. explain the ideal gas law.
2. state Boyle's law.
3. state some useful consequences of the compressibility of gases.
4. state Charles' law.

The Pressure of Gases

- A fixed mass of gas can be modelled as an ideal gas if two assumptions are made.
  - The interactive forces between molecules are negligible.
  - The volume that the molecules themselves occupy is negligible.
- All gases exert pressure on the inside surface of the containers that hold them.

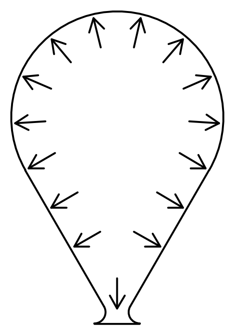

Figure 3.3.2.1

- A fixed mass of an ideal gas in a container can undergo a process, during which any combination of its pressure, volume and temperature changes.
- The **ideal gas law** states that the product of a fixed mass of gas' pressure and volume is proportional to its absolute temperature.

$$PV \propto \theta$$

$$PV = k\theta \qquad (3.3.2.1)$$

Where
  - P is the pressure the gas applies on the inside of its container;
  - V is the volume the container encloses;
  - k is a constant of proportionality;
  - θ is the absolute temperature of the gas, in K.

The distinction between absolute K and $^\circ C$ is not treated until form 4. It is however critical that all calculations involving Charles' law and Gay-Lussac's law use absolute degrees of temperature. All relevant sample problems in this section have been adapted such that no conversion between K and $^\circ C$ is necessary. If it is necessary to treat such conversions at this point, one can use the following equation.

$$\theta_K = \theta_{\circ C} + 273 \quad \text{or} \quad \theta_K - 273 = \theta_{\circ C}$$

- There exists three laws which concern the relationship of two out of an ideal gas' three properties while a third is held constant.

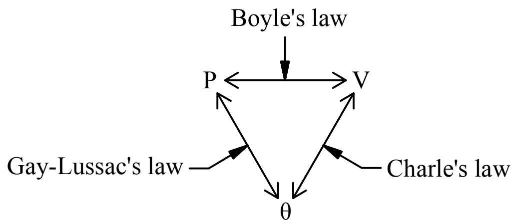

Figure 3.3.2.2

  - Boyle's law relates a fixed mass of gas' pressure and volume while its temperature is held constant.
  - Charles' law relates a fixed mass of gas' volume and temperature while its pressure is held constant.
  - Gas-Lussac's law relates a fixed mass of gas' pressure and temperature as its volume is held constant.

Boyle's Law

- **Boyle's law** states that the pressure of a fixed mass of gas is inversely proportional to its volume, provided its temperature is constant.

$$P \propto \frac{1}{V}$$

$$P = \frac{k}{V} \tag{3.3.2.2}$$

Where
  - P is the gas' pressure throughout a process;
  - k is a constant of proportionality throughout a process;
  - V is the gas's volume throughout a process.

- An **isothermal** process is one in which the body's temperature is held constant.

- Boyle's law can be applied to an isothermal process in order to relate the properties of pressure and volume before and after a compression or expansion.

$$\text{turning k into subject from Boyle's law: } k = PV$$

$$\text{considering initial properties: } k_i = P_i V_i$$

$$\text{considering final properties: } k_f = P_f V_f$$

$$\text{constant of proportionality does not change: } k_i = k_f$$

$$\text{considering a given isothermal process: } P_i V_i = P_f V_f \tag{3.3.2.3}$$

$$\text{turning pressure ratio and volume ratio into subjects: } \frac{P_f}{P_i} = \frac{V_i}{V_f} \tag{3.3.2.4}$$

Where
  - P_i is the gas' initial pressure;
  - V_i is the gas' initial volume;
  - P_f is the gas' final pressure;
  - V_f is the gas' final volume.

- It should be noted that equations 3.3.2.3 and 3.3.2.4 hold for any units of pressure and volume, as long as the same units are used between P_i and P_f as well between V_i and V_f.

- Air compressed inside a closed syringe at constant temperature is a demonstration of Boyle's law.

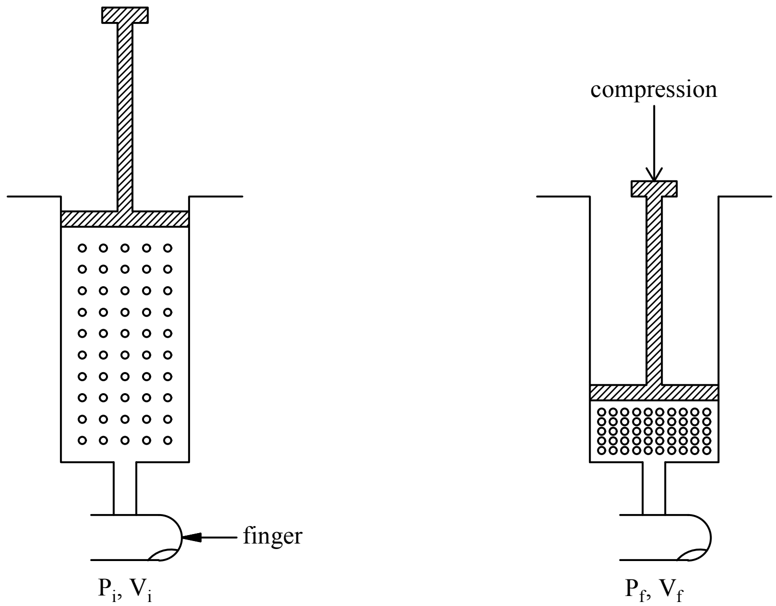

Figure 3.3.2.3

  - Before a compression, the gas inside the syringe has an initial pressure (P_i) and an initial volume (V_i).
  - After compression, the same mass of gas has a final, higher pressure (P_f) given a lower, final volume (V_f).

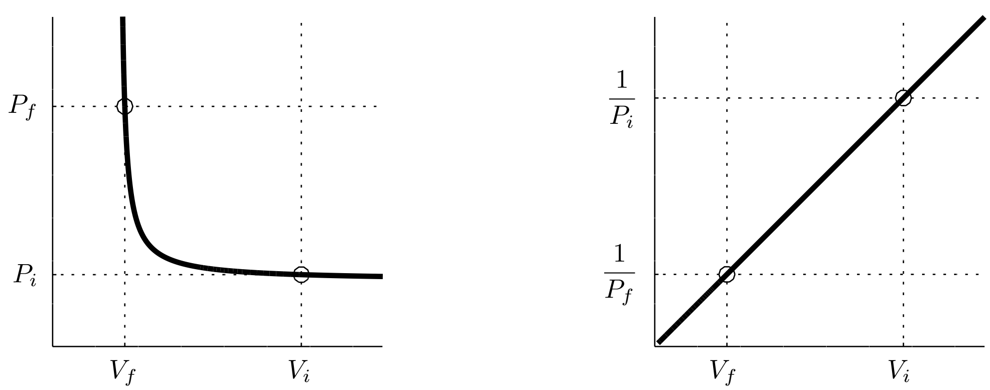

Figure 3.3.2.4

Useful Consequences of the Compressibility of Gases

- Vehicle tyres can be filled with air.
- Large amounts of gases used for industrial purposes can be stored in small, portable containers.

Charles' Law

- **Charles' law** states that the volume of a fixed mass of gas is directly proportional to its absolute temperature at constant pressure.

$$V \propto \theta$$

$$V = k\theta \tag{3.3.2.5}$$

- An **isobaric** process is one in which the body's pressure is held constant.
- Charles' law can be applied to an isobaric process in order to relate the properties of absolute temperature and volume before and after a process.

$$\text{turning k into subject from Charles' law: } k = \frac{V}{\theta}$$

$$\text{considering initial properties: } k_i = \frac{V_i}{\theta_i}$$

$$\text{considering final properties: } k_f = \frac{V_f}{\theta_f}$$

$$\text{constant of proportionality does not change: } k_i = k_f$$

$$\text{considering a given isobaric process: } \frac{V_i}{\theta_i} = \frac{V_f}{\theta_f} \qquad (3.3.2.6)$$

$$\text{turning volume ratio and temperature ratio into subjects: } \frac{V_f}{V_i} = \frac{\theta_f}{\theta_i} \qquad (3.3.2.7)$$

Where
  - V_i is the gas' initial volume;
  - θ_i is the gas' initial absolute temperature, in K;
  - V_f is the gas' final volume;
  - θ_f is the gas' final absolute temperature, in K.

- It should be noted that equations 3.3.2.6 and 3.3.2.7 hold for any units of volume as long as the same units are used between V_i and V_f .
- This equation does however require the temperature scale to be in absolute degrees, which is why the units are specified as K for θ_i and θ_f.
- A balloon expanding as it warms with a constant interior pressure is a demonstration of Charles' law.
  - Before heating, the balloon has an initial temperature of θ_i and an initial volume of V_i.
  - After heating, the balloon has a final temperature of θ_f and a final volume of V_f.

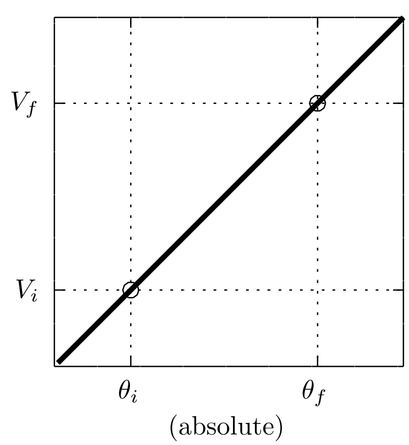

Figure 3.3.2.5

GCE Paper 1 Questions

1. When the temperature of a fixed mass of gas reduces at constant pressure,

   A its volume stays the same.
   B the gas becomes warmer.
   C its volume reduces.
   D its density reduces.

2. At constant temperature, the volume of a fixed mass of gas is inversely proportional to its pressure. This is known as

   A the pressure law
   B the law of conservation of momentum
   C Boyle's law
   D Charles' law

3. At constant pressure, the volume of a fixed mass of gas is proportional to its temperature. This is known as

   A the pressure law B the law of the land C Boyle's law D Charles' law

4. A fixed mass of gas has a pressure of 60 atm. If, after an isothermal compression, its final volume is one third that of its initial volume, its final pressure is

   A 20 atm B 180 atm C 60 atm D 120 atm

5. A fixed mass of gas undergoes an isobaric compression. Which of its properties stays the same?

   A pressure B volume C temperature D density

6. A fixed mass of gas undergoes an isothermal expansion. Which of its properties stays the same?

   A pressure B volume C temperature D density

7. Pushing down on a syringe plunger causes the interior pressure to increase with no noticeable change in temperature. This is most likely a demonstration of which law?

   A Charles' law B Boyle's law C martial law D the density law

8. A fixed mass of gas has the initial properties $P_i = 1\ atm$, $V_i = 3\ cm^3$ and $\theta_i = 298K$. After an expansion, its final properties are $P_f = 0.25\ atm$, $V_f = 12\ cm^3$ and $\theta_f = 298K$. This process is

   A isothermal B isographic C isotropic D isobaric

9. A fixed mass of gas has the initial properties 3 MPa, $V_i = 7\ m^3$, $\theta_i = 500K$. After a change in volume, its final properties are 3 MPa and $V_f = 3.5\ m^3$ and $\theta_f = 250K$. This process is

   A isothermal B isographic C isotropic D isobaric

10. In the equation $PV = k\theta$, if the quantities of pressure, volume and temperature are in their base SI units, k has which units?

    A $N\ m^{-1}\ K^{-1}$ B $N\ m\ K^{-1}$ C $N^2\ m\ K^{-3}$ D $N\ m^{-1}\ K$

GCE Paper 1 Solutions

1. C 2. C 3. D 4. B 5. A 6. C 7. B 8. A 9. D 10. B

GCE Paper 2 Questions

1. The piston of a car engine compresses a fixed mass of a gaseous mixture of air and petrol inside a bore. As the turning engine shaft pushes the piston, the gas' volume reduces to one quarter of its initial value. The initial pressure is $1.2156 \times 10^5\ Pa$.

   (a) Calculate the gas' initial pressure in *atm*. **(2 mks)**
   (b) Calculate the gas's final pressure, in *atm*, after being compressed. **(3 mks)**
   (c) Explain what happens to the gas' final pressure if its final volume becomes so small that its value is very close to zero. **(2 mks)**
   (d) Explain why the gas' final volume can never become zero. **(2 mks)**

. . . .

Solution

(a) *The unit equivalence* $1\ atm = 1.013 \times 10^5\ Pa$ *is used.*

$$\text{given quantity: } P_i = 1.2156 \times 10^5\ Pa$$

$$\text{applying conversion factor: } P_i = 1.2156 \times 10^5\ Pa \left(\frac{1\ atm}{1.013 \times 10^5\ Pa}\right)$$

$$\text{final answer: } \boxed{P_i = 1.2\ atm}$$

(b) *Boyle's law is applied to this isothermal process.*

$$\text{applying Boyle's law for a process: } P_iV_i = P_fV_f$$

$$\text{turning final pressure into subject: } P_f = P_i\left(\frac{V_i}{V_f}\right)$$

$$\text{inverting volume ratio to accept known value: } P_f = P_i\left(\frac{V_f}{V_i}\right)^{-1}$$

$$\text{subtituting known values: } P_f = (1.2\ atm)\left(\frac{1}{4}\right)^{-1}$$

$$\text{final answer: } \boxed{P_f = 4.8\ atm}$$

(c) *Such a compression is still assumed to be isothermal.*

$$\text{applying Boyle's law for a process: } P_iV_i = P_fV_f$$

$$\text{turning final pressure into subject: } P_f = \frac{P_iV_i}{V_f}$$

$$\text{approximating final volume as zero: } P_f = \frac{P_iV_i}{0}$$

$$\text{final pressure become infinite: } P_f = \frac{k}{0} = \infty$$

(d) It is physically impossible to have an infinite pressure. Therefore, it is just as impossible for the volume to be compressed completely to zero.

2. A deflated, light-weight plastic bag initially holds a volume of 7.5 L of air. It is then held over a flame such that it warms slightly. Its initial temperature is 298 K and its final temperature is 350 K. Its pressure remains constant throughout the process.

   (a) State the type of process the bag has undergone and which gas law applies to this process. **(2 mks)**
   (b) Express the bag's initial volume in cubic centimetres. **(2 mks)**
   (c) Calculate the bag's final volume in cubic centimetres. **(2 mks)**
   (d) If the bag were placed in a box of ice, would it size reduce or increase? Explain. **(2 mks)**

....

Solution

(a) This process is isobaric, given that the pressure remains constant throughout. Therefore, Charles' law applies.

(b) *The unit equivalence* 1000 $L = 1\ m^3$ *is used.*

$$\text{given quantity: } V_i = 7.5\ L$$

$$\text{applying conversion factors: } V_i = 7.5\ L\left(\frac{1\ m^3}{1000\ L}\right)\left(\frac{100\ cm}{1\ m}\right)^3$$

$$\text{final answer: } \boxed{V_i = 7500\ cm^3}$$

(c) *Given that the value of the initial volume is already known in* cm^3*, that will be the value substituted in order to find the final volume in the same units.*

$$\text{applying Charles' law for a process: } \frac{V_i}{\theta_i} = \frac{V_f}{\theta_f}$$

$$\text{turning final volume into subject: } V_f = V_i\left(\frac{\theta_f}{\theta_i}\right)$$

$$\text{substituting known values: } V_f = \left(7500\ cm^3\right)\left(\frac{350\ K}{298\ K}\right)$$

$$\text{final answer: } \boxed{V_f \approx 8808.72\ cm^3}$$

(d) Its size would decrease. This is because, assuming the pressure remains constant, Charles' law states that the volume of a gas is proportional to its temperature. That is, at constant P, $V \propto \theta$. Therefore, a reduction in temperature will cause a reduction in volume.

3.3.3 Gay-Lussac's Law and The Ideal Gas Law

Objectives

By the end of the lesson, students should be able to

1. state Gay-Lussac's law.
2. discuss the advantages of pressure cookers.
3. solve problems involving the pressure, volume and temperature of a fixed mass of gas before and after a process.

Gay-Lussac's law

- **Gay-Lussac's law** states that the pressure of a fixed mass of gas is directly proportional to its absolute temperature at constant volume.

$$P \propto \theta$$

$$P = k\theta \tag{3.3.3.1}$$

Where
  - P is the pressure the gas applies on the inside of its container;
  - k is a constant of proportionality;
  - θ is the absolute temperature of the gas, in K.

- An **isometric or isochoric** process is one in which the body's volume is held constant.

- Gay-Lussac's law can be applied to an isometric process in order to relate the properties of pressure and temperature before and after a process.

$$\text{turning k into subject from Gay-Lussac's law: } k = \frac{P}{\theta}$$

$$\text{considering initial properties: } k_i = \frac{P_i}{\theta_i}$$

$$\text{considering final properties: } k_f = \frac{P_f}{\theta_f}$$

$$\text{constant of proportionality does not change: } k_i = k_f$$

$$\text{considering a given isometric process: } \frac{P_i}{\theta_i} = \frac{P_f}{\theta_f} \tag{3.3.3.2}$$

$$\text{turning pressure ratio and temperature ratio into subjects: } \frac{P_i}{P_f} = \frac{\theta_i}{\theta_f} \tag{3.3.3.3}$$

Where
  - P_i is the gas' initial pressure;
  - T_i is the gas' initial absolute temperature, in K;
  - P_f is the gas' final pressure;
  - T_f is the gas' final absolute temperature, in K.

- It should be noted that equations 3.3.3.2 and 3.3.3.3 hold for any units of pressure as long as the same units are used between P_i and P_f . The equation does however require the temperature scale to be absolute, which is why the units are specified as K for θ_i and θ_f.

Pressure Cookers

- Pressure cookers are a demonstration of Gay-Lussac's law.
  - Before heating, the gas inside the pot has an initial temperature of θ_i and an initial pressure of P_i.
  - After heating, the pot has a final temperature of θ_f and a final pressure of P_f.
  - As the final pressure is above atmospheric conditions, the boiling temperature of the water is increased.
  - This allows for water to be maintained in its liquid phase at higher temperatures, which facilitates cooking.

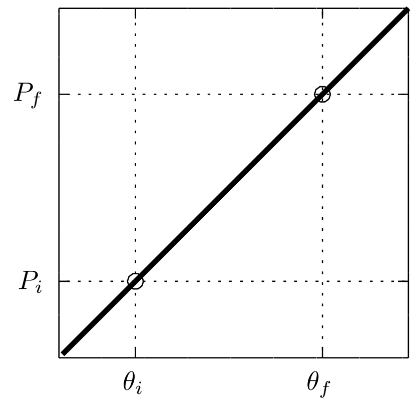

Figure 3.3.3.1

Relating All Three Properties in the Ideal Gas Law

- If the pressure, volume and temperature of a fixed mass of gas all change during a process, they can be related using the ideal gas law.

$$\text{considering ideal gas law: } PV = k\theta$$

$$\text{turning k into subject from ideal gas law: } k = \frac{PV}{\theta}$$

$$\text{considering initial properties: } k_i = \frac{P_i V_i}{\theta_i}$$

$$\text{considering final properties: } k_f = \frac{P_f V_f}{\theta_f}$$

$$\text{constant of proportionality does not change: } k_i = k_f$$

$$\text{considering a given process: } \frac{P_i V_i}{\theta_i} = \frac{P_f V_f}{\theta_f} \quad (3.3.3.4)$$

$$\text{turning pressure ratio into subject: } \frac{P_f}{P_i} = \frac{V_i \theta_f}{V_f \theta_i} \quad (3.3.3.5)$$

$$\text{turning volume ratio into subject: } \frac{V_f}{V_i} = \frac{P_i \theta_f}{P_f \theta_i} \quad (3.3.3.6)$$

$$\text{turning temperature ratio into subject: } \frac{\theta_f}{\theta_i} = \frac{P_f V_f}{P_i V_i} \quad (3.3.3.7)$$

Where
- P_i is the gas' initial pressure;
- V_i is the gas' initial volume;
- θ_i is the gas' initial absolute; temperature, in K;
- P_f is the gas' final pressure;
- V_f is the gas' final volume;
- θ_f is the gas' initial absolute; temperature, in K.

- It should be noted that equations 3.3.3.4, 3.3.3.5, 3.3.3.6 and 3.3.3.7 hold for any units of pressure and volume, as long as the same units are used between P_i and P_f as well between V_i and V_f. The equation does however require that the temperature scale to be absolute, which is why the units are specified as K for both θ_i and θ_f.

Example: A Tangui bottle has a volume of 1.5 L. It is opened, filled only with air at 298 K and 1 atm, and then closed again. Calculate its final pressure if it is compressed to 1.0 L and heated to 350 K.

$$\text{applying the ideal gas law for a process: } \frac{P_i V_i}{\theta_i} = \frac{P_f V_f}{\theta_f}$$

$$\text{turning final pressure into subject: } P_f = \frac{P_i V_i \theta_f}{V_f \theta_i}$$

$$\text{subtituting known values: } P_f = \frac{(1\ atm)\,(1.5\ L)\,(350\ K)}{(1.0\ L)\,(298\ K)}$$

$$\text{final answer: } \boxed{P_f \approx 1.76\ atm}$$

Example: Considering the same initial conditions, calculate the final temperature if the bottle was again compressed to 1 L but somehow ended with a pressure of 0.5 atm.

$$\text{applying the ideal gas law for a process: } \frac{P_i V_i}{\theta_i} = \frac{P_f V_f}{\theta_f}$$

$$\text{turning final temperature into subject: } \theta_f = \frac{P_f V_f \theta_i}{P_i V_i}$$

$$\text{substituting known values: } \theta_f = \frac{(0.5\ atm)(1\ L)(298\ K)}{(1\ atm)(1.5\ L)}$$

$$\text{final answer: } \boxed{\theta_f \approx 99.33\ K}$$

Gas Law Summary

- There are four separate laws relating the three properties of pressure, volume and temperature of a fixed mass of an ideal gas during a process.

- Their applicability depends on which properties, if any, are constant.

property being held constant	applicable law	type of process	equation
temperature	Boyle's law	isothermal	$PV = k$
pressure	Charles' law	isobaric	$V = k\theta$
volume	Gay-Lussac's law	isometric	$P = k\theta$
-	the ideal gas law	-	$PV = k\theta$

Table 3.3.3.1

GCE Paper 1 Questions

1. A pressure pot is useful because it cooks food at a

   A lower temperature. B lower pressure. C higher cost. D higher temperature.

   Questions 2 through 6 refer to the graphs shown in figure 3.3.3.2 below.

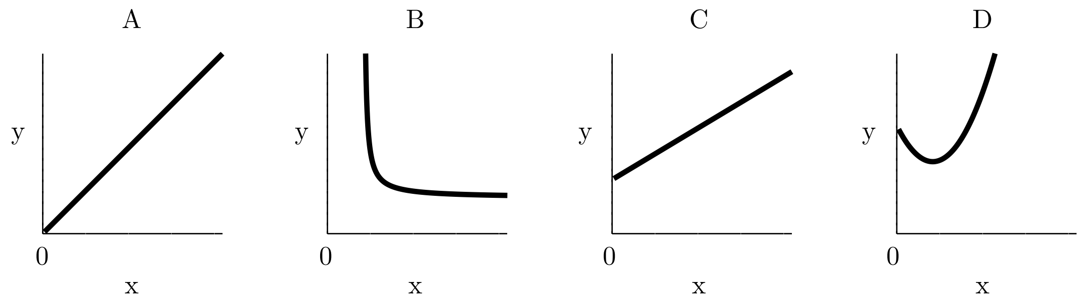

Figure 3.3.3.2

Which of these graphs

2. shows how the volume, y, of a fixed mass of gas at constant pressure varies with its temperature x, in K?
3. shows how the pressure, y, of a fixed mass of gas at constant temperature varies with its volume, x?
4. shows how the pressure, y, of a fixed mass of gas at constant volume varies with its temperature, x, in K?
5. shows how the volume, y, of a fixed mass of gas at constant temperature varies with its pressure, x?
6. shows how the pressure, y, of a fixed mass of gas at constant temperature varies with the reciprocal of its volume, x?
7. Gay-Lusscac's law best applies to which type of process?

   A isometric B isobaric C isothermal D isolated

8. During an isometric process, which of the following properties remains unchanged?

   A pressure B volume C temperature D pressure inverse

9. During an isometric process involving an ideal gas, which of the following properties is most likely to change?

   A temperature B density C volume D mass

10. If, during a process, both the pressure and volume of a fixed mass of gas are doubled, the absolute temperature

    A doubles B is halved C is tripled D quadruples

GCE Paper 1 Solutions

1. D 2. A 3. B 4. A 5. B 6. A 7. A 8. B 9. A 10. D

GCE Paper 2 Questions

1. A driver checks the pressure in his car tyres in Bamenda to be 3 Pa when the temperature is 297 K. He then drives to Yaoundé and finds that the pressure there is 4 Pa. Assuming the volume of the tyres remains unchanged, calculate the absolute temperature of the air inside in Yaoundé. **(3 mks)**

....

Solution

Gay-Lussac's law is applied to this isometric process.

$$\text{applying Gay-Lussac's law for a process: } \frac{P_i}{\theta_i} = \frac{P_f}{\theta_f}$$

$$\text{turning final temperature into subject: } \theta_f = \theta_i \left(\frac{P_f}{P_i}\right)$$

$$\text{substituting known values: } \theta_f = (297\ K)\left(\frac{4\ Pa}{3\ Pa}\right)$$

$$\text{final answer: } \boxed{\theta_f = 396\ K}$$

---

2. (a) What effect does doubling the temperature of a gas have on its pressure at constant volume? **(3 mks)**
   (b) An enclosed cylinder contains 150 dm^3 of oxygen at a temperature of 303 K and a pressure of 1.5 atm. The gas is then compressed by a piston to a volume of 60 dm^3 while the temperature increases to 313 K. What is the final pressure of the gas? **(4 mks)**

....

Solution

(a) *Gay-Lussac's law is applied to this isometric process.*

$$\text{applying Gay-Lussac's law for a process: } \frac{P_i}{\theta_i} = \frac{P_f}{\theta_f}$$

$$\text{substituting final temperature as double initial temperature : } \frac{P_i}{\theta_i} = \frac{P_f}{2\theta_i}$$

$$\text{turning final pressure into subject: } P_f = 2\theta_i \left(\frac{P_i}{\theta_i}\right)$$

$$\text{cancelling initial temperature: } P_f = 2P_i$$

$$\text{final answer: } \boxed{\text{pressure is doubled}}$$

(b) *Given that the pressure, volume and temperature change during this process, the ideal gas law is used.*

$$\text{applying the ideal gas law for a process: } \frac{P_iV_i}{\theta_i} = \frac{P_fV_f}{\theta_f}$$

$$\text{turning final pressure into subject: } P_f = \frac{\theta_f}{V_f}\left(\frac{P_iV_i}{\theta_i}\right)$$

$$\text{substituting known values: } P_f = \frac{313\ K}{60\ dm^3}\left(\frac{(1.5\ atm)(150\ dm^3)}{303\ K}\right)$$

$$\text{final answer: } \boxed{P_f \approx 3.87\ atm}$$

---

3. Before being used to cook fufu corn, the air locked inside a pressure cooker pot has a pressure of 1 atm, a volume of 10 L and a temperature of 300 K. After cooking, before the pot is opened, the same mass of air has a pressure of 1.30 atm.

   (a) State and explain whether the air has undergone an isothermal, isobaric or isometric process. Ignore any changes in the volume of the fufu. **(2 mks)**
   (b) Calculate the air's final temperature in K. **(3 mks)**

. . . .

Solution

(a) The process is isometric. Since the air expands to the volume of its container, and the size of the container is the same throughout cooking, the volume of the air itself is also constant. Therefore, $V_i = V_f$.

(b) *Gay-Lussac's law is applied to this isometric process.*

$$\text{applying Gay-Lussac's law for a process: } \frac{P_i}{\theta_i} = \frac{P_f}{\theta_f}$$

$$\text{turning final temperature into subject: } \theta_f = \theta_i \left(\frac{P_f}{P_i}\right)$$

$$\text{substituting known values: } \theta_f = (300\ K)\left(\frac{1.30\ atm}{1\ atm}\right)$$

$$\text{final answer: } \boxed{\theta_f = 390\ K}$$

---

4. (a) A fixed mass of gas has a volume of 600 cm^3, a temperature of 300 K and a pressure of 1.2 atm. Calculate its volume when its pressure rises to 2 atm and its temperature rises to 373 K. **(4 mks)**
   (b) Once the gas is at this temperature of 373 K, explain how it could be cooled without having heat removed from it. **(3 mks)**

. . . .

Solution

(a) *Given that the pressure, volume and temperature change during this process, the ideal gas law is used.*

$$\text{applying the ideal gas law for a process: } \frac{P_iV_i}{\theta_i} = \frac{P_fV_f}{\theta_f}$$

$$\text{turning final volume into subject: } V_f = \frac{\theta_f}{P_f}\left(\frac{P_iV_i}{\theta_i}\right)$$

$$\text{substituting known values: } V_f = \frac{373\ K}{2\ atm}\left(\frac{(1.2\ atm)(600\ cm^3)}{300\ K}\right)$$

$$\text{final answer: } \boxed{V_f = 447.6\ cm^3}$$

(b) Given the ideal gas law $PV \propto \theta$, the gas can be cooled by reducing the pressure while keeping the volume the same or by decreasing the volume while keeping the pressure the same.

3.3.4 Fluid Pressure and Depth

Objectives

By the end of the lesson, students should be able to
1. explain how pressure in a fluid increases with depth.
2. explain the effects of fluid pressure on atmospheric pressure.
3. relate changes in elevation to pressure differences.
4. understand atmospheres as a unit of pressure.
5. describe an experiment to demonstrate atmospheric pressure.
6. explain the relationship between pressure and weather.

Fluid Pressure

- **Fluids** are any matter, particularly in the liquid or gaseous phase, that can flow.
- All fluids exert pressure on themselves as well as on the interior surface of their container.
- This pressure increases in magnitude with increasing depth below the surface of the fluid.
- As shown in figure 3.3.4.1, at any particular depth h, the pressure in the fluid is caused by the repulsive force of the molecules at its own depth pushing out as well as the force of the weight of the molecules above.

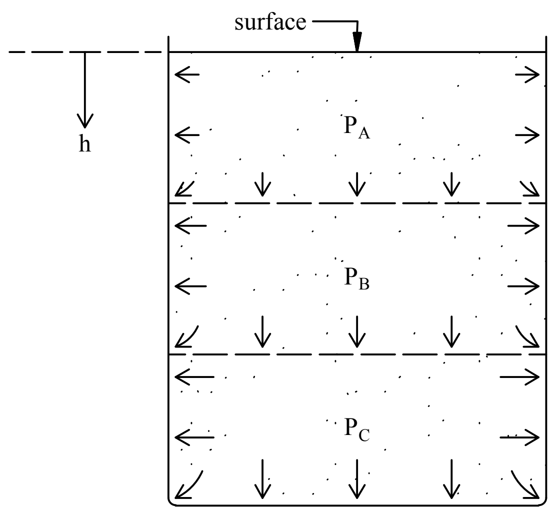

Figure 3.3.4.1

- Fluid pressure increases both with depth as well as the fluid's density.

$$P_f \propto h$$

$$P_f \propto \rho$$

$$P_f = \rho_f g h_f \tag{3.3.4.1}$$

Where
- P_f is the fluid pressure at particular depth, in Pa;
- ρ_f is the fluid's density, in $kg\ m^{-3}$;
- g is the acceleration of gravity, in $m\ s^{-2}$;
- h_f is the depth considered in the fluid, in m.

The acceleration of gravity, g, may be unfamiliar to students at this level. A brief explanation of the approximation $g = 10\ m\ s^{-2}$ should suffice, especially if the units are checked for dimensional homogeneity in equation 3.3.4.1.

Atmospheric Pressure

- Air is a gaseous fluid with a small but significant density.
- Therefore, given the height of the layer of air that composes the earth's atmosphere, there is a significant pressure near the surface of the earth.
- This is called **atmospheric pressure**.

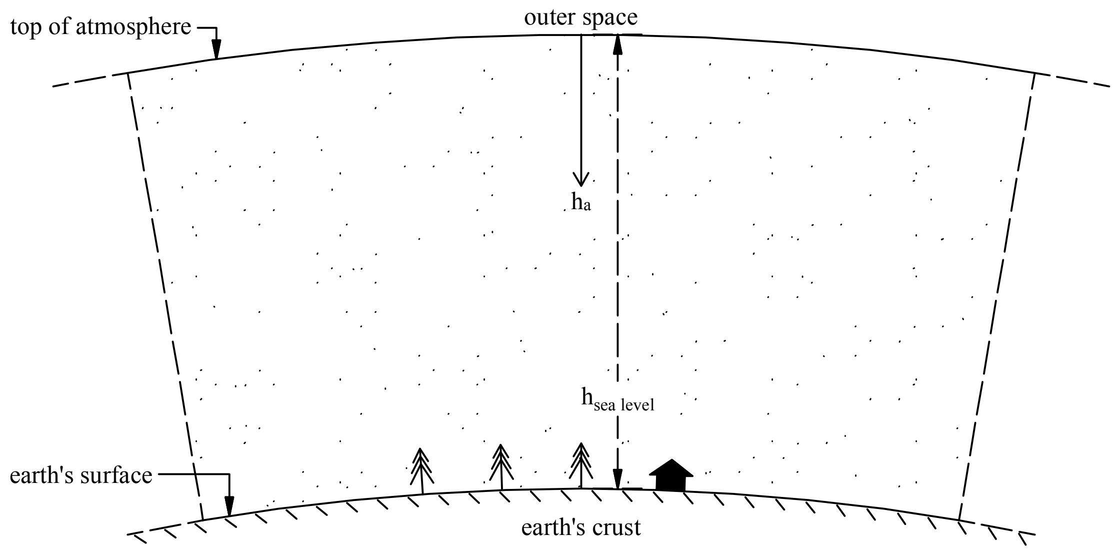

Figure 3.3.4.2

- This pressure increases with increasing depth in the atmospheric layer.
- That is, the highest atmospheric pressure is found at the lowest elevations.

$$P_a = \rho_a g h_a \tag{3.3.4.2}$$

Where
- P_a is atmospheric pressure at a particular depth, in Pa;
- ρ_a is the density of air, in $kg\ m^{-3}$;
- g is the acceleration of gravity, in $m\ s^{-2}$;
- h_a is the depth considered in the atmosphere, in m.

- The density of air is approximately 1.3 $kg\ m^{-3}$.
- The depth in the atmosphere increases with the opposite of elevation from sea level.

$$h_a = k - E$$

Where
- h_a is the depth considered in the atmosphere, in m;
- k is a constant of difference;
- E is the elevation, in m.

- Therefore, atmospheric pressure increases with the opposite of elevation.

given equation for atmospheric pressure: $P_a = \rho_a g h_a$

substituting equation for atmospheric depth: $P_a = \rho_a g(k - E)$

multiplying out: $P_a = \rho_a g k - p_a g E$

combining constant: $P_a = k - \rho_a g E$

- The typical value of P_a at sea level is 1.013×10^5 Pa, which is equivalent to one unit of atmospheres (atm).

- Considering a difference in atmospheric pressure removes the constant of difference.

$$\text{given equation for atmospheric pressure: } P_a = k - \rho_a g E$$

$$\text{considering a difference: } (P_a)_f - (P_a)_i = (k - \rho_a g E_f) - (k - \rho_a g E_i)$$

$$\text{simplifying: } (P_a)_f - (P_a)_i = -(\rho_a g)(E_f - E_i)$$

$$\text{final equation for difference: } \Delta P_a = -\rho_a g \Delta E \qquad (3.3.4.3)$$

Where
- ΔP_a is the difference in atmospheric pressure, in m;
- ρ_a is the density of air, in $kg\ m^{-3}$;
- g is the acceleration of gravity, in $m\ s^{-2}$;
- ΔE is the difference in elevation, in m.

Example: If the atmospheric pressure at sea level in Limbe is 1 atm and that in Ntumbaw is 0.743 atm, calculate the height of Ntumbaw above sea level.

$$\text{given equation for difference in atmospheric pressure: } \Delta P_a = -\rho_a g \Delta E$$

$$\text{turning elevation difference into subject: } \Delta E = -\frac{\Delta P_a}{\rho_a g}$$

$$\text{substituting known values: } \Delta E = -\frac{(0.743\ atm) - (1\ atm)}{(1.3\ kg\ m^{-3})(10\ m\ s^{-2})}$$

$$\text{applying conversion factor:} \Delta E = -\frac{-0.257\ atm}{(1.3\ kg\ m^{-3})(10\ m\ s^{-2})}\left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right)$$

$$\text{final answer: } \boxed{\Delta E \approx 2002.62\ m}$$

Experiment to Demonstrate Atmospheric Pressure

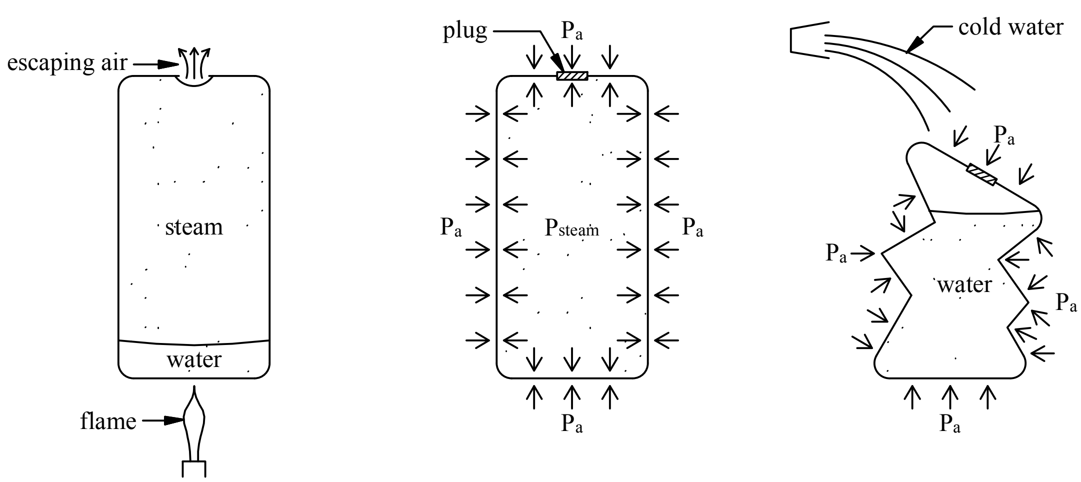

Figure 3.3.4.3

I Procedure
1 A small amount of water is placed in a metal can.
2 The can is heated, causing the water to turn to steam.
3 The expanding steam pushes out any air.
4 The can is then plugged.
5 Cold water is poured over the can.

II Observation
- The can collapses when the cold water is poured over it.

III Explanation
- Before the water is poured, the interior fluid pressure of the steam in the can was equal to the atmospheric pressure on the outside, so the can keeps its form.
- After the water is poured, the steam turns back to liquid, reducing the interior fluid pressure.
- This causes the exterior atmospheric pressure to be greater than the can's interior pressure.
- This difference in pressure pushes inwards on the can, causing it to implode.

IV Conclusion
- The can is crushed by some pressure pushing inwards on its exterior surface.
- This pressure must be atmospheric pressure.

Relationship between Pressure and Weather

- **Relative humidity**, or $\mathbf{R_H}$, is a measurement of the amount of water vapour in air.

- Relative humidity is an indication of the probability of rain.
  - A higher humidity indicates a higher chance of rain.
  - A lower humidity indicates a lower chance of rain.

- Atmospheric pressure is inversely proportional to humidity.

$$P_a = \frac{k}{R_H}$$

$$P_a \propto \frac{1}{R_H}$$

  Where
  - P_a is atmospheric pressure, in Pa;
  - k is a constant of proportionality;
  - R_H is the relative humidity (unit-less).

- Atmospheric pressure reduces significantly when rain is likely.

- Atmospheric pressure is also directly proportional to temperature.

$$P_a = k\theta$$

$$P_a \propto \theta$$

  Where
  - P_a is atmospheric pressure, in Pa;
  - k is a constant of proportionality;
  - θ is absolute temperature, in K.

- **Barometers** are instruments that measure atmospheric pressure.

- **Barometric altimeters** are devices that use the opposite - proportionality between height and atmospheric pressure to measure elevation.

- Barometric altimeters are useful tools for measuring large elevation differences. They are not accurate for smaller differences, such as the height of a building.

- Atmospheric pressure is rarely a good measurement of temperature because of the many other factors at play, such as elevation and relative humidity.

GCE Paper 1 Questions

1. The pressure of one atmosphere (1 atm) at sea level is the same as

   A 760 $mmHg$ B 700 $mmHg$ C 1 $mmHg$ D 750 $mmHg$

2. A form three student seeks to estimate the static pressure exerted by air on the floor of her bedroom. Besides knowing the density of air, what other measurement must she take?

   A the weight of air in the room
   B the surface area of the floor
   C the volume of the room
   D the height of the room

3. Air pressure at the foot of Mount Cameroon is equal 76 $cm\ Hg$ while at its top it is 36 $cm\ Hg$. The height of the mountain above sea level is:

   A 4080 m B 2280 m C 4100 m D 4000 m

4. If an instrument measuring atmospheric pressure shows a sudden drop in its reading with no change in elevation or temperature, which natural phenomenon is likely to occur?

   A earthquake B rain C landslide D water shortage

5. The instrument mentioned in question 4 is likely a

   A micrometre B balance scale C barometer D metre rule

6. Velma climbs a very tall mountain with a barometer. As she ascends, she is most likely to see the device's measurement

   A increase B decrease C remain unchanged D fluctuate cyclically

7. 76 $cm\ Hg$ is equal to all except for which of the following quantities of pressure?

   A $1.013 \times 10^5\ Pa$ B 1 atm C 0.1 atm D 760 $mmHg$

8. The atmospheric pressure in a village is 1.1 atm. Which of the following is least likely to be true?

   A Its elevation is less than that of the sea.
   B Its temperature is relatively high.
   C The density of its air is relatively high.
   D It is about to rain in there soon.

9. If the atmospheric pressure at the top of a house in Nkambe is 0.7000 atm and at the bottom it is 0.7004 atm, what is the approximate height of the building?

   A 3.117 m B 3.407 m C 4.117 m D 2.071 m

10. What's the best instrument to measure the height of the building in question 9?

    A tape measure B barometric altimeter C voltmeter D digital scale

GCE Paper 1 Solutions

1. A 2. D 3. C 4. B 5. C 6. B 7. C 8. D 9. A 10. A

GCE Paper 2 Questions

1. At a village just above sea level, the atmospheric pressure is 0.9 atm. State this value

   (a) in units of Pa. **(3 mks)**
   (b) in units of $mmHg$. **(2 mks)**
   (c) in units of $cmHg$. **(2 mks)**
   (d) in units of $inHg$, given $1\ in = 2.54\ cm$. **(2 mks)**

. . . .

Solution

(a) *The unit equivalence of* $1\ atm = 1.013 \times 10^5\ Pa$ *is used.*

$$\text{given value in atm: } P_a = 0.9\ atm$$

$$\text{applying conversion factor: } P_a = 0.9\ atm\left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right)$$

$$\text{final answer: } \boxed{P_a = 9.117 \times 10^4\ Pa}$$

(b) *The unit equivalence of* $1\ atm = 760\ mmHg$ *is used.*

$$\text{given value in atm: } P_a = 0.9\ atm$$

$$\text{applying conversion factor: } P_a = 0.9\ atm\left(\frac{760\ mmHg}{1\ atm}\right)$$

$$\text{final answer: } \boxed{P_a = 684\ mmHg}$$

(c) *A quantity in $mmHg$ is converted into $cmHg$ simply by converting mm to cm.*

$$\text{given value in mmHg: } P_a = 684\ mmHg$$

$$\text{applying conversion factor: } P_a = (684\ mmHg)\left(\frac{1\ cm}{10\ mm}\right)$$

$$\text{final answer: } \boxed{68.4\ cmHg}$$

(d) *This is again simply a length conversion.*

$$\text{given value in cmHg: } P_a = 68.4\ cmHg$$

$$\text{applying conversion factor: } P_a = (68.4\ cmHg)\left(\frac{1\ in}{2.54\ cm}\right)$$

$$\text{final answer: } \boxed{P_a \approx 26.93\ inHg}$$

---

2. The atmospheric pressure at the top of one mountain is 0.6 atm. The same property on the top of another mountain is 0.5 atm. Calculate the difference in elevation between the mountains' summits

   (a) in metres. **(3 mks)**
   (b) in kilometres. **(2 mks)**
   (c) in millimetres. **(2 mks)**

. . . .

Solution

(a) *Any differences in temperature and relative humidity between the two summits are assumed negligible.*

$$\text{given equation for difference in atmospheric pressure: } \Delta P_a = -\rho_a g \Delta E$$

$$\text{turning elevation difference into subject: } \Delta E = -\frac{\Delta P_a}{\rho_a g}$$

$$\text{substituting known values: } \Delta E = -\frac{(0.6\ atm - 0.5\ atm)}{(1.3\ kg\ m^{-3})(10\ m\ s^{-2})}$$

$$\text{applying conversion factor: } \Delta E = -\frac{0.1\ atm}{(1.3\ kg\ m^{-3})(10\ m\ s^{-2})}\left(\frac{1.013 \times 10^5\ Pa}{1\ atm}\right)$$

$$\text{final answer in metres: } \boxed{\Delta E \approx -779.231\ m}$$

That is, the summit with the atmospheric pressure of 0.6 atm *is about* 779.231 m *<u>lower</u> than the summit whose atmospheric pressure is* 0.5 atm.

(b) *This is simply a length conversion.*

$$\text{given elevation difference in metres: } \Delta E \approx -779.231\ m$$

$$\text{applying conversion factor: } \Delta E \approx (-779.231\ m)\left(\frac{1\ km}{10^3\ m}\right)$$

$$\text{final answer in kilometres: } \boxed{\Delta E \approx 0.779231\ km}$$

(c) *This is again a length conversion, but to a smaller unit.*

$$\text{given elevation difference in metres: } \Delta E \approx -779.231\ m$$

$$\text{applying conversion factor: } \Delta E \approx (-779.231\ m)\left(\frac{10^3\ mm}{1\ m}\right)$$

$$\text{final answer in millimetres: } \boxed{\Delta E \approx -7.79231 \times 10^5 mm}$$

It should be emphasized which units are most and least convenient for measuring distances this large.

3.3.5 Liquid Pressure and Depth

Objectives

By the end of the lesson, students should be able to

1. define liquid density.
2. describe an experiment to demonstrate that liquid pressure increases with depth.
3. be able to solve problems relating liquid pressure to height/depth, liquid density and the acceleration of gravity.
4. list and explain hazards of high altitudes and deep sea diving and state appropriate precautions.
5. explain the difference between absolute, liquid and atmospheric pressure.

Liquid Pressure

- **Liquid density**, or $\rho_\mathbf{l}$, is the mass of a given body of liquid per its volume.

- **Liquid pressure**, or $\mathbf{P_l}$, is the product of depth in a liquid, the liquid's density and the acceleration of gravity.

$$P_l = \rho_l g h_l \tag{3.3.5.1}$$

  Where
  - P_l is the liquid pressure, in Pa;
  - ρ_l is the liquid's density, in $kg\ m^{-3}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - h_l is the depth considered in the liquid, in m.

- Liquid pressure is caused by the force of a liquid's molecules against each other or the interior surface of the liquid's container per unit area.

- **Hydrostatic pressure** is the fluid pressure a liquid exhibits while at rest.

- The hydrostatic pressure
  - in a given liquid increases with depth.
  - in different liquids at the same depth increases with density.
  - at any point in a liquid is the same in all directions.
  - is transmitted, undiminished, throughout its body.
  - does not depend on the shape of the container.

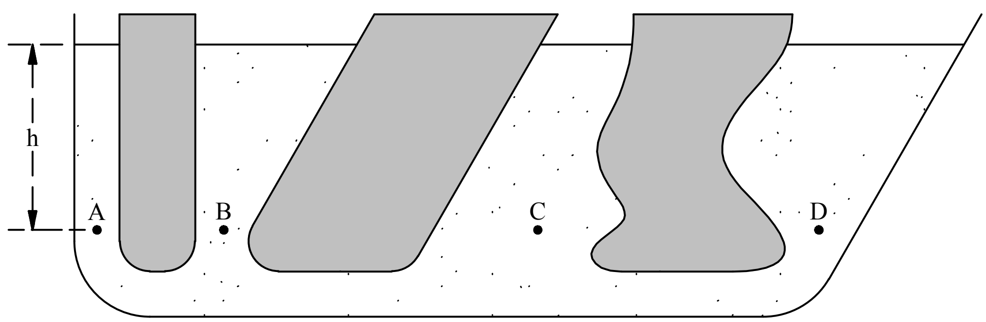

Figure 3.3.5.1

- Regarding figure 3.3.5.1, the hydrostatic pressure is the same at points A, B, C and D given their similar depth in the liquid.

$$P_{lA} = P_{lB} = P_{lC} = P_{lD}$$

$$\rho_l g h_A = \rho_l g h_B = \rho_l g h_C = \rho_l g h_D$$

$$h_A = h_B = h_C = h_D$$

Applications of Liquid Pressure's Proportionality to Depth

- A dam is often built such that its base is thicker than its top. This is because the water it retains applies more liquid pressure at its foot than at its tip.
- Deep sea divers must wear specialised suits to avoid being crushed by the extensive liquid pressure at lower depths.
- In multiple-floor building, water runs faster from taps on lower floors than higher ones.

Experiment to Demonstrate that Liquid Pressure Increases with Depth

I Procedure
   1 Three holes are cut at three different heights in a tall vessel.
   2 The vessel is continually filled with water such that the column of water inside maintains a constant height.

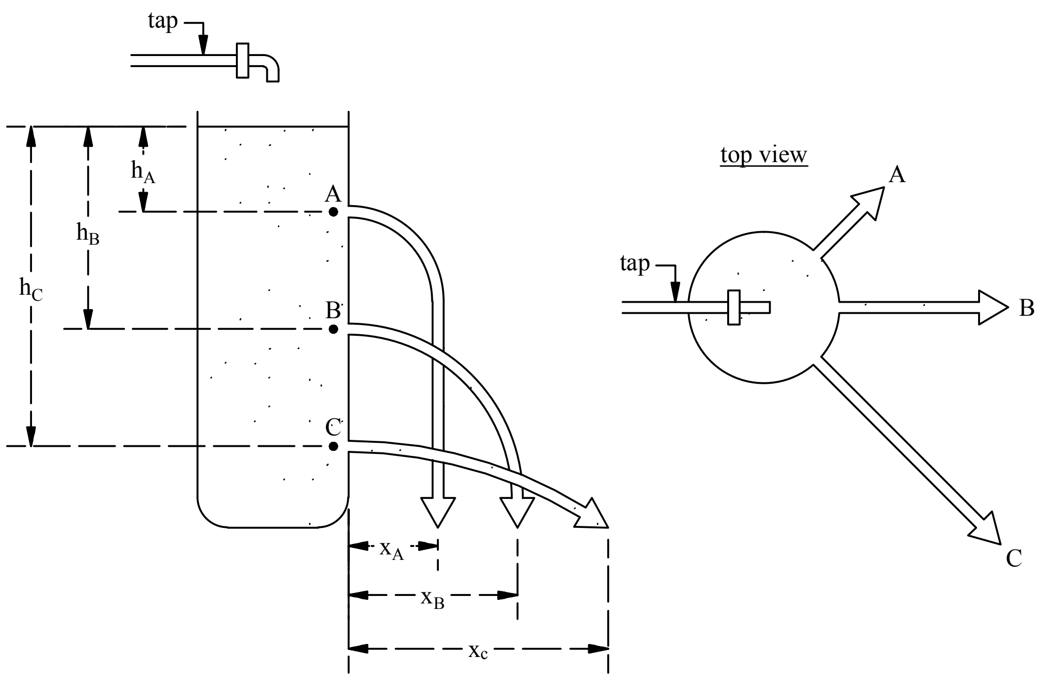

Figure 3.3.5.2

II Observations
- The highest hole (A) at the shallowest depth below the surface (h_A) produces a weak jet of water that shoots out a short horizontal distance (x_A).
- The next hole down (B) at the next greatest depth below the surface (h_B) produces a stronger jet that shoots out a farther distance (x_B).
- The lowest hole (C) at the greatest depth below the water's surface (h_C), produces the strongest jet that shoots out the farthest distance (x_C).

III Conclusion
- The liquid pressure increases with the depth in the liquid.

IV Precaution
- A shown on the right-hand side of figure 3.3.5.2, the holes in the vessel are staggered so as to avoid a collision of water jets.

Calculations of Liquid Pressure

- Problems involving liquid pressure can require that one determine the liquid pressure, liquid density, or depth with the other two properties being given.

- Unless otherwise specified,
  - the density of water is 1000 $kg\ m^{-3}$.
  - the acceleration of gravity is $g = 10\ m\ s^{-2}$.

  Example: Calculate the liquid pressure a diver experiences 7 m under the water's surface. Express this value in both Pascals as well as atmospheres.

  $$\text{given equation for liquid pressure: } P_l = \rho_l g h_l$$

  $$\text{substituting known values: } P_l = \left(1000\ kg\ m^{-3}\right)\left(10\ m\ s^{-2}\right)(7\ m)$$

  $$\text{final answer in Pascals: } \boxed{P_l = 7 \times 10^4\ Pa}$$

  $$\text{applying conversion factor: } P_l = \left(7 \times 10^4\ Pa\right)\left(\frac{1\ atm}{1.013 \times 10^5\ Pa})\right)$$

  Example: At a certain depth below the water's surface in a lake, the liquid pressure is measured to be 30000 Pa. Calculate the depth at which this measurement was taken.

  $$\text{given equation for liquid pressure: } P_l = \rho_l g h_l$$

  $$\text{turning liquid depth into subject: } h_l = \frac{P_l}{\rho_l g}$$

  $$\text{substituting known values: } h_l = \frac{30000\ Pa}{\left(1000\ kg\ m^{-3}\right)\left(10\ m\ s^{-2}\right)}$$

  $$\text{final answer: } \boxed{h_l = 3\ m}$$

Hazards of High Altitudes and Deep Sea Diving

- High Altitudes
  - Mountain climbers must bring additional oxygen in pressurized tanks so that they can breath despite the low air pressure at high altitudes. Without these tanks, the climbers would faint and/or suffocate.
  - An aircraft must be sealed tightly in order to maintain higher, sea-level pressures at greater altitudes where the atmospheric pressure is significantly less. Without this seal, passengers would faint and/or suffocate.

- Deep Sea Diving
  - Deep sea divers must where special suits capable to withstanding the high pressures encountered at low depths. Without these suits, a deep sea diver would be crushed to death.
  - Submarines must be well-sealed in order to maintain lower, sea-level pressures as lower depths where the liquid pressure is significantly greater. Without this seal, water would rush in violently into the vessel.

Absolute Pressure

- The **absolute pressure**, or $\mathbf{P_0}$, in a liquid, as shown in figure 3.3.5.3, is the sum of the hydrostatic/liquid pressure as a function of depth as well as the atmospheric pressure at the liquid's surface.

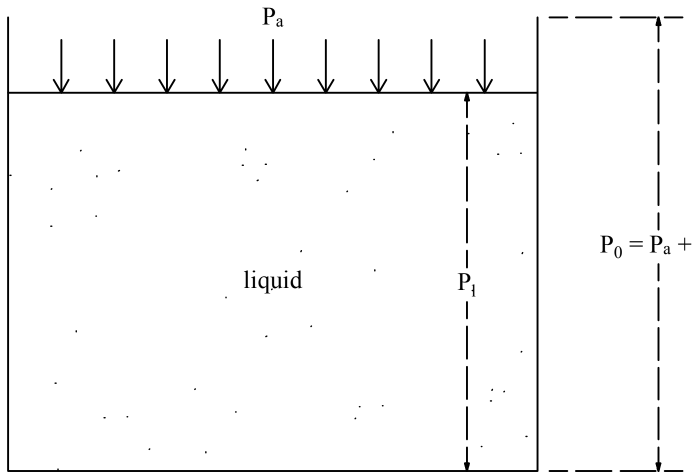

Figure 3.3.5.3

$$P_0 = P_a + P_l \tag{3.3.5.2}$$

Where
- P_0 is the absolute pressure in the liquid, in Pa;
- P_a is the atmospheric pressure at the liquid's surface, in Pa;
- P_l is the liquid pressure, in Pa.

GCE Paper 1 Questions

1. In administering a drip to a patient in a hospital, the drip has to be raised to a certain height above the patient where the drip pressure will

   A be far above the blood pressure.
   B just be above the blood pressure.
   C be less than the blood pressure.
   D be equal to the blood pressure.

2. Figure 3.3.5.4 shows an oil jar with holes P and Q at different heights from the base.

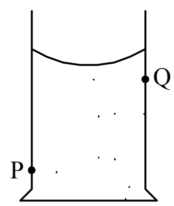

Figure 3.3.5.4

Which of the following statements is true?

   A The pressure at P is greater than that at Q.
   B The pressure at Q is lower than atmospheric.
   C Atmospheric pressure does not affect the pressure at P.
   D An oil jet from Q falls further from the base than an oil jet from P.

Questions 3 through 4 refer to figure 3.3.5.5, which shows a large tank partially filled with mercury and left open to atmospheric pressure such that the absolute pressure at P is equal to 100000 Pa. The density of mercury is given to be 13600 $kg\ m^{-3}$.

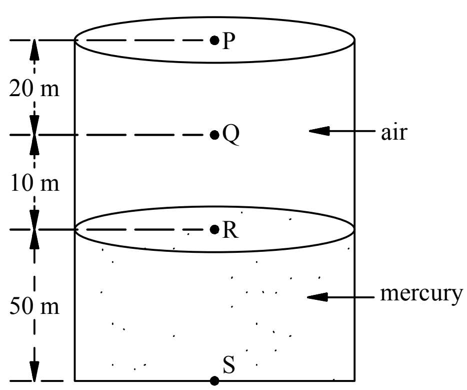

Figure 3.3.5.5

3. Which of the following statements is true? The absolute pressure at

   A P is greater than that at Q.
   B P is greater than that at S.
   C P is equal to that at Q.
   D Q is greater than that at P.

4. The absolute pressure at S is approximately

   A 13.6 kPa
   B 10 kPa
   C 6900 kPa
   D 6810 kPa

5. Given the acceleration of gravity is 10 $m\ s^{-2}$, the hydrostatic pressure exerted at a point 6 m below the surface of fresh water of density 1000 $kg\ m^{-3}$ is approximately

   A 60000 Pa B 6000 Pa C 600000 Pa D 10000 Pa

6. Which of the following best describes the pressure of a liquid at rest?

   A electric B hydrostatic C fluid D atmospheric

7. A bath of water varies in depth from 0.2 m at the shallow end to 0.3 m at the deep, plug-hole end. If the density of the water inside is 1000 $kg\ m^{-3}$, and the acceleration due to gravity is 10 $m\ s^{-2}$, the pressure of water acting on the plug in Pa is

   A 5000 B 1000 C 3000 D 2000

8. If an aircraft reaches a cruising altitude (typical flying height), and it isn't sealed properly,

   A the interior pressure will be maintained at a value of $20,000$ atm.
   B no serious problems will occur.
   C outside air will rush in.
   D inside air will burst out.

9. If a submarine dives deep underwater and it isn't sealed properly,

   A it will get attacked by a shark.
   B no serious problems will occur.
   C outside water will rush in.
   D inside air will burst out.

10. The atmospheric pressure in a given location is measured to be 1.1 atm. Assuming temperature, density and humidity effects are negligible, the location is likely

   A above sea level.
   B below sea level.
   C on the surface of the moon.
   D on top of Mount Cameroon.

GCE Paper 1 Solutions

1. B 2. A 3. D 4. C 5. A 6. B 7. C 8. D 9. C 10. B

GCE Paper 2 Questions

1. (a) Figure 3.3.5.6 shows a tall drum containing water. On it are four equally-sized holes, labelled A, B, C and D as shown.

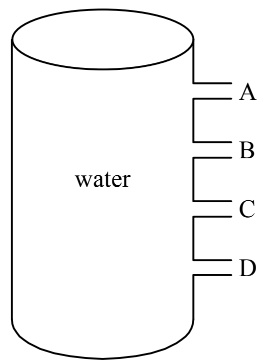

Figure 3.3.5.6

(i) State from which of these holes, A, B, C or D, will the water shoot out the furthest horizontally from the drum. Explain why. **(2 mks)**

(ii) State and explain an application of this. **(4 mks)**

(b) From your understanding of liquid pressure, explain why dams are constructed such that they are thicker at the base than at the top. **(2 mks)**

. . . .

Solution

(a) (i) The jet will shoot the furthest from hole D. This is because hydrostatic pressure is proportional to depth, and D is the hole at the lowest depth in the drum. This highest pressure will cause the the liquid to shoot from the drum with the highest force, resulting in the longest horizontal jet.

(ii) *The following is a non-exhaustive list of examples of applications of increasing hydrostatic pressure with depth. Only one is required.*

- Damns must be built such that their bases are thicker than their tops, or crests. This is because the pressure in the liquid they retain increases with their depth, so their strength must increase accordingly.

- Deep sea divers must wear pressurized suits with diving bells which warn them when they have reached a certain depth, below which they may be crushed by the higher pressure.

- In high-rise buildings, water rushes out of the lower floor's taps than those on the higher floors. This must be corrected with special plumbing equipment.

(b) The hydrostatic pressure of the liquid retained by a dam is higher at its base than at its top since the depth of the water is greater at its bottom. This requires a damn to have additional strength at its bottom, which is accomplished by having a thicker base.

---

2. A student placed water in an open can and heated it until it began to boil. He then removed the can from the heat source, sealed it tightly by screwing on its cap and then poured cold water over it.

(a) What does the steam do to the air inside the can and to the walls of the can? **(2 mks)**

(b) State and explain what happens to the can as cold water is poured on it. **(3 mks)**

(c) A 0.84 m tall jar is 75% full of a liquid of density 1000 $kg\ m^{-3}$. Calculate the liquid pressure exerted on the bottom of the jar. **(2 mks)**

(d) The jar is then emptied and filled with paraffin wax having a density of 900 $kg\ m^{-3}$. It is observed that the bottom of the jar experiences the same liquid pressure as in (c) above. The depth of the wax occupies what percentage of the jars height?

. . . .

Solution

(a) The steam exerts pressure on both the air as well as the interior surface of the can's walls.

(b) As cold water is poured over it, the steam inside the can turns back into liquid. This causes the interior pressure to decrease to a value lower than the exterior pressure. This inward-facing pressure difference causes the can to collapse.

(c) *The depth used to calculate the hydrostatic pressure is the depth of the liquid, not the jar.*

$$\text{given equation for liquid pressure: } P_l = \rho_l g h_l$$

$$\text{considering liquid's depth: } P_l = \rho_l g \left(\left(\frac{75}{100} \right) h_{\text{jar}} \right)$$

$$\text{substituting known values: } P = \left(1000\ kg\ m^{-3}\right)\left(10\ m\ s^{-2}\right)\left(\left(\frac{75}{100} \right) (0.84\ m) \right)$$

$$\text{final answer: } \boxed{\rho = 6300\ Pa}$$

(d) *Given the lower density, a taller liquid column is needed to achieve the same pressure at the bottom.*

$$\text{given equation for liquid pressure: } P_l = \rho_l g h_l$$

$$\text{turning depth into subject: } h_l = \frac{P_l}{\rho_l g}$$

$$\text{considering ratio of wax depth and total jar height: } \%\ \text{filled} = \frac{h_l}{h_{jar}}$$

$$\text{substituting equation for wax's depth: } \%\ \text{filled} = \frac{\frac{P_l}{\rho_l g}}{h_{jar}}$$

$$\text{substituting known values: } \%\ \text{filled} = \frac{\frac{6300\ Pa}{(900\ kg\ m^{-3})\,(10\ m\ ^{-2})}}{0.84\ m}$$

$$\text{final answer: } \boxed{\%\ \text{filled} \approx 83\%}$$

3. The hydrostatic pressure was measured at different depths in a certain liquid and the results were recorded in the table below.

pressure / $N\ m^{-2} \times 10^5$	1.08	1.16	1.24	1.32	1.40	1.48	1.56	1.64
depth / m	1.0	2.0	3.0	4.0	5.0	6.0	7.0	8.0

(a) Plot a graph of pressure along the y-axis against depth along the x-axis. **(4 mks)**
(b) Find the gradient of the graph. **(2 mks)**
(c) Determine the pressure when the depth is zero and state what this represents. **(2 mks)**
(d) Find the density ρ of the liquid if the acceleration of gravity, g, is given to be 10 $m\ s^{-2}$. **(2 mks)**

Solution

(a) *See figure 3.3.5.7*

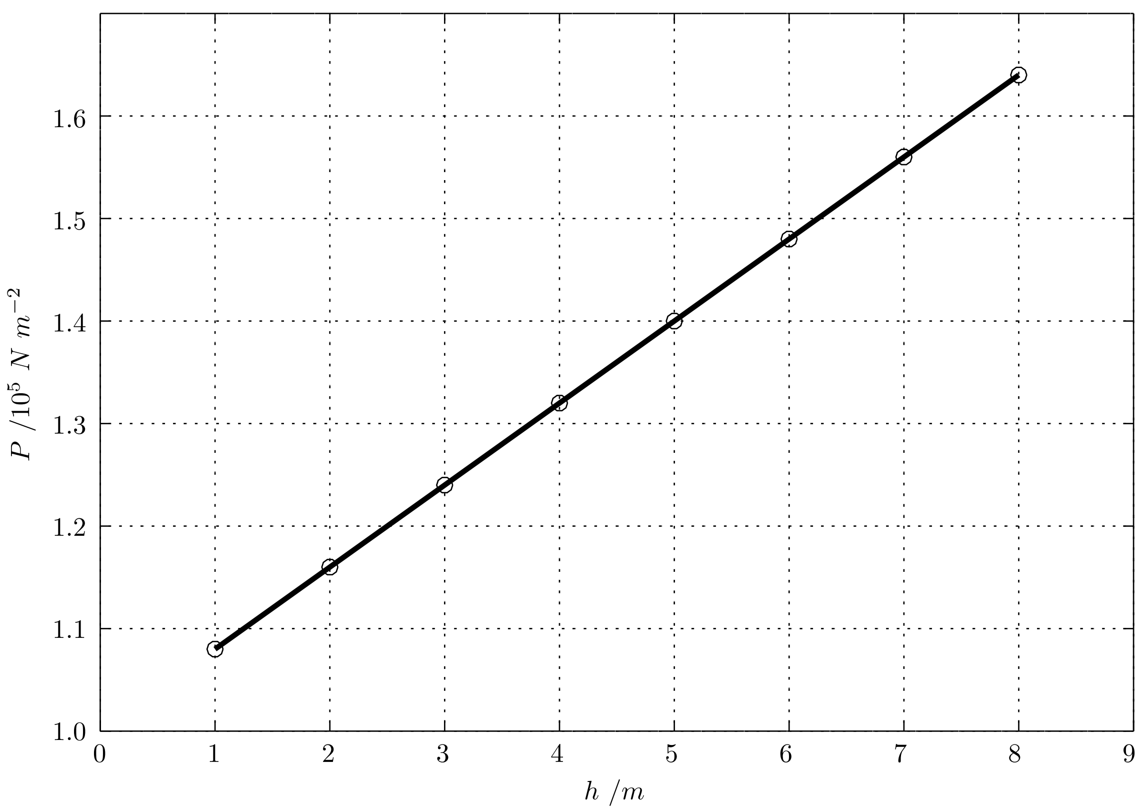

Figure 3.3.5.7

(b) *The gradient, slope, or proportionality is most easily calculated as the difference in the first and last pressure values per the difference in the first and last depths.*

$$\text{calculating gradient:} \quad k = \frac{\Delta P}{\Delta h}$$

$$\text{substituting first and last data points:} \quad k = \frac{(1.64 \times 10^5\ N\ m^{-2}) - (1.08 \times 10^5\ N\ m^{-2})}{8.0\ m - 1.0\ m}$$

$$\text{final answer:} \quad \boxed{\text{gradient} = k = 8000\ N\ m^{-3}}$$

(c) *While any data point works for the solution, the point $h = 1\ m$ and $P = 1.08 \times 10^5\ N\ m^{-2}$ is used.*

$$\text{assuming a linear relationship: } P = kh + P_{h=0m}$$

$$\text{turning zero-depth pressure into subject: } P_{h=0m} = P - kh$$

$$\text{substituting known slope/gradient: } P_{h=0m} = P - \left(8000\ N\ m^{-3}\right) h$$

$$\text{substituting first data point: } P_{h=0m} = 1.08 \times 10^5\ N\ m^{-2} - \left(8000\ N\ m^{-3}\right)(1.0\ m) + P_0$$

$$\text{final answer: } \boxed{P_{h=0m} = 1 \times 10^5\ N\ m^{-2} \text{ or } 100000\ Pa}$$

$P_{h=0m}$ represents the atmospheric pressure at the liquid's surface.

(d) *Given the equation for liquid density $P_l = \rho_l g h_l$, the gradient of a graph of P_l vs h_l is equal to ρg. That is, $k = \rho_l g$. Thus, the density is found by taking this equation and turning ρ_l into the subject.*

$$\text{gradient of graph: } k = \rho_l g$$

$$\text{turning density into subject: } \rho_l = \frac{k}{g}$$

$$\text{substituting known values: } \rho_l = \frac{8000\ N\ m^{-3}}{10\ m\ s^{-2}}$$

$$\text{final answer: } \boxed{\rho = 800\ kg\ m^{-3}}$$

3.3.6 Pressure inside Containers

Objectives

By the end of the lesson, students should be able to
1. explain how mercury barometers work.
2. explain how u-tube manometers work.
3. state Pascal's principle.
4. describe how Pascal's principle applies to hydraulic jacks.

Liquid Barometers

- When a liquid is placed in a scaled cylinder and any gaseous material such as air is removed, it can be inverted and used to measure atmospheric pressure.

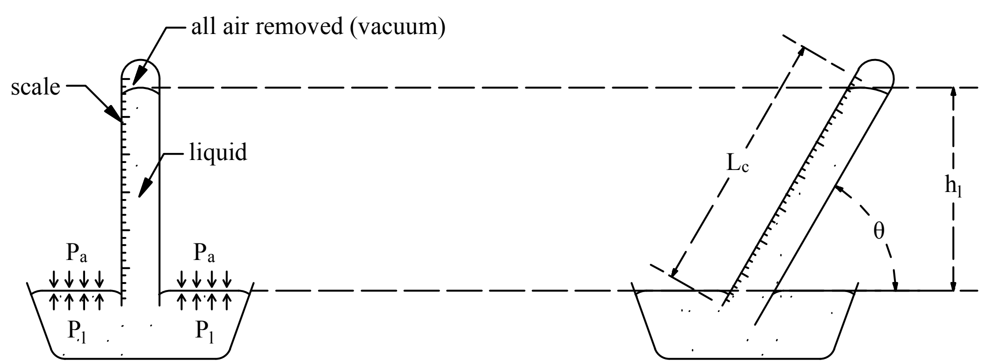

Figure 3.3.6.1

- As shown on the left of figure 3.3.6.1, since the pressure at the bottom of liquid column is assumed to be entirely due to the liquid's pressure, with no additional gaseous pressure.

assuming equilibrium at bottom pool's surface: $P_a = P_l$

substituting given equation for liquid pressure: $P_a = \rho_l g h_l$

scale on side of column can be used to measure atmospheric pressure: $P_a \propto h_l$

$$P_a = kh_l$$

Where
  - P_a is the surrounding atmospheric pressure, in Pa;
  - ρ_l is the column's liquid density, in $kg\ m^{-3}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - k is the proportionality constant;
  - h_l is the vertical height of the liquid column, in m.

- If the liquid inside inside such a barometer is mercury, and the surrounding atmospheric pressure is 1 atm, the column's height is 0.76 m, 760 mm or 76 cm.

- Thus, both millimetres of mercury ($mmHg$) and centimetres of mercury ($cmHg$) are units of pressure, with the unit equivalence 1 atm = 760 $mmHg$ = 76 $cmHg$ = $1.013 \times 10^5 Pa$.

- As shown on the right of figure 3.3.6.1, the scale on the side of a liquid barometer is only valid if the column is perfectly vertical.

- If the liquid column is not vertical, both its vertical height and the atmospheric pressure must be calculated considering the angle.

$$\text{applying trigonometry to column length: } h_l = \sin(\theta) L_c$$

$$\text{substituting adjusted column length into pressure formula: } P_a = \rho_l g \sin(\theta) L_c$$

Where
  - P_a is the surrounding atmospheric pressure, in Pa;
  - ρ_l is the column's liquid density, in $kg\ m^{-3}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - θ is the angle of the column against horizontal, in degrees;
  - L_c is the total length of the liquid column, in m.

U-tube Manometers

- U-tube manometers use the principle of hydrostatic pressure to measure a pressure difference.

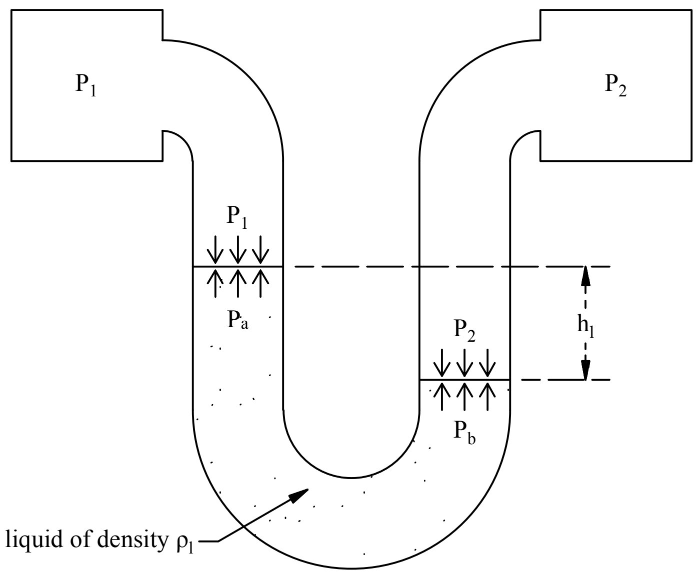

Figure 3.3.6.2

- As shown in figure 3.3.6.2 the difference in absolute pressures between two regions is directly proportional to the height difference of a liquid in a U-tube.

$$\text{considering difference in surface pressures as difference in liquid pressure: } P_b = P_a + \rho_l g h_l$$

$$\text{assuming equilibrium at both liquid surfaces: } P_a = P_1, \quad P_b = P_2$$

$$\text{applying pressure equivalences at surfaces: } P_2 = P_1 + \rho_l g h_l$$

$$\text{turning pressure difference into subject: } \Delta P = P_2 - P_1 = \rho_l g h_l \quad (3.3.6.1)$$

Where
  - ΔP is the pressure difference between the liquid's two surfaces, in Pa;
  - P_1 is the pressure to which one end of the U-tube is connected, in Pa;
  - P_2 is the pressure to which the other end of the U-tube is connected, in Pa;
  - ρ_l is the density of the liquid used in the manometer, in $kg\ m^{-3}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - h_l is the difference in heights of the liquid's two surfaces, in m.

- NB: The lower surface indicates the higher pressure. For example, in figure 3.3.6.2, the pressure labelled P_2 is greater than that labelled P_1.

- NB: Any difference in the chambers' heights has no effect on the measurement read. For example, all three pressure differences shown in figure 3.3.6.3, have the same value. That is $P_2 - P_1 = P_4 - P_3 = P_6 - P_5 = \rho_l g h_l$.

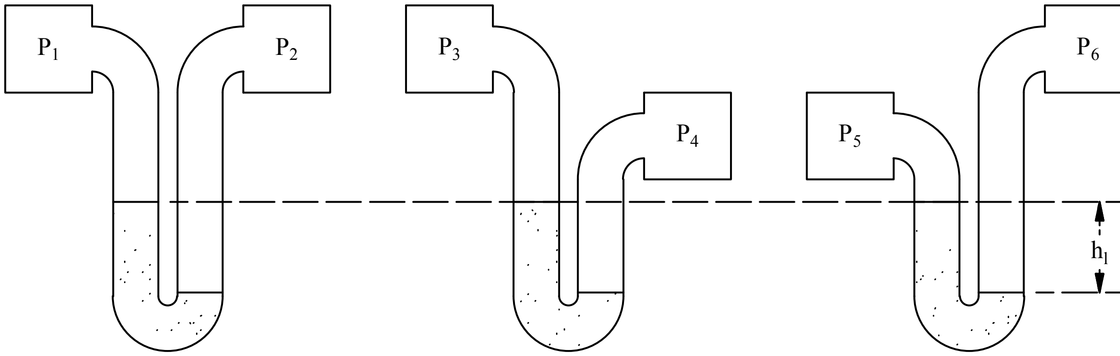

Figure 3.3.6.3

- A u-tube manometer can be used to measure the pressure difference between a container and the atmosphere.

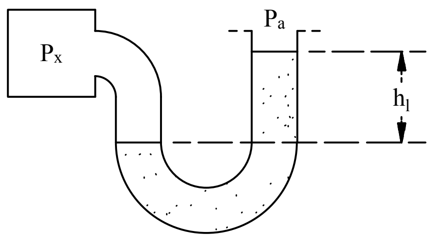

Figure 3.3.6.4

- The set-up shown in figure 3.3.6.4, can therefore be used to measure a container's pressure against the atmosphere.

$$\text{considering pressure difference: } \Delta P = \rho_l g h_l = P_x - P_a$$

$$\text{turning container's unknown pressure into subject: } P_x = \rho_l g h_l + P_a \quad (3.3.6.2)$$

Where
  - P_x is the unknown pressure being measured, in Pa;
  - ρ_l is the density of the liquid used in the manometer, in $kg\ m^{-3}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - h_l is the difference in heights of the liquid's two surfaces, in m;
  - P_a is the atmospheric pressure, in Pa.

- P_x, which includes atmospheric pressure, is the absolute pressure.

- Often, a pressure measuring instrument will only display the difference between the pressure of some enclosed container and that of the atmosphere. This is called gauge pressure.

$$P_g = P_x - P_a \quad (3.3.6.3)$$

Where
  - P_g is the gauge pressure displayed, in Pa;
  - P_x is the absolute pressure of the container, in Pa;
  - P_a is the atmospheric pressure, in Pa.

- NB: While absolute pressure can never be negative, gauge pressure certainly can. This occurs when the pressure inside a container is less than atmospheric pressure.

Incompressibility of Liquids

- Liquid is almost entirely incompressible.
- **Pascal's principle** states that the pressure applied at one point on an enclosed fluid is transmitted, undiminished, to every part of the fluid and to the walls of the container.
- Practical devices that make use of a liquid's incompressibility include
  - hydraulic jacks
  - hydraulic brakes
  - medical syringes
  - U-tube manometers

Hydraulic Jacks

- As shown in figure 3.3.6.5, a hydraulic jack is one application of Pascal's principle.

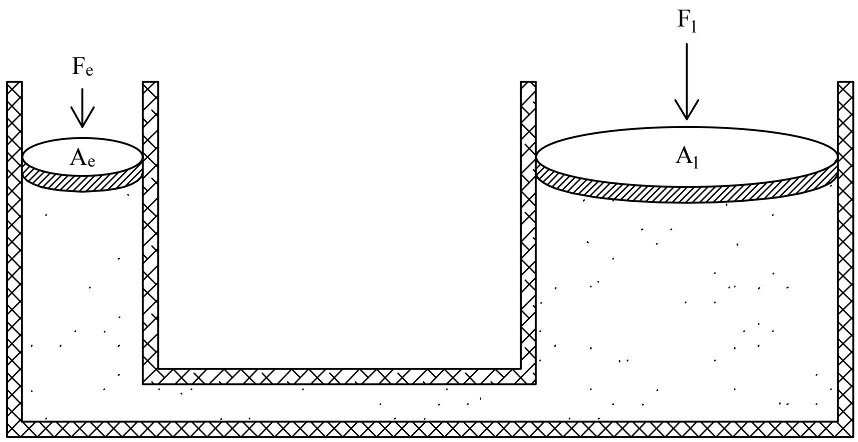

Figure 3.3.6.5

- A force applied on the effort piston creates a pressure which is applied, undiminished, to the load piston.
- The subscript $_e$ is used for effort properties and while the subscript $_l$ is used for load properties.
- This allows for the load force to be significantly greater than the effort force.

$$\text{applying Pascal's principle: } P_e = P_l$$

$$\text{substituting equation for pressure: } \frac{F_e}{A_e} = \frac{F_l}{A_l}$$

$$\text{turning force ratio into subject: } \frac{F_l}{F_e} = \frac{A_l}{A_e}$$

- Thus, a larger load area allows for a larger lifting (load) force.
- Liquid are preferred as the fluid in a hydraulic jack as opposed to gases. This is because liquids
  - are almost entirely incompressible, allowing almost all effort energy to be used at the load.
  - have a density that varies very little with depth.
  - transmit pressure from effort to load instantly.
  - expand very little with increasing temperature.
- Oil is preferred as the liquid in a hydraulic jack as opposed to water. This is because oil
  - forms less bubbles than water.
  - causes less rust then water.
  - is more viscous (thicker) than water.
  - is even less compressible than water.

GCE Paper 1 Questions

1. Hydraulic machines use oil instead of water as the interior liquid

   A oil is a lubricant B oil prevents rust C oil is incompressible D oil is viscous

2. Which of the following is useful in a hydraulic lift?

   A Archimedes' principle
   B the principle of moments
   C the principle of work
   D Pascal's principle

3. In a hydraulic jack, an effort of 20 N is applied to an area of 0.0001 m^2 to raise an object resting on a load piston whose area is 0.002 m^2. If the jack is not faulty, the pressure on the load is

   A $1 \times 10^4\ Pa$ B $2 \times 10^5\ Pa$ C $1 \times 10^8\ Pa$ D $4 \times 10^{-6}\ Pa$

4. Which of the following is not true about liquid pressure in an open container?

   A Pressure depends on the shape of the container.
   B Pressure acts in all directions.
   C Pressure increases with depth.
   D Pressure depends on the density of the liquid.

Questions 5 through 7 refer to figure 3.3.6.6, which shows a hydraulic jack with an effort and a load piston.

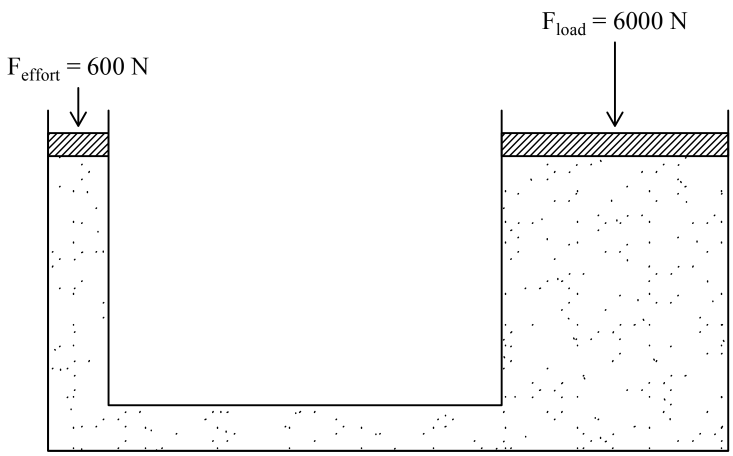

Figure 3.3.6.6

5. What is the area-ratio between the two pistons?

   A $\left(\frac{A_{\text{load}}}{A_{\text{effort}}}\right) = \frac{1}{6}$ B $\left(\frac{A_{\text{load}}}{A_{\text{effort}}}\right) = \frac{6}{1}$ C $\left(\frac{A_{\text{load}}}{A_{\text{effort}}}\right) = \frac{1}{10}$ D $\left(\frac{A_{\text{load}}}{A_{\text{effort}}}\right) = \frac{10}{1}$

6. If the area of the load piston is 0.3 cm^2, what is the area of the effort piston?

   A 0.3 cm^2 B 0.03 cm^2 C 3 cm^2 D 1.8 cm^2

7. Given the area of the effort piston in question 6, the pressure in the hydraulic jack's fluid is

   A $2 \times 10^3\ Pa$ B $2 \times 10^6\ Pa$ C $2 \times 10^7\ Pa$ D $2 \times 10^8\ Pa$

8. The diagram shown in figure 3.3.6.7 is used to estimate the atmospheric pressure in a particular location.

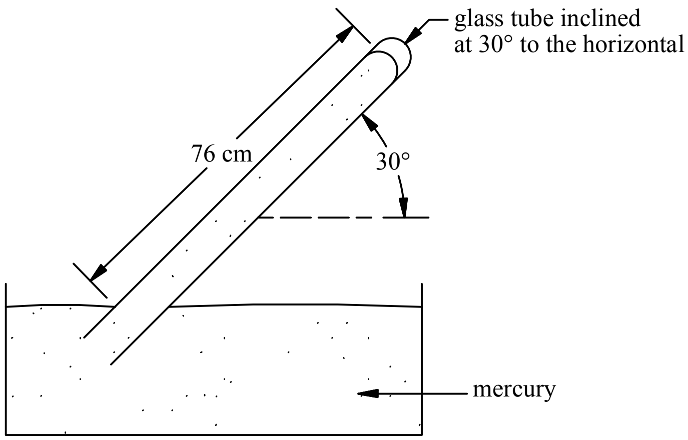

Figure 3.3.6.7

What is the atmospheric pressure measured?

A 38 $cmHg$ B 30 $cmHg$ C 76 $cmHg$ D 2280 $cmHg$

Questions 9 through 10 refer to figure 3.3.6.8, which shows a u-tube manometer with one end connected to an enclosed container and the other end left open to the atmospheric pressure.

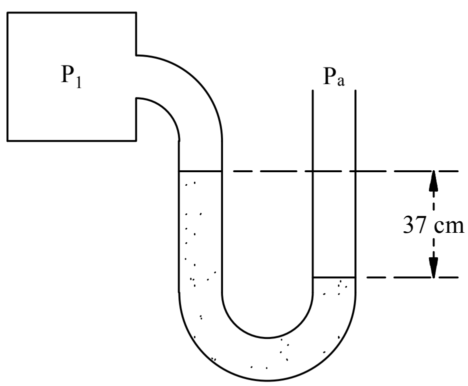

Figure 3.3.6.8

9. How does the container's interior pressure relate to the outside atmospheric pressure?

A $P_1 > P_a$ B $P_1 = P_a$ C $P_1 = 2(P_a)$ D $P_1 < P_a$

10. If P_a is equal to 1 atm and the liquid inside is mercury with a density of 13600 $kg\ m^{-3}$, what is the approximate value of P_1? (Given 1 $atm = 1.013 \times 10^5\ Pa$).

A 0.503 atm B 2 atm C 0.2 atm D 5.03 atm

GCE Paper 1 Solutions

1. B 2. D 3. B 4. A 5. D 6. C 7. C 8. A 9. D 10. A

GCE Paper 2 Questions

1. Figure 3.3.6.9 shows water of density 1000 $kg\ m^{-3}$ in a u-tube. A student places his mouth at X and causing the difference in water height between the two ends as shown.

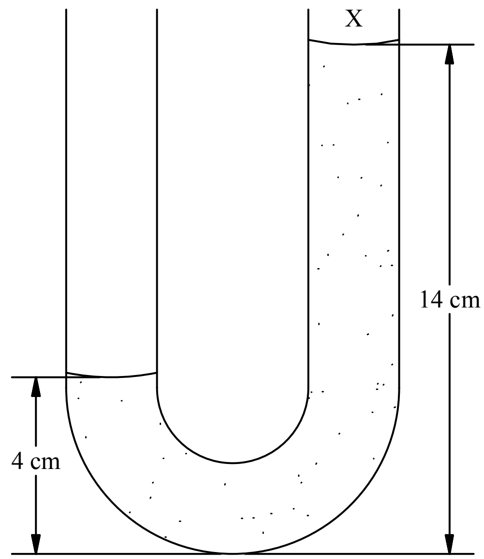

Figure 3.3.6.9

(a) Is the student blowing or sucking? Provide an explanation in the response. **(5 mks)**
(b) Determine the pressure difference created by the student. **(5 mks)**

. . . .

Solution

(a) The student is sucking because the higher liquid column near X indicates a vacuum, or lower pressure than on the open end on the left. This vacuum is achieved when the student draws air into his mouth.

(b) *This problem is very similar to a manometer, allowing its corresponding equation to be used.*

$$\text{given equation for manometers: } \Delta P = \rho_l g h_l$$

$$\text{substituting known values: } \Delta P = \left(1000\ kg\ m^{-3}\right)\left(10\ m\ s^{-2}\right)\left(14\ cm - 4cm\right)$$

$$\text{applying conversion factor: } \Delta P = \left(1000\ kg\ m^{-3}\right)\left(10\ m\ s^{-2}\right)\left(10\ cm\right)\left(\frac{1\ m}{100\ cm}\right)$$

$$\text{final answer: } \boxed{\Delta P = 1000\ Pa}$$

2. Figure 3.3.6.10 can be modified to function as a hydraulic lift. The master (smaller) cylinder has a cross sectional area of 0.02 m^2, while the slave (larger) cylinder has a cross sectional area of 0.6 m^2.

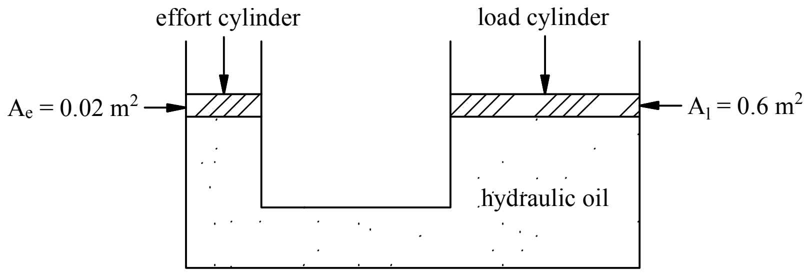

Figure 3.3.6.10

(a) If the effort applied is 150 N, what is the pressure transmitted in the fluid? **(2 mks)**
(b) Given this effort force, what is the force applied by the load piston? **(2 mks)**
(c) State two properties of liquid which make it suitable for this machine. **(2 mks)**

. . . .

Solution

(a) *Pascal's principle can be used to assume that the pressure applied at the effort piston is equal to that which is transmitted in the liquid.*

$$\text{considering pressure at effort piston: } P_e = \frac{F_e}{A_e}$$

$$\text{applying Pascal's principle to liquid pressure throughout jack: } P = \frac{F_e}{A_e}$$

$$\text{substituting known values: } P = \frac{150\ N}{0.02\ m^2}$$

$$\text{final answer: } \boxed{P = 7500\ Pa}$$

(b) *Given Pascal's principle, it is assumed that the pressure created by the effort piston is transmitted, undiminished, to the load piston.*

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{considering load piston: } P_l = \frac{F_l}{A_l}$$

$$\text{applying Pascal's principle to liquid pressure throughout jack: } P = \frac{F_l}{A_l}$$

$$\text{turning load force into subject: } F_l = P\,(A_l)$$

$$\text{substituting known values: } F_l = (7500\ Pa)\left(0.6\ m^2\right)$$

$$\text{final answer: } \boxed{F_l = 4500\ Pa}$$

(c) *The following is a non-exhaustive list of the advantages of liquids over gases. Only two are required.*
- Liquids transmit pressure undiminished from the effort to the load.
- Liquids transmit pressure instantly from the effort to the load.
- Liquids are almost entirely incompressible.
- A liquid expands and contracts less with changes in temperature.

3. In a simple hydraulic machine, a force of 60 N was applied at an effort piston of area 0.02 m^2.

   (a) State two characteristics of the fluid used in such a machine. **(2 mks)**
   (b) Calculate the pressure acting on the effort piston. **(2 mks)**

. . . .

Solution

(a) *The following is a non-exhaustive list of characteristics preferred of a fluid in a hydraulic jack. Only two are required.*

- It does not cause rust.
- Its incompressible.
- It does not form bubbles.
- Its pressure isn't affected by temperature.

(b) *The pressure acting on the effort piston is simply the effort force per the piston's area.*

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{substituting known values: } P = \frac{60\ N}{0.02m^2}$$

$$\text{final answer: } \boxed{P = 3000\ Pa}$$

3.3.7 Buoyancy

Objectives

By the end of the lesson, students should be able to
1. state Archimedes' principle.
2. explain upthrust.
3. explain buoyancy.
4. solve problems involving a body's density, volume, and upthrust.

Upthrust

- **Archimedes' principle** states that a body totally or partially immersed in a fluid is subject to an upward force, or upthrust, equal in magnitude to the weight of fluid it displaces.
- **Upthrust**, or $\mathbf{F_u}$, is the force a solid body experiences when placed in in a fluid.
- Like any force, upthrust is a vector with the Newton as its SI unit.

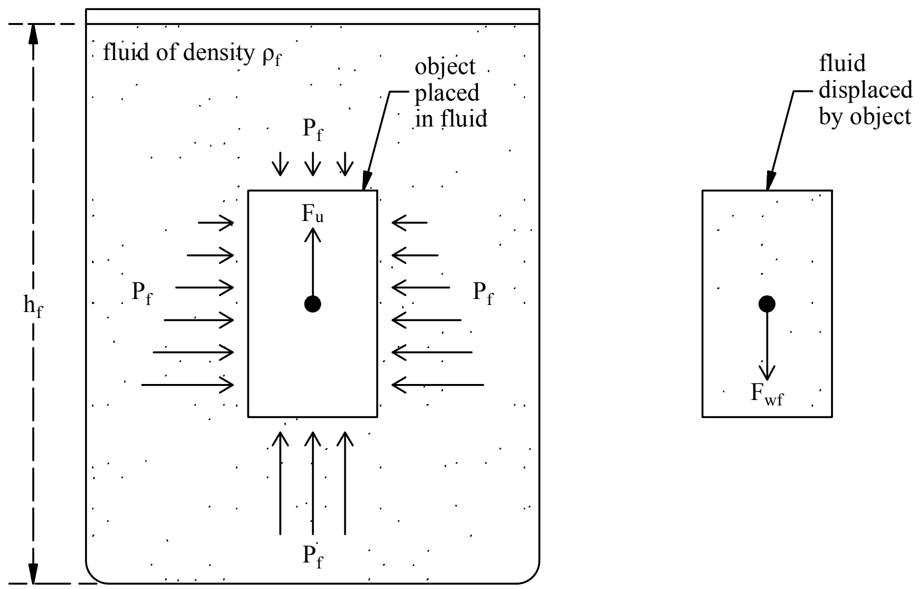

Figure 3.3.7.1

  - The object displaces some volume of the fluid.
  - The fluid pressure P_f, increases with fluid depth, h_f.
  - This causes a varying pressure along the height of the body.
  - This vertically-varying pressure causes a force that also varies along the body's height.
  - This force difference causes a net force upward, called upthrust.
  - The upthurst is always equal to the weight of the displaced fluid.

$$F_u = F_{wf}$$

Where
  - F_u is the upthrust force, in N;
  - F_{wf} is the force of the weight of the displaced fluid, in N.

- The volume of the fluid displaced is equal to the volume of the body submerged.

$$V_b = V_f$$

Where
  - V_b is the volume of the body submerged, in m^3;
  - V_f is the volume of the displaced fluid, in m^3.

The following derivations use the equation for weight $F_w = mg$. The relationship between weight, mass and the acceleration of gravity is not treated in detail until form 5. It is recommended to state/review this relationship briefly before continuing.

- If this volume and the density of the fluid displaced is known, and the body is not moving, the resulting upthrust force can be calculated.

$$\text{considering Archimedes' principle: } F_u = F_{wf}$$

$$\text{substituting equation for weight: } F_u = m_f g$$

$$\text{considering fluid density: } \rho_f = \frac{m_f}{V_f}$$

$$\text{turning fluid's mass into subject: } m_f = \rho_f V_f$$

$$\text{substituting equation for mass into upthrust equation: } F_u = \rho_f V_f g$$

$$\text{assuming volume equivalence between submerged body and displaced fluid: } V_b = V_f$$

$$\text{substituting volume, giving equation for upthrust: } F_u = \rho_f V_b g \qquad (3.3.7.1)$$

Where
- F_u is the upthrust force on the body, in N;
- ρ_f is the density of the displaced fluid, in $kg\ m^{-3}$;
- V_b is the volume of the body submerged, in m^3;
- g is the acceleration due to gravity, in $m\ s^{-2}$.

Buoyancy

- **Buoyancy** is the tendency of a solid body to float in a fluid.
- A body is buoyant if, when it is fully submerged, its upthrust in the fluid is greater than its force of weight.
- A body's buoyancy is ultimately determined by its density in relation to that of the fluid in which its placed.

$$\text{considering condition for buoyancy: } F_u > F_{wb}$$

$$\text{substituting equation for upthrust: } \rho_f V_b g > F_{wb}$$

$$\text{substituting equation for weight: } \rho_f V_b g > m_b g$$

$$\text{considering body's density: } \rho_b = \frac{m_b}{V_b}$$

$$\text{turning body's mass into subject: } m_b = \rho_b V_b$$

$$\text{substituting equation for body's mass into equation for body's weight: } \rho_f V_b g > \rho_b V_b g$$

$$\text{simplifying, giving condition for buoyancy: } \rho_f > \rho_b$$

Where
- F_u is the upthrust, in N;
- F_{wb} is the force of the body's weight, in N;
- ρ_f is the density of the fluid displaced, in $kg\ m^{-3}$;
- ρ_b is the density of the body, in $kg\ m^{-3}$;
- m_b is the mass of the body, in kg;
- V_b is the volume of the body, in m^3.

- Likewise, a body will sink if its density is greater than that of the fluid in which its placed.

$$\text{condition for sinking: } \quad \rho_b > \rho_f$$

Buoyancy of a Solid Body on a Liquid Surface

- Often times, a body floating on the surface of a liquid will become only partially submerged.
- This is because the body will only displace enough liquid such that its own force of weight is equal to the weight of the liquid it displaces.
- Thus, the body displaces only enough liquid to create an upthrust equal to its own force of weight.

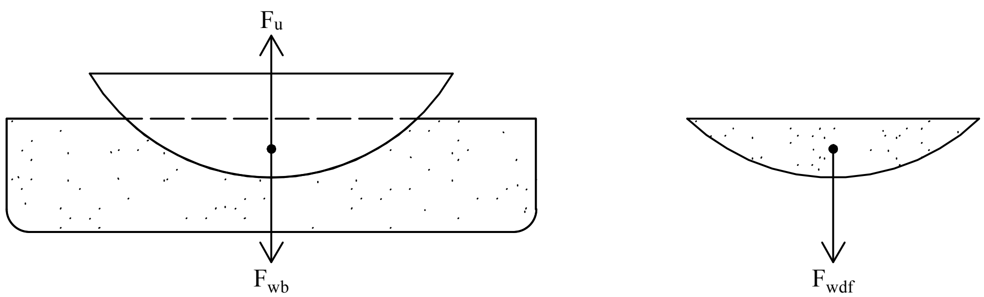

Figure 3.3.7.2

Buoyancy of a Solid Body Rising in a Fluid

- If the density of a solid body is less than that of a fluid in which it is surrounded, it will rise.
- For example, a balloon filled with helium has a density less than that of air.
- As shown in figure 3.3.7.3, the upthrust this balloon experiences is greater than its weight.

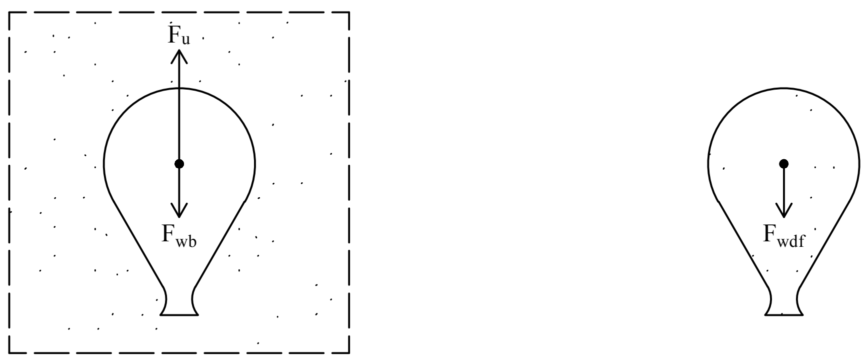

Figure 3.3.7.3

- This net upward force causes the balloon to rise in air.

$$\text{adding all vertical forces: } F_{net} = F_u - F_{wb}$$

$$\text{substituting equations for upthrust and weight: } F_{net} = \rho_f V_b g - m_b g$$

$$\text{considering the body's density: } F_{net} = \rho_f V_b g - \rho_b V_b g$$

$$\text{simplifying: } F_{net} = V_b g\left(\rho_f - \rho_b\right) \tag{3.3.7.2}$$

- Thus, a body experiences a net upward force ($F_{net} > 0$) if its density is less than that of its surrounding fluid.
- A body experiences a net downward force ($F_{net} < 0$) if its density is greater than its surrounding fluid.
- A solid body floating on the surface of a liquid experiences no net vertical force ($F_{net} = 0$).

GCE Paper 1 Questions

1. An object of mass 2.5 kg is completely immersed in water. The mass of the water displaced is 0.5 kg. Find the upthrust on the object. Assume acceleration due to gravity (g) is 10 $N\ kg$.

   A 5 N B 20 N C 25 N D 30 N

2. A hot-air balloon moving upwards has a total weight of 200 N and a volume of 20 m^3. Assuming the density of the surrounding air is 1.2 $kg\ m^3$ and the acceleration of gravity (g) is 9.81 $m\ s^{-2}$, the net upward force on the balloon is

   A 24 N B 40 N C 36 N D 240 N

   Questions 3 through 10 refer to figure 3.3.7.4, which shows four different solid bodies, A, B, C and D in a fluid. For each body, the mass and volume is shown. The following material densities are given.

   $\rho_{\text{liquid water}} = 1000\ kg\ m^3$ $\rho_{\text{air}} = 1.3\ kg\ m^3$ $\rho_{\text{petrol}} = 800\ kg\ m^3$

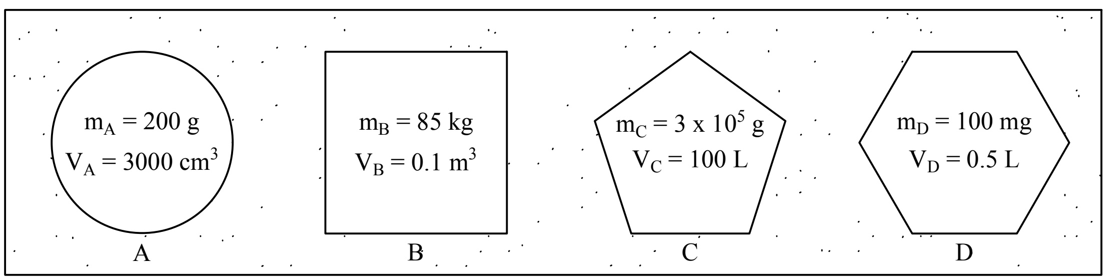

Figure 3.3.7.4

3. Which body has a density approximately three times that of liquid water?
4. If the surrounding fluid is liquid water, which body sinks?
5. Which body has a density less than that of air?
6. If the the surrounding fluid is air, which body rises?
7. Which body has a density greater than that of petrol but less than that of water?
8. Which body sinks in petrol but floats in water?
9. Which body has a density equal to approximately 6.7% that of water.
10. Which of the following statements is true regarding the bodies' shapes?

    A The circular shape of A causes it to float
    B The square shape of B causes it to sink in water.
    C Only shapes with an even number of sides can float.
    D There is no relationship between the shape of the solid bodies and their buoyancy.

GCE Paper 1 Solutions

1. A   2. B   3. C   4. C   5. D   6. D   7. B   8. B   9. A   10. D

GCE Paper 2 Questions

1. A solid body is placed in a pool of liquid water having a density of 1000 $kg\ m^3$. It is measured that the upthrust on the body is 20 N.

   (a) Calculate body's volume, in cubic metres. **(3 mks)**
   (b) Calculate body's volume, in litres. **(2 mks)**
   (c) If the liquid water is then replaced with liquid mercury having a density of 13600 $kg\ m^{-3}$, calculate the upthrust when the same body is submerged. **(3 mks)**
   (d) If it is given that the body's mass is 10 kg, state and explain whether it will sink in the water, the mercury, neither, or both. **(2 mks)**

. . . .

Solution

(a) *No conversion is necessary to give the body's volume in cubic metres.*

$$\text{given equation for upthrust: } F_u = \rho_f V_b g$$

$$\text{turning body's volume into subject: } V_b = \frac{F_u}{\rho_f g}$$

$$\text{substituting known values: } V_b = \frac{20\ N}{(1000\ kg\ m^{-3})\,(10\ m\ s^{-2})}$$

$$\text{final answer: } \boxed{V_b = 0.002\ m^3}$$

(b) *The unit equivalence* $1000\ L = 1\ m^3$ *is used.*

$$\text{known value for body's volume: } V_b = 0.002\ m^3$$

$$\text{applying conversion factor: } V_b = \left(0.002\ m^3\right)\left(\frac{1000\ L}{1\ m^3}\right)$$

$$\text{final answer: } \boxed{V_b = 2\ L}$$

(c) *An increase in the fluid density should cause an increase in the upthrust.*

$$\text{given equation for upthrust: } F_u = \rho_f V_b g$$

$$\text{substituting known values: } F_u = \left(13600\ kg\ m^{-3}\right)\left(0.002\ m^3\right)\left(10\ m\ s^{-2}\right)$$

$$\text{final answer: } \boxed{F_u = 272\ N}$$

(d) *The relationship between the body's density and either liquid's density determines its buoyancy in each. Since the body's volume is already known, its density is easily calculated.*

$$\text{considering the body's density: } \rho_b = \frac{m_b}{V_b}$$

$$\text{substituting known values: } \rho_b = \frac{10\ kg}{0.002\ m^3}$$

$$\text{final value for body's density: } \rho_b = 5000\ kg\ m^{-3}$$

When the body is placed in liquid water, its density is greater than that of the surrounding fluid, so it sinks. If it is placed in liquid mercury, its density is less than that of fluid in which it is placed, so it floats on the pool's surface.

2. Figure 3.3.7.5 shows a metal block of mass m_2 sitting atop a floating foam block of mass m_1. The fluid on which the foam floats is liquid water having a density of 1000 $kg\ m^{-3}$.

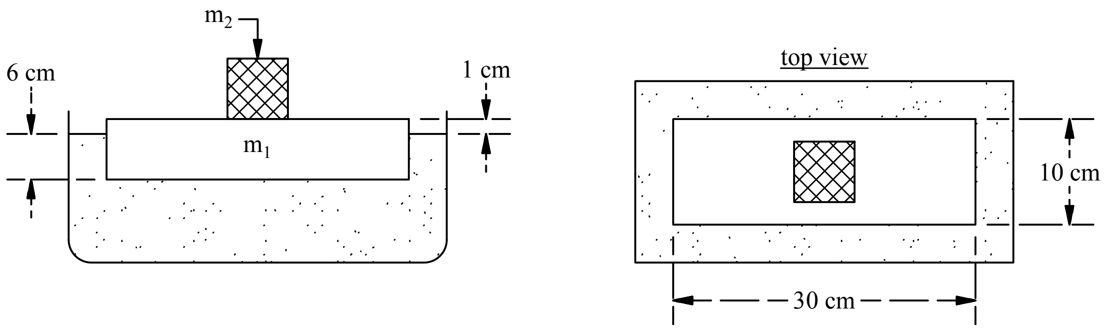

Figure 3.3.7.5

(a) Calculate the upthrust on the foam block. **(3 mks)**
(b) If the foam block is given to have a mass of 1 kg, calculate the mass of the metal block, in kg. **(2 mks)**
(c) More mass is slowly added to the metal block until the foam block becomes fully submerged. Calculate the metal block's new mass just as this occurs. **(3 mks)**
(d) If the metal block, with the new mass acquired in (c), has a volume of 2 cm^3, state and explain whether or not it would float in a pool of liquid mercury having a density of 13600 $kg\ m^{-3}$. **(3 mks)**

....

Solution

(a) *The volume of the submerged body is calculated as a simple, rectangular-based prism having a cross sectional area indicated by the top view and a height of the block's depth of submergence.*

given equation for upthrust: $F_u = \rho_f V_b g$

substituting volume: $F_u = \rho_f\,[(l)(w)(h)]\,g$

substituting known values: $F_u = \left(1000\ kg\ m^{-3}\right)[(30\ cm)(10\ cm)(6\ cm)]\left(10\ m\ s^{-2}\right)$

applying conversion factor: $F_u = \left(1000\ kg\ m^{-3}\right)\left[(30\ cm)(10\ cm)(6\ cm)\left(\frac{1\ m}{100\ cm}\right)^3\right]\left(10\ m\ s^{-2}\right)$

final answer: $\boxed{F_u = 18\ N}$

(b) *It is assumed that the total force of weight is equal to the upthrust.*

considering the block to be in equilibrium: $F_u = F_{wb}$

considering the weight of both blocks: $F_u = F_{wb1} + F_{wb2}$

substituting equation for weight: $F_u = m_1 g + m_2 g$

turning mass of metal block into subject: $m_2 = \dfrac{F_u - m_1 g}{g}$

substituting known values: $m_2 = \dfrac{18\ N - (1\ kg)\left(10\ m\ s^{-2}\right)}{10\ m\ s^{-2}}$

final answer: $\boxed{m_2 = 0.8\ kg}$

(c) *The foam block becomes fully submerged just as its entire 7 cm height goes below the water's surface, at which point it displaces a volume of water equal to its own volume.*

given equation for upthrust: $F_u = \rho_f V_b g$

considering block to be in equilibrium: $F_u = F_{wb}$

substituting force equivalence: $F_{wb} = \rho_f V_b g$

substituting equation for weight: $m_b g = \rho_f V_b g$

substituting mass of both blocks: $(m_1 + m_2)\, g = \rho_f V_b g$

turning metal block mass into subject: $m_2 = \rho_f V_b - m_1$

substituting volume: $m_2 = \rho_f [(l)(w)(h)] - m_1$

substituting known values: $m_2 = \left(1000\ kg\ m^{-3}\right)[(30\ cm)(10\ cm)(7\ cm)] - 1\ kg$

applying conversion factor: $m_2 = \left(1000\ kg\ m^{-3}\right)\left[(30\ cm)(10\ cm)(7\ cm)\left(\frac{1\ m}{100\ cm}\right)^3\right] - 1\ kg$

final answer: $\boxed{m_2 = 1.1\ kg}$

(d) *The metal block's given volume must be converted to SI units.*

given equation for density: $\rho_{\text{block}} = \frac{m_{\text{block}}}{V_{\text{block}}}$

substituting known values: $\rho_{\text{block}} = \frac{1.1\ kg}{2\ cm^3}$

applying conversion factor: $\rho_{\text{block}} = \frac{1.1\ kg}{2\ cm^3}\left(\frac{100\ cm}{1\ m}\right)^3$

final value of block's density: $\rho_{\text{block}} = 5.5 \times 10^5\ kg\ m^{-3}$

comparing densities: $\rho_{\text{block}} > \rho_{\text{mercury}}$

The metal block sinks in liquid mercury because its density is greater than that fluid.

3.4 Elasticity

3.4.1 Hooke's Law

Objectives

By the end of the lesson, students should be able to
1. state Hooke's Law.
2. define elasticity.
3. define elastic limit.
4. solve problems involving the loading force on a body, its extension and its spring constant.
5. name an instrument that depends on Hooke's law.

Elasticity

- **Elasticity** is the tendency of a body, after being deformed by some force, to regain its original shape an size after the deforming force is removed.
- **Hooke's law** states that the force applied to an elastic material is directly proportional to its extension, provided the proportional limit is not exceeded.

$$F_l \propto e$$

$$F_l = ke \tag{3.4.1.1}$$

Where
- F_l is the load force applied to the material, in N;
- k is the stiffness of the material, in $N\ m^{-1}$;
- e is the extension of the material, in m.

- Like any force, the load force is a vector with the Newton as its SI unit.

Extension

- **Extension**, or **e**, is the difference between a body's extended length under a load force and its unloaded length.
- It is a vector.
- Its SI unit is the metre, abbreviated m.

$$e = l - l_0 \tag{3.4.1.2}$$

Where
- e is the material's extension, in m;
- l is the materials extended length, in m;
- l_0 is the materials unloaded length, in m.

Stiffness

- A sping's **stiffness**, **spring constant**, or **k**, is the load force required to cause a unit extension.
- It is a scalar.
- Its SI units are Newtons per metre, abbreviated $N\ m^{-1}$.

$$k = \frac{F_l}{e}$$

Where
- k is the material's stiffness, in $N\ m^{-1}$;
- F_l is the load force applied to the material, in N;
- e is the material's extension, in m.

Extension and Loading Force as Vectors

- The loading force and the extension of a material are both vectors oriented in the same direction.
- A material under **tension** experiences a positive extension by a "stretching" loading force.
- A material under **compression** experiences a negative extension by a "squeezing" loading force.
- As shown in figure 3.4.1.1, a rightward loading force causes a rightward extension while a leftward loading force causes a leftward extension.

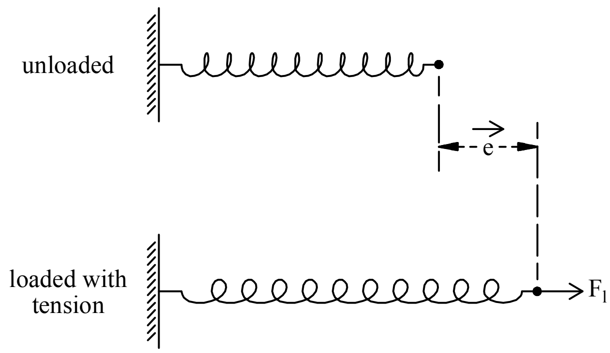

Figure 3.4.1.1

- NB: Not all materials have the same stiffness under tension as they do under compression. This chapter focuses entirely on materials under tensile loading forces, where $e > 0$.

Force vs. Extension Plots

- Force vs. extension plots are commonly used to analyse the elastic properties of different materials.
- Such plots show a material's proportionality limit, elastic limit and breaking point.

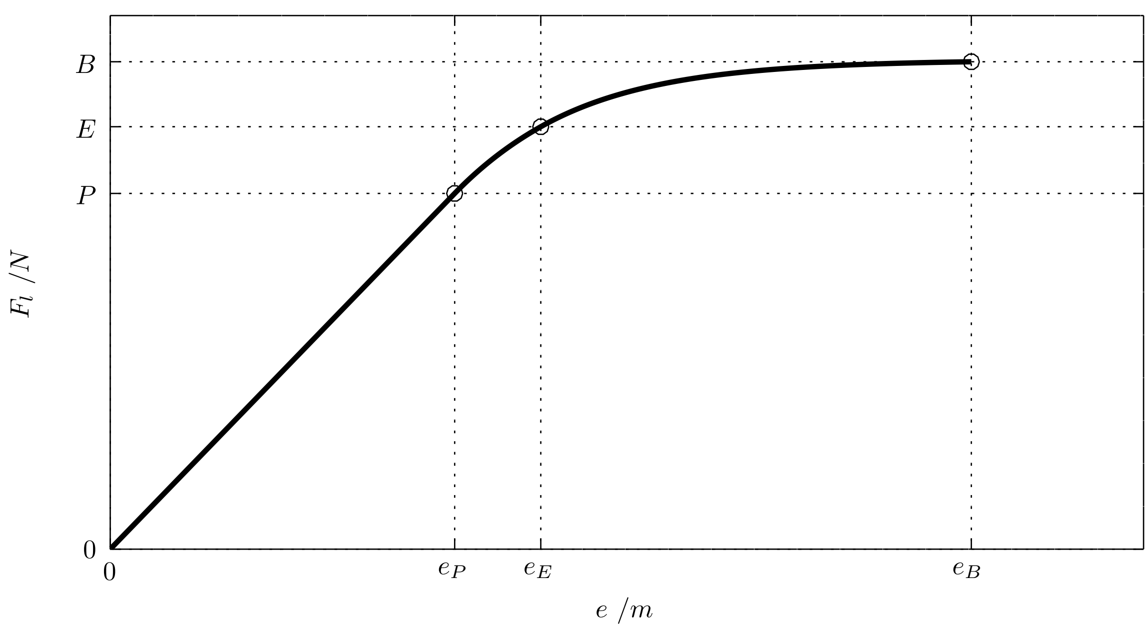

Figure 3.4.1.2

- A body's **proportionality limit**, or **P**, is the maximum load force it can withstand while still maintaining a linear relationship between its extension and its load force. That is,

$$F = ke \text{ for } F_l < P$$

- A body's **elastic limit**, **E**, is the maximum load force applied it can withstand beyond which it does not recover its original length when the force is removed. That is,

$$e \text{ returns to 0 when } F_l \text{ returns to 0 for } F_l < E$$

- NB: As shown, a material's elastic limit is often just above its proportionality limit.
- A body's **breaking limit**, or **B**, is the maximum load force it can withstand beyond which it breaks or fails.

$$\text{no breaking occurs for } F_l < B$$

- **Elastic deformation** is any extension occurring before the elastic limit is reached.

$$\text{elastic deformation for } e < e_E$$

  - If F_l is less than E and then removed, the body returns to its original length.
  - Therefore, elastic deformation or temporary deformation causes only temporary extension.

- **Plastic deformation** is any extension occurring after the elastic limit is exceeded but before the breaking point is reached.

$$\text{plastic deformation for } e_E < e < e_B$$

  - If F_l exceeds E but not B and is then removed, the body does not return to its original length.
  - Therefore, plastic deformation or permanent deformation causes permanent extension.

Spring Balances

- A **spring balance** can be used to measure the weight or mass of a body.

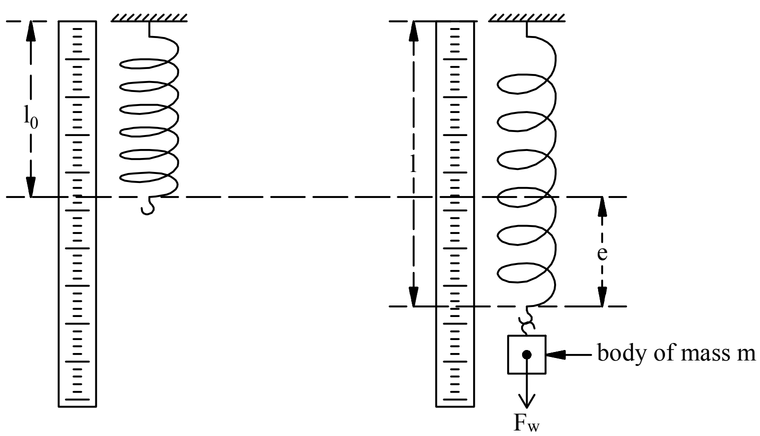

Figure 3.4.1.3

- When a body is suspended vertically, the spring's extension is proportional to the body's mass and weight.

$$\text{given equation for Hooke's law: } F_l = ke$$

$$\text{substituting body's force of weight for loading force: } F_w = ke$$

$$\text{substituting equation for weight: } mg = ke$$

$$\text{turning mass into subject: } m = \frac{ke}{g}$$

Where
- F_w is the weight of the suspended body, in N;
- k is the spring's stiffness, in $N\ m^{-1}$;
- e is the spring's extension, in m;
- m is the mass of the body, in kg;
- g is the acceleration of gravity, in $m\ s^{-2}$.

- A body's stiffness can be calculated from any points of its force vs. extension plot or F_l vs. e data as long as all values of F_l are less than P and all values of e are less than e_P.

$$k = \frac{\Delta F_l}{\Delta e} = \frac{F_{lf} - F_{li}}{e_f - e_i} \text{ for all } F_l < P \text{ and } e < e_P$$

Where
- k is the body's stiffness, in $N\ m^{-1}$;
- ΔF_l is the change in loading force applied, in N;
- Δe is the change in extension, in m;
- F_{lf} is the final loading force considered, in N;
- F_{li} is the initial loading force considered, in N;
- e_f is the final extension considered, in m;
- e_i is the initial extension considered, in m.

- If a body's original length is unknown, its stiffness can still be calculated from other values of its loaded force and extension. This is again as long as all these values are taken below the elastic limit.

$$\text{given equation for stiffness: } k = \frac{F_{lf} - F_{li}}{e_f - e_i}$$

$$\text{substituting equation for extension: } k = \frac{F_{lf} - F_{li}}{(l_f - l_0) - (l_i - l_0)}$$

$$\text{simplifying: } k = \frac{F_{lf} - F_{li}}{l_f - l_i}$$

Where
- k is the body's stiffness, in $N\ m^{-1}$;
- F_{lf} is the final loading force considered, in N;
- F_{li} is the initial loading force considered, in N;
- l_f is the body's final extended length considered, in m;
- l_i is the body's initial extended length considered, in m.

- If if a spring balance's stiffness is unknown, it can still be used to measure a body's weight or mass as long as other reliable values of F_w and l or F_w and e are available.

$$\text{with two known force and extension values: } F_w = \left(\frac{F_{lf} - F_{li}}{e_f - e_i}\right) e$$

$$\text{with two known force and length values: } F_w = \left(\frac{F_{lf} - F_{li}}{l_f - l_i}\right)(l - l_i) + F_{li}$$

$$\text{or: } F_w = \left(\frac{F_{lf} - F_{li}}{l_f - l_i}\right)(l - l_f) + F_{lf}$$

Where
- F_w is the weight of the suspended body, in N;
- F_{lf} is the final loading force considered, in N;
- F_{li} is the initial loading force considered, in N;
- e is the extension causes by the suspended body, in m;
- l_f is the body's final extended length considered, in m;
- l_i is the body's initial extended length considered, in m;
- l is the spring's length when the body is suspended, in m.

GCE Paper 1 Questions

1. The elastic limit of a material is

   A its maximum extension after it is permanently deformed.
   B the maximum force applied after it is permanently deformed.
   C its maximum extension before it is permanently deformed.
   D the maximum force applied before it is permanently deformed.

2. The ability of a material to return to its original shape and size upon removal of an applied force is known as

   A plasticity B elasticity C elastic limit D breaking limit

3. A spring fixed at one end can be stretched by hanging standard masses from the other end. Assuming an acceleration due to gravity (g) of 10 $N\ kg$, the mass needed to achieve a stretched force of 0.5 N is

   A 0.05 kg B 0.5 kg C 5 kg D 20 kg

4. The length of a spring is 25 cm when a load of 4 N hangs on it and 30 cm when a load of 8 N is hung. Assuming Hooke's law is obeyed, what is the length of the spring when the load is 6 N?

   A 20 cm B 26.5 cm C 27.5 cm D 28.5 cm

5. Figure 3.4.1.4 shows the length, in cm of a spring before and after a 6 N weight is suspended from it.

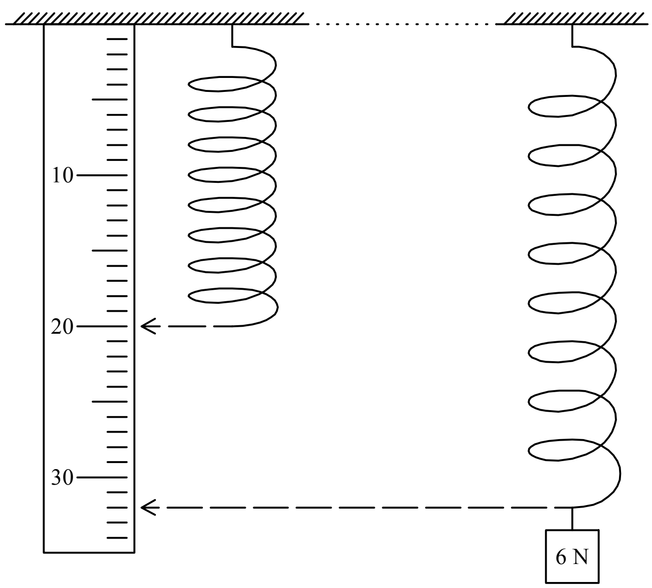

Figure 3.4.1.4

   If the 6 N weight is now replaced by a 5 N weight, the new length will be

   A 10 cm B 24 cm C 28 cm D 30 cm

6. A spring balance reads m_1 when a mass hangs from it in air and m_2 when it hangs in a liquid. The upthrust on the mass in the liquid, in Newtons, is

   A $(m_1 - m_2) \times 10\ m\ s^{-2}$
   B $(m_2 - m_1) \times 10\ m\ s^{-2}$
   C $m_1 \times 10\ m\ s^{-2}$
   D $m_2 \times 10\ m\ s^{-2}$

Questions 7 through 10 refer to a body with the following properties.

$k = 40\ N\ m^{-1}$ $P = 4\ N$ $E = 5\ N$ $B = 100\ N$

7. If a loading force of 4.5 N is applied and then removed, the body will

   A return to a length shorter than its original length.
   B return to its original length.
   C return to a length greater than its original length.
   D break completely.

8. If a loading force of 200 N is applied and then removed, the body will

   A return to a length shorter than its original length.
   B return to its original length.
   C return to a length greater than its original length.
   D break completely.

9. If a loading force of 5.5 N is applied and then removed, the body will

   A return to a length shorter than its original length.
   B return to its original length.
   C return to a length greater than its original length.
   D break completely.

10. Before being removed, the loading force applied in question 7 creates an extension of

    A 0.1125 cm B 1.125 cm C 11.25 cm D 1.125 m

GCE Paper 1 Solutions

1. D 2. B 3. A 4. C 5. D 6. A 7. B 8. D 9. C 10. C

GCE Paper 2 Questions

1. The length of a wire that obeys Hooke's law increases from 80 mm to 83 mm when a mass of 0.30 kg is suspended from it. When an additional mass is also suspended, the the wire's length increases to 94 mm. Find the mass of this additional body. **(3 mks)**

. . . .

Solution

It is assumed that the weight of both suspended masses do not surpass the wire's elastic limit. The extension when the second mass is applied is due to the force of the weight of both masses. The second to last step of the following solution requires no unit conversion since one quantity of a distance in units of mm is divided by another quantity in the same units.

given equation for extension: $F = ke$

assuming weight to be only loading force: $F_w = ke$

substituting equation for force of weight: $mg = ke$

substituting equation for extension: $mg = k\left(l - l_0\right)$

considering extension of first mass: $m_1 g = k\left(l_1 - l_0\right)$

turning stiffness into subject: $k = \dfrac{m_1 g}{l_1 - l_0}$

substituting stiffness into previous equation: $mg = \left(\dfrac{m_1 g}{l_1 - l_0}\right)\left(l - l_0\right)$

considering both masses: $\left(m_1 + m_2\right) g = \left(\dfrac{m_1 g}{l_1 - l_0}\right)\left(l_2 - l_0\right)$

simplifying: $m_1 + m_2 = \left(\dfrac{l_2 - l_0}{l_1 - l_0}\right)\left(m_1\right)$

turning second mass into subject: $m_2 = \left(\dfrac{l_2 - l_0}{l_1 - l_0}\right) m_1 - m_1$

substituting known values: $m_2 = \left(\dfrac{94\ mm - 80\ mm}{83\ mm - 80\ mm}\right)\left(0.30\ kg\right) - 0.30\ kg$

final answer: $\boxed{m_2 = 1.1\ kg}$

---

2. Figure 3.4.1.5 shows the force vs. extension plot of a material obeying Hooke's law.

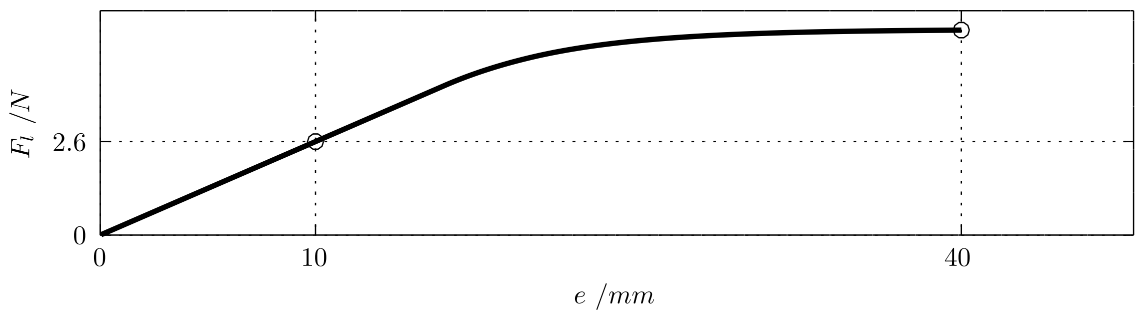

Figure 3.4.1.5

With the data shown,

(a) determine the materials stiffness in that property's typical units. **(3 mks)**
(b) determine the material's extension, in cm, resulting from a loading force of 1.3 N. **(3 mks)**
(c) State and explain if one could calculate the loading force causing an extension of 100 mm. **(2 mks)**

Solution

(a) *The typical units of stiffness are* $N\ m^{-1}$.

$$\text{given equation for stiffness: } k = \frac{F_l}{e}$$

$$\text{substituting known values: } k = \frac{2.6\ N}{10\ mm}$$

$$\text{applying conversion factor: } k = \left(\frac{2.6\ N}{10\ mm}\right)\left(\frac{1000\ mm}{1\ m}\right)$$

$$\text{final answer: } \boxed{k = 260\ N\ m^{-1}}$$

(b) *Since the indicated loading force is less than the loading force shown, the resulting extension should also be less than the coordinating value in the plot.*

$$\text{given equation for Hooke's law: } F = ke$$

$$\text{turning extension into subject: } e = \frac{F}{k}$$

$$\text{substituting known values: } e = \frac{1.3\ N}{260\ N\ m^{-1}}$$

$$\text{final answer in metres: } e = 0.005\ m$$

$$\text{applying conversion factor: } e = (0.005\ m)\left(\frac{100\ cm}{1\ m}\right)$$

$$\text{final answer: } \boxed{e = 0.5\ cm}$$

(c) One could not calculate the F_l that would cause an e of 100 mm. This is because, according to the given plot, such an extension is far greater than the one associated with the materials breaking point.

3.4.2 Hookean and Non-Hookean Materials

Objectives

By the end of the lesson, students should be able to

1. name some materials that obey Hooke's law.
2. name some materials that do not obey Hooke's law
3. sketch the force-extension graph of a rubber band.
4. describe an experiment to assess the applicability of Hooke's law to both springs and rubber bands.

Hookean and Non-Hookean Materials

- **Hookean materials**, which obey Hooke's law when a loading force is applied, include
  - helical steel springs
  - steel wire
  - copper wire
  - guitar string
- **Non-Hookean materials**, which do not obey Hooke's law for any loading force, include
  - rubber bands
  - nylon
  - polypropylene
  - meat
- Figure 3.4.2.1 shows the force extension graph of a rubber band.

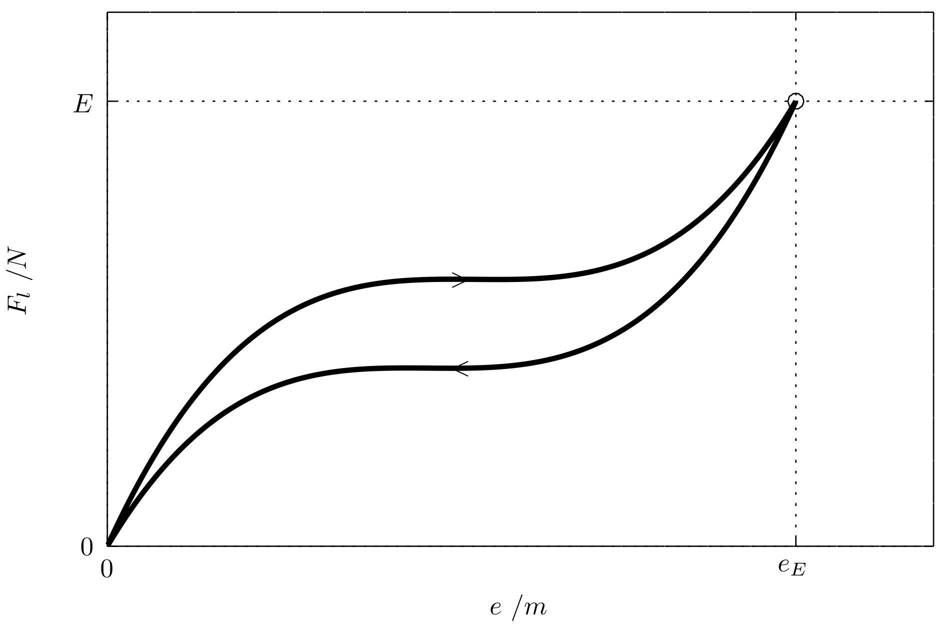

Figure 3.4.2.1

- As the rubber band is gradually stretched, its extension has a non-linear relationship with the ever-increasing loading force.
- The absence of any linear relationship between the rubber band's extension and loading force along any range of F_l or e causes it to be non-Hookean.
- Also, as the loading force is gradually reduced, the relation between its expansion and force is again non-linear, but also not similar to the relationship while the force was increasing.

Experiment to Verify Hooke's Law for a Spring

I Procedure

1 A helical spring is suspended vertically next to a metre rule.
2 A light-weight pointer is added to the spring's bottom so that it's length can be read from the metre rule.
3 The spring initial, unloaded length, l_0, is read from the metre rule.
4 A slotted standard mass m is suspended from the spring.
5 The spring's extended length l_a is read from the metre rule.
6 Steps 4 through 5 are repeated with more and more slotted masses, increasing the load incrementally.

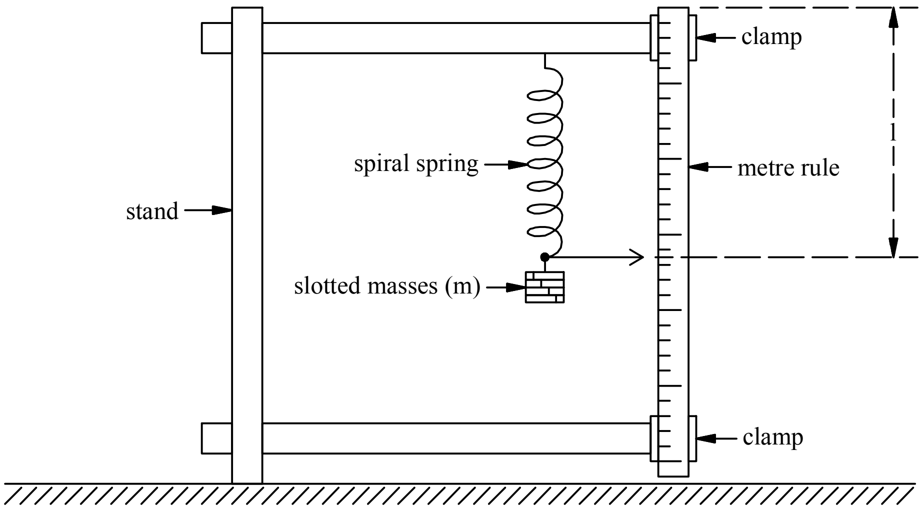

Figure 3.4.2.2

7 After reaching a maximum load, the slots are incrementally removed.
8 For each slot removed, and the spring's shortened length l_b is read from the metre rule.

II Calculations

The force of weight after each mass added is calculated as $F_l = F_w = nmg$

(where n is the number of slotted masses)

For each length, the extension is calculated as $e = l - l_0$

The average of each length while extending and shortening are calculated as $l = \frac{1}{2}(l_a + l_b)$

- The data is then recorded in a tabular format.

n	**m /kg**	**F_l /N**	**l_a /m**	**l_b /m**	**l /m**	**e /m**
1	$1 \times m$	$1 \times mg$	l_{a1}	l_{b1}	$\frac{1}{2}(l_{a1} + l_{b1})$	$l_1 - l_0$
2	$2 \times m$	$2 \times mg$	l_{a2}	l_{b2}	$\frac{1}{2}(l_{a2} + l_{b2})$	$l_2 - l_0$
3	$3 \times m$	$3 \times mg$	l_{a3}	l_{b3}	$\frac{1}{2}(l_{a3} + l_{b3})$	$l_3 - l_0$
4	$4 \times m$	$4 \times mg$	l_{a4}	l_{b4}	$\frac{1}{2}(l_{a4} + l_{b4})$	$l_4 - l_0$

Table 3.4.2.1

- Values of the loading force, F_l, are plotted against values of extension, e.

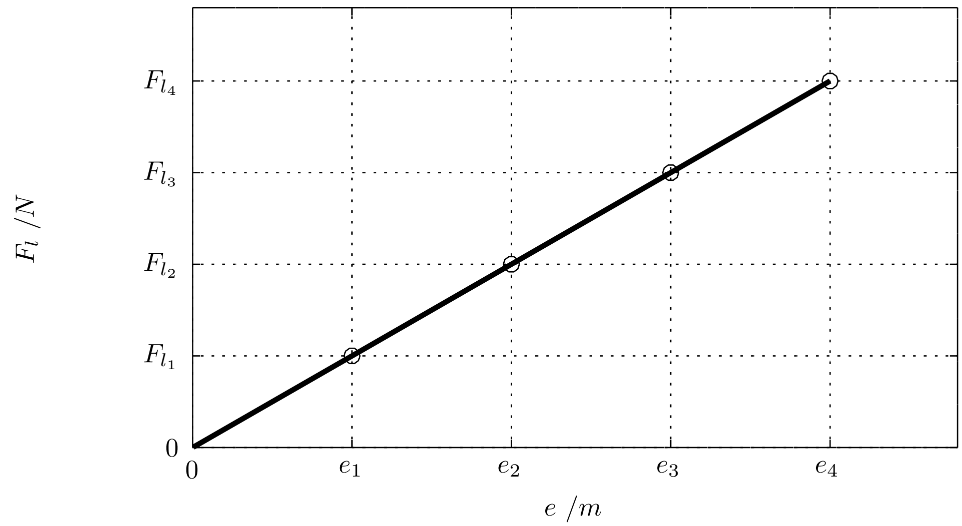

Figure 3.4.2.3

- The graph of F_l vs e is a straight line passing through the origin, verifying Hooke's law for the helical spring. That is, the helical spring is confirmed Hookean.

III Precautions

- As the slots are being removed in steps 7 through 8, the coordinating values of l_a and l_b are checked to have a minimal difference. This ensures that the elastic limit was not exceeded during loading.

Experiment to Verify Hooke's Law for a Rubber Band

- In order to assess the applicability of Hooke's law to a rubber band, the same experiment is carried out with the band in place of the spring.

- As shown in figure 3.4.2.4, plotting the resulting values of F_l vs. e yields a non-linear relationship, which confirms that Hooke's law does not apply to the rubber band. The rubber band is confirmed non-Hookean.

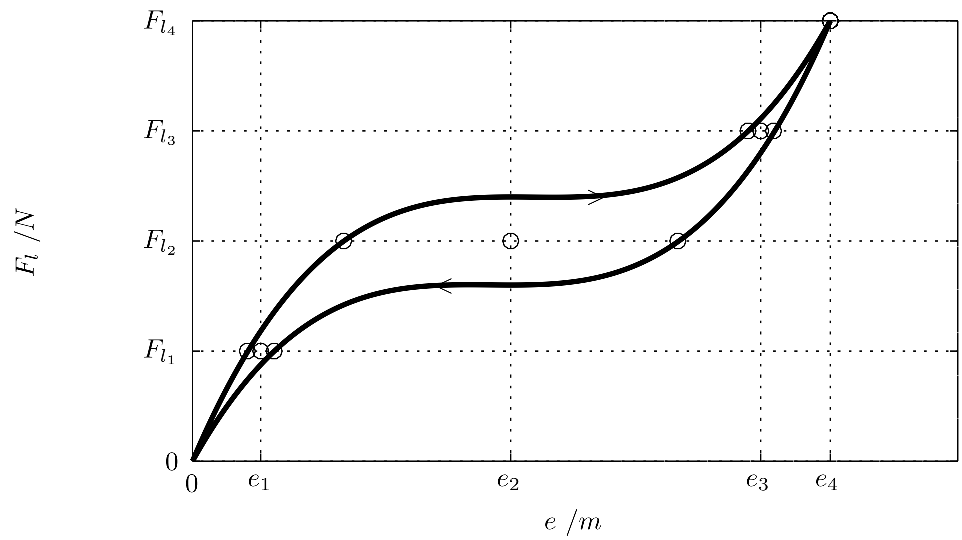

Figure 3.4.2.4

GCE Paper 1 Questions

1. The force beyond which a stretched spring does not return to its original length and/or and size after the stretching force is applied is called the spring's

   A yield point. B breaking point. C elastic limit. D Hooke's law force.

Questions 2 through 10 refer to table 3.4.2.2, which shows the loading force for several values of extension for a material under tension.

extension /cm	0.35	0.45	0.55	0.65	0.75	0.85	0.95	1.05	1.15
force /N	0.8225	1.0575	1.2925	1.5275	1.7625	1.9975	2.2215	2.3619	2.4005

Table 3.4.2.2

2. Which of the following terms describes this material?

   A Hookean B non-Hookean C fluid D absolute

3. The difference between the least two forces shown is

   A 2.35 N B 0.235 N C 1.0 N D 0.1 N

4. The difference between the least two extensions shown is

   A 2.35 cm B 0.235 cm C 0.1 m D 0.001 m

5. The quotient of the answer to question 3 over the answer to question 4 is

   A 2.35 $N\ m^{-1}$ B 235 $N\ m^{-2}$ C 235 $N\ m^{-1}$ D 0.425 $N\ m^{-1}$

6. What is the stiffness of this material?

   A 2.35 $N\ m^{-1}$ B 235 $N\ m^{-2}$ C 235 $N\ m^{-1}$ D 0.425 $N\ m^{-1}$

7. Which of the following values of force is most likely for an extension of 0.2 cm?

   A 4.7 N B 0.47 N C 47 cm D 0.47 cm

8. Which of the following values of force is most likely for an extension of 2 mm?

   A 4.7 N B 0.47 N C 47 cm D 0.47 cm

9. The material's proportionality limit is most likely

   A 1 N B 1.5 N C 2 N D 3 N

10. If the material's original length is 5.15 cm, then its total length under a loading force of 1.0575 N is

    A 0.45 cm B 5.60 cm C 0.45 m D 5.60 m

GCE Paper 1 Solutions

1. C 2. A 3. B 4. D 5. C 6. C 7. B 8. B 9. C 10. B

GCE Paper 2 Questions

1. A girl subjects a wire and a rubber band, both of equal lengths, to the same series of different stretching forces. With the first stretching force being 10 N, she notes the following observations:

   – For every 10 N of stretching force added, the wire exhibits an additional 0.002 m of extension .
   – Up to a certain force, both the wire and the rubber band return to their original, unstretched lengths when the force is removed. This maximum force is 250 N for the wire and 500 N for the rubber band.

   (a) What term can be used to describe the behaviour of both the wire and the rubber band? **(1 mk)**
   (b) Determine the wire's stiffness. **(2 mks)**
   (c) Using a sketches or graphs, explain whether or not each material obeys Hooke's law. **(4 mks)**
   (d) Determine the elastic limits of both the wire and the rubber band. **(3 mks)**

....

Solution

(a) Both material's are elastic. That is, they both return to their original shape after having the force removed.

(b) *The wire's stiffness is its force per extension.*

$$\text{given equation for Hooke's law: } F = ke$$

$$\text{turning stiffness into subject: } k = \frac{F}{e}$$

$$\text{substituting known values: } k = \frac{10\ N}{0.002\ m}$$

$$\text{final answer: } \boxed{k = 5000\ N\ m^{-1}}$$

(c) *See figures 3.4.2.5 and 3.4.2.6. The wire, which is already known to be Hookean, has a straight F_l vs e graph with a slope equal to the previously-calculated stiffness. Assuming its P is just below its given E of* 250 N, *its graph is a straight line connecting the origin to its maximum e at $F_l = 240\ N$. The rubber band's graph resembles figure 3.4.2.1 with a max force of* 500 N *and no max extension specified.*

$$\text{wire's maximum extension: } e_{max} = \frac{F_{l_{max}}}{k} = \frac{240\ N}{5000\ N\ m^{-1}} = 0.048\ m$$

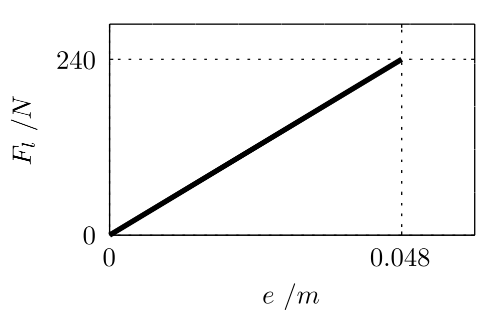

Figure 3.4.2.5

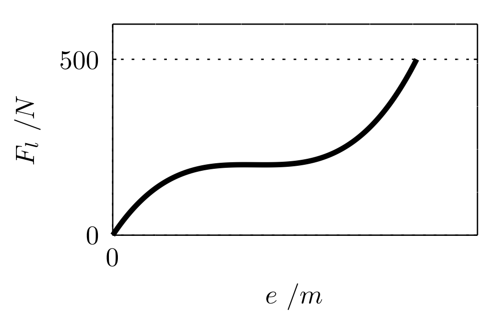

Figure 3.4.2.6

As shown with the left graph, the force applied to the wire is directly proportional to its extension for all forces below a limit. That is, the wire is Hookean. As shown on the right, the relationship between the rubber band's force and extension is not linear. That is, it is non-Hookean.

(d) *Both materials' elastic limit is the given maximum loads applied without permanent deformation.*

$$\boxed{E_{\text{wire}} = 250\ N, \quad E_{\text{rubber band}} = 500\ N}$$

2. In an experiment to verify Hooke's law, the length (l) of a spring for corresponding stretching forces (F) were measured and recorded in the table below.

force F /N	0	10	20	30	40	50	60	70
length l /cm	20.0	21.6	23.2	24.8	26.4	28.0	29.6	31.2
extension e /cm				4.8	6.4			

(a) Copy and complete the table **(2 mks)**
(b) Plot a graph with force along the y-axis against extension along the x-axis. **(5 mks)**
(c) Determine the slope of your graph. **(2 mks)**
(d) State the significance of the slope. **(1 mk)**

Solution

(a) *Solutions in bold*

force F /N	0	10	20	30	40	50	60	70
length l /cm	20.0	21.6	23.2	24.8	26.4	28.0	29.6	31.2
extension e /cm	**0.0**	**1.6**	**3.2**	4.8	6.4	**8.0**	**9.6**	**11.2**

(b) *See figure 3.4.2.7*

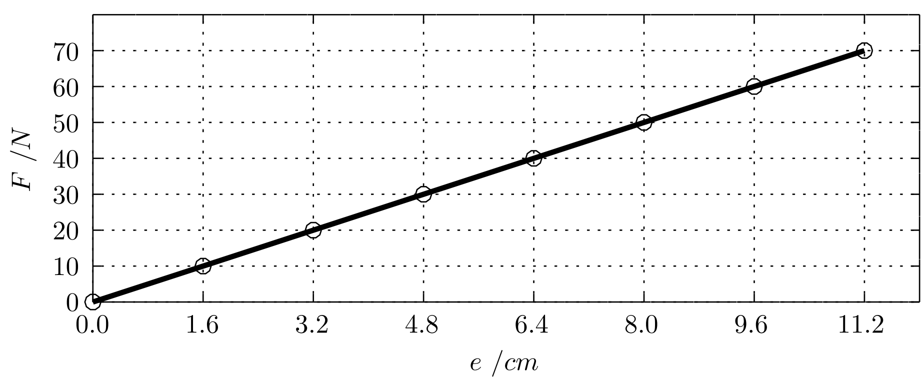

Figure 3.4.2.7

(c) *Since the graph passes through the origin, the slope is easily calculated as the maximum force per the maximum extension.*

$$\text{calculating slope as change in force per extension: slope} = \frac{\Delta F}{\Delta e}$$

$$\text{substituting origin and data point furthest from origin: slope} = \frac{70\ N - 0\ N}{11.2\ cm - 0\ cm}$$

$$\text{final answer: } \boxed{\text{slope} = 6.25\ N\ cm^{-1}}$$

(d) *The graph shows the proportionality of $F \propto e \rightarrow F = ke$. Therefore, the slope is equivalent to $k = \frac{F}{e}$, which is the definition of a material's stiffness.*

The slope is the stiffness of the material in units of $N\ cm^{-1}$ as opposed the standard units of $N\ m^{-1}$.

3. In an experiment using a spiral spring, a student records the following values.

F_l /N	0.0	0.2	0.4	0.6	0.8	1.0	1.2
e /cm	0.0	1.8	3.6	5.4	6.2	9.0	10.8

(a) State the law which governs the extension of this spring as it is incrementally loaded. **(2 mks)**
(b) Plot a graph of the load along the y-axis against the extension along the x-axis. **(4 mks)**
(c) Indicate any erroneous or unideal readings. **(2 mks)**
(d) Determine the spring constant, k. **(2 mks)**
(e) After correcting for any unideal behaviour or readings, determine the extension, in m, that would be expected for a load of 0.7 N. **(3 mks)**
(f) Express the extension determined in (e) in cm. **(3 mks)**

. . . .

Solution

(a) The extension of this spring as it is incrementally loaded is governed by Hook's law.

(b) See figure 3.4.2.8

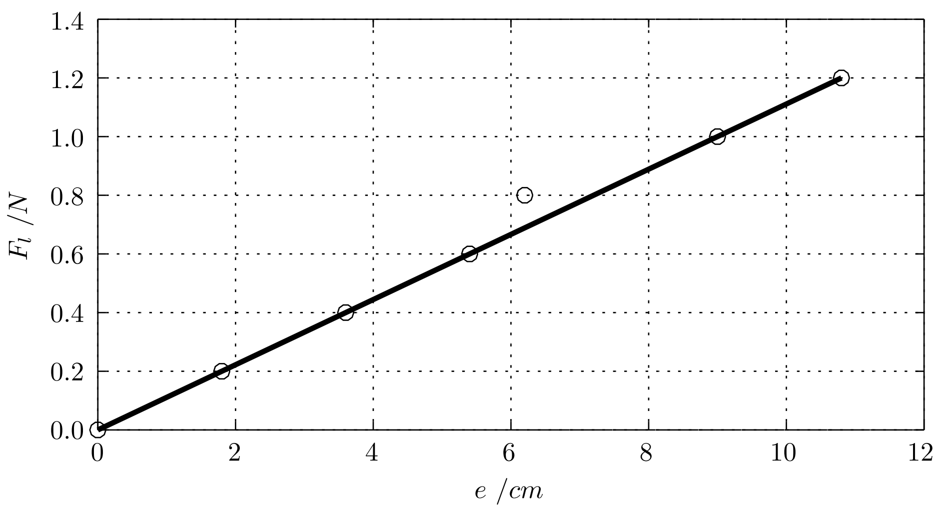

Figure 3.4.2.8

(c) After plotting the values of F_l vs. e, it can be seen that the value for $e = 6.2\ cm$ and $F_l = 0.8\ N$ does not fall along the same straight line as the other values. This is taken to be erroneous or non-ideal reading.

(d) *The spring constant can be determined from any of the non-erroneous values.*

$$\text{given equation for extension: } F = ke$$

$$\text{turning stiffness into subject: } k = \frac{F}{e}$$

$$\text{subtituting known values: } k = \frac{1.2\ N}{10.8\ cm}$$

$$\text{applying conversion factor: } k = \frac{1.2\ N}{10.8\ cm}\left(\frac{100\ cm}{m}\right)$$

$$\text{final answer: } \boxed{k \approx 11.1\ N\ m^{-1}}$$

(e) *It is assumed that the previously calculated value of k has been determined using entirely corrected readings.*

$$\text{given equation for extension: } F = ke$$

$$\text{turning extension into subject: } e = \frac{F}{k}$$

$$\text{substituting known values: } e = \frac{0.7\ N}{11.1\ N\ m^{-1}}$$

$$\text{final answer: } \boxed{e \approx 0.063\ m}$$

(f) *The unit equivalence* $1\ m = 100\ cm$ *is used.*

$$\text{given value in metres: } e \approx 0.063\ m$$

$$\text{applying conversion factor: } e \approx 0.063\ m\left(\frac{100\ cm}{1\ m}\right)$$

$$\text{final answer: } \boxed{e \approx 6.3\ cm}$$

---

4. (a) Give one example each of a Hookean and a non-Hookean material. **(2 mks)**
   (b) Name an instrument whose functioning is based on Hooke's Law **(1 mk)**

....

Solution

(a) *The following is a non-exhaustive list of examples of materials that obey Hooke's law. Only one is required.*

- steel wire
- helical steel springs
- copper wire
- guitar string

The following is a non-exhaustive list of examples of materials that don't obey Hooke's law. Only one is required.

- rubber bands
- nylon
- polypropylene
- meat

(b) A spring balance

3.5 Mechanics

3.5.1 Energy and Work

Objectives

By the end of the lesson, students should be able to

1. define work, stating its unit.
2. define energy and states its unit.
3. solve problems involving work and one-dimensional force and displacement.
4. describe energy transformation with appropriate transducers.
5. distinguish between renewable and non-renewable energy sources.
6. state some sources of energy.

Work and Energy

- **Work**, or **W**, is the action of a force producing movement.
- It is a scalar.
- Its SI unit is the Joule, abbreviated J.
- It is calculated as the product of a force's magnitude and the distance along which it is applied or overcome.

$$W = Fd \tag{3.5.1.1}$$

  Where
  - W is the work done by the force on the object, in N;
  - F is the force, in N;
  - d is the distance parallel to the force's line of action along which the force is applied or overcome, in N.

- NB: d is strictly aligned with the force's line of action.
- As shown in figure 3.5.1.1, d is the distance along which the force is applied, not necessarily the distance the affected body moves along a surface.

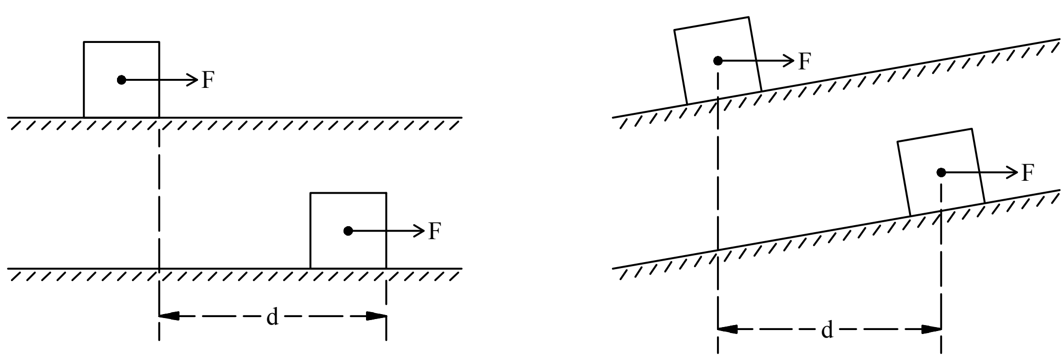

Figure 3.5.1.1

- **Energy**, or **E**, is the ability to do work.
- Like work, energy is a scalar.
- Its SI unit is the Joule, abbreviated J.
- The SI unit of work and energy, the Joule, is equivalent to the product of one Newton and one metre

$$J = N\ m$$

- Problems involving work can require that one of the following three properties be calculated while the other two are given - work, force and distance.

  Example: A girl drags a box across the ground with a force of 20 N, parallel to the floor. If the distance moved is 3 m, calculate the work she does on the box.

$$\text{given equation for work: } W = Fd$$

$$\text{subtituting known values: } W = (20\ N)(3\ m)$$

$$\text{final answer: } \boxed{W = 60\ J}$$

  Example: While the same girl drags a different box with a force of a different magnitude but same direction, she does 130 J of work in moving it a distance of 0.5 m. Calculate the force she applies.

$$\text{given equation for work: } W = Fd$$

$$\text{turning force into subject: } F = \frac{W}{d}$$

$$\text{subtituting known values: } F = \frac{130\ J}{0.5\ m}$$

$$\text{final answer: } \boxed{F = 260\ N}$$

  Example: If a farmer does 6000 J of work while pushing a wheelbarrow with a force that is parallel to the ground and has a magnitude of 300 N, how far does the wheelbarrow move along the ground?

$$\text{given equation for work: } W = Fd$$

$$\text{turning distance into subject: } d = \frac{W}{F}$$

$$\text{subtituting known values: } d = \frac{6000\ J}{300\ N}$$

$$\text{final answer: } \boxed{d = 20\ m}$$

Common Units of Energy

unit	**unit abbreviation**	**equivalence in base**
kiloJoule	kJ	$10^3\ J$
megaJoule	MJ	$10^6\ J$
British thermal unit	btu	$1.054 \times 10^3\ J$
electron volt	eV	$1.602 \times 10^{-19}\ J$
kiloWatt hour	kWh	$3.60 \times 10^6\ J$
calorie	cal	$4.186\ J$
kilocalorie	$kcal$	$4.186 \times 10^3\ J$

Table 3.5.1.1

Examples of Different Types of Energy

- **Mechanical energy** is the sum of all energy associated with a body's movement.

  NB: While other fields of physics consider other forms of energy to also be mechanical, this chapter focuses only on the mechanical energy associated with a body's movement.

- **Thermal energy** is the energy associated with the movement of a body's molecules.
- **Sound energy** is the energy associated with the propagation of sound waves through a material.
- **Elastic energy** is the energy associated with a body's temporary deformation.
- **Light energy** or **radiative energy** is the energy associated with the propagation of photons through space.
- **Electrical energy** is the energy associated with the work required to move charged particles.
- **Chemical energy** is associated with the energy lost or gained when atoms, molecules and compounds undergo changes or reactions.
- **Nuclear energy** is the energy associated with the interaction between the particles inside an atom.

Converting Energy

- Energy can neither be created nor destroyed, only converted from one form to another.
- **Transducers** convert energy from one form to another.
- The conversion of energy is sometime referred to as "transduction".
- Though there are many devices which engage in transduction, they are rarely referred to as transducers.

transducer name	input energy type	output energy time
electric iron	electric	thermal
light bulb	electric	light
electric fan	electric	mechanical
loudspeaker radio	electric	sound
charging phone battery	electric	chemical
torch powered by batteries	chemical	light
microphone	sound	electric
solar panel	light	electric
thermocouple	thermal	electric
electric motor	electric	kinetic
petrol-powered electric generator	chemical	electric
turbine	mechanical	electric
flower	light	chemical
television	electrical	light
kerosene stove	chemical	thermal
nuclear power plant	nuclear	thermal

Table 3.5.1.2

Typical Quantities of Energy

description	typical amount	typical amount in base
energy of a brick's motion when thrown	-	$75\ J$
chemical energy in a 250 ml bottle of planet	139.3 $kcal$	$5.83 \times 10^5\ J$
thermal energy needed to boil a kettle of water	700 kJ	$7 \times 10^5\ J$
electrical energy stored in a fully-charged vehicle battery	2 MJ	$2 \times 10^6\ J$
chemical energy in total food consumed in a day	11 MJ	$1.1 \times 10^7\ J$
chemical energy stored in 1 L of petrol	35 MJ	3.5×10^7

Table 3.5.1.3

Renewable Sources of Energy

- Almost all of the energy available for human use on the earth originally comes from the sun.
  - Humans eat plants like corn which get their energy from photosynthesis, a process powered by light.
  - Humans also eat meat like cows which get their energy by eating plants.
  - Petrol is refined petroleum, which has formed over millions of years naturally from decaying plants.
  - Hydroelectric power comes from water which flows because of climate cycles powered by the sun.
- **Renewable energy sources** are sources of energy that cannot be exhausted.
- While the primary advantage of renewable energy sources is that they can be reused endlessly, the primary disadvantage is the cost, difficulty and/or unreliability of their use.
- **Biofuel** or **biomass** energy is energy harvested from growing plants as they produce fuel during the process of photosynthesis.
  - Examples include
    * wood
    * sawdust
    * leaves
    * composted food
  - Advantages include
    * long-term affordability
    * availability
  - Disadvantages include
    * environmental pollution
    * depletion of the ozone layer
    * poor manageability of bulky materials
- **Wind energy** is energy harvested from the bulk movement of air with wind turbines.
  - Advantages include
    * freedom from grid connection
    * long-term affordability
  - Disadvantages include
    * low efficiency
    * dependence of productivity on climate type
    * high installation cost
    * dependence of productivity on weather

- **Hydroelectric energy** is energy harvested from the bulk movement of water with water turbines in dams.
  - Hydroelectric energy is often sourced from large dams that are built on high-flow rivers.
  - Advantages include
    - availability of water in certain areas
    - long-term affordability
    - convenience of maintenance
    - convenience of distribution through grid
  - Disadvantages include
    - dependence on seasonal variations in water flow
    - large cost of dam construction
    - long-term environmental impacts like erosion
    - environmental changes by backed-up water.
    - displacement of populations by backed-up water
    - noise pollution
- **Solar energy** is energy produced by photovoltaic "solar" panels when they are placed in the sun's light.
  - Advantages include
    - freedom from grid connection
    - long-term affordability
  - Disadvantages include
    - high installation cost
    - difficultly in storing once captured
    - productivity dependent on daylight
    - productivity dependent on weather
- **Geothermal energy** is the energy harvested from the bulk movement of steam and hot water from springs and volcanoes with steam turbines.
  - Geothermal energy involves very high pressures.
  - Each time energy is harvested, the hot rock that heats the steam cools temporarily, causing a delay.
  - Advantages include
    - long-term affordability
    - low consumption of land
  - Disadvantages include
    - corrosiveness of liquids involved.
    - short-term non renewability.
- **Tidal energy** is the energy harvested from the bulk movement of a water's elevation near coastal areas.
  - **Tides** are the daily rise and fall of a water's elevation near a shore.
  - The rising water is stored in a container and released through water turbines.
  - Tides are powered by the slight increase and decrease of the moon's distance from the earth each day.
  - Advantages include
    - no negative environmental impact
    - does not take up land
  - Disadvantages include
    - low productivity per day
    - dependence of water movement on seasons
    - high cost per useful energy produced

Non-Renewable Sources of Energy

- **Non-renewable energy sources** are sources of energy that can be exausted.
- While the primary advantage of non-renewable energy sources is their affordability and reliability, they are scare and often pose serious environmental risks.
- **Fossil fuel energy** is the energy harvested from the burning of materials that have formed in the earth naturally over millions of years.
  - Examples include
    - petrol
    - oil
    - kerosene
    - propane
    - natural gas
    - coal
  - Advantages include
    - short-term affordability
    - high energy density per volume and mass
    - portability
    - reliability
  - Disadvantages include
    - environmental pollution
    - depletion of the ozone layer
- **Nuclear fission energy** is the energy harvested from the heat released by certain radioactive particles.
  - The heat is used to boil water into steam which expands and turns steam turbines.
  - Advantages include
    - ability to be installed anywhere
    - high productivity
    - usability of some radioactive by-products in medicine
    - relative affordability of fuel (uranium and/or plutonium)
  - Disadvantages include
    - environmental pollution of some radioactive by-products
    - high installation cost
    - risk of radioactive poisoning to workers
    - risk of catastrophic radioactive meltdown

GCE Paper 1 Questions

1. Most of Earth's energy comes from

   A coal B the sun C oil D natural gas

2. Which of the following energy sources is renewable?

   A coal B oil C hydroelectric power D natural gas

3. Geothermal energy results in

   A heavy rainfall B folding C volcanic eruption D high pressure

Questions 4 through 7 refer to the following sources energy

A tidal B hydroelectric C geothermal D solar

Which of these sources of energy

4. uses the energy carried by rushing water?
5. depends on the motion of the moon?
6. uses the energy in hot rocks?
7. absorbs energy radiated by the sun?

8. Which of the following transducers is capable of converting electrical energy to kinetic energy?

   A a radio B a torch C an electric fan D an electric iron

9. Which of the following transducers converts mechanical energy to electrical energy?

   A an electric bulb B a transformer C an electric motor D a generator

10. A frictionless cart of mass 300 kg is pushed on a straight, horizontal path using a horizontal force of 80 N over a distance of 5 m. The work done on the cart, in joules, is

    A 400 B 600 C 1900 D 19000

GCE Paper 1 Solutions

1. B 2. C 3. D 4. B 5. A 6. C 7. D 8. C 9. D 10. A

GCE Paper 2 Questions

1. A horizontal force of 42.16 N moves an object a horizontal distance of 5 km.

   (a) Calculate the work done on the object, in Joules. **(3 mks)**
   (b) Calculate the work done on the object, in $kcal$. **(2 mks)**
   (c) Calculate the work done on the object, in btu. **(3 mks)**
   (d) Calculate the distance moved, in km, if the same force is applied in order to accomplish 1 MJ of work on the object. **(3 mks)**

. . . .

Solution

(a) *The unit equivalence* $1\ km = 1000\ m$ *is used.*

$$\text{given equation for work: } W = Fd$$

$$\text{substituting known values: } W = (42.16\ N)(5\ km)$$

$$\text{applying conversion factor: } W = (42.16\ N)(5\ km)\left(\frac{1000\ m}{1\ km}\right)$$

$$\text{final answer: } \boxed{W = 2.108 \times 10^5\ J}$$

(b) *The unit equivalence* $1\ kcal = 4.186 \times 10^3\ J$ *is used.*

$$\text{given value of work in Joules: } W = 2.108 \times 10^5\ J$$

$$\text{applying conversion factor: } W = \left(2.108 \times 10^5\ J\right)\left(\frac{1\ btu}{4.186 \times 10^3\ J}\right)$$

$$\text{final answer: } \boxed{W \approx 50.358\ kcal}$$

(c) *The unit equivalence* $1\ btu = 1.054 \times 10^3\ J$ *is used.*

$$\text{given value of work in Joules: } W = 2.108 \times 10^5\ J$$

$$\text{applying conversion factor: } W = \left(2.108 \times 10^5\ J\right)\left(\frac{1\ btu}{1.054 \times 10^3\ J}\right)$$

$$\text{final answer: } \boxed{W = 200\ btu}$$

(d) *The unit equivalence* $1\ MJ = 10^6\ J$ *is used.*

$$\text{given equation for work: } W = Fd$$

$$\text{turning distance into subject: } d = \frac{W}{F}$$

$$\text{substituting known values: } d = \frac{1\ MJ}{42.16\ N}$$

$$\text{applying conversion factors: } d = \left(\frac{1\ MJ}{42.16\ N}\right)\left(\frac{1 \times 10^6\ J}{1\ MJ}\right)\left(\frac{1\ km}{1000\ m}\right)$$

$$\text{final answer: } \boxed{d \approx 23.7192\ km}$$

---

2. An body having a mass of 15 kg is lifted from the ground by a vertical force that is large enough only to offset its weight. A work of 600 J is done in this process.

   (a) Calculate the vertical distance through which the object is lifted. **(3 mks)**
   (b) Identify all the forces acting on this object. **(2 mks)**
   (c) Calculate the work done in lifting a 300 g object through the same vertical distance. **(3 mks)**
   (d) Calculate the vertical distance if the work in (a) is done in lifting the object in (c). **(3 mks)**

. . . .

Solution

(a) *The unit equivalence* $1\ kg = 1000\ g$ *is used.*

$$\text{given equation for work: } W = Fd$$

$$\text{turning distance into subject: } d = \frac{W}{F}$$

$$\text{substituting equation for weight: } d = \frac{W}{mg}$$

$$\text{substituting known values: } d = \frac{600\ J}{(15\ kg)\,(10\ m\ s^{-2})}$$

$$\text{final answer: } \boxed{d = 4\ m}$$

(b) The downward force of weight and the upward force of tension are the two forces acting on this object.

(c) *The unit equivalence* $1\ kg = 1000\ g$ *is used.*

$$\text{given equation for work: } W = Fd$$

$$\text{substituting equation for weight: } W = (mg)(d)$$

$$\text{substituting known values: } W = \left[(300\ g)\left(10\ m\ s^{-2}\right)\right](4\ m)$$

$$\text{applying conversion factor: } W = \left[(300\ g)\left(\frac{1\ kg}{1000\ g}\right)\left(10\ m\ s^{-2}\right)\right](4\ m)$$

$$\text{final answer: } \boxed{W = 12\ J}$$

(d) *The unit equivalence* $1\ kg = 1000\ g$ *is used again.*

$$\text{given equation for work: } W = Fd$$

$$\text{turning distance into subject: } d = \frac{W}{F}$$

$$\text{substituting equation for weight: } d = \frac{W}{mg}$$

$$\text{substituting known values: } d = \frac{600\ J}{(300\ g)\,(10\ m\ s^{-2})}$$

$$\text{applying conversion factor: } d = \frac{600\ J}{(300\ g)\left(\frac{1\ kg}{1000\ g}\right)(10\ m\ s^{-2})}$$

$$\text{final answer: } \boxed{d = 200\ m}$$

3.5.2 Potential Energy

Objectives

By the end of the lesson, students should be able to

1. define potential energy and state its unit.
2. solve problems relating a body's potential energy to its mass, height and the acceleration of gravity.

Effort Actions and Potential Energy

- **Effort** is the action that one inputs into a process in order to achieve a result.

- When work is done on a body to move it horizontally, the the force of friction is often the only force that must be overcome by the effort force in order to achieve body's horizontal displacement.

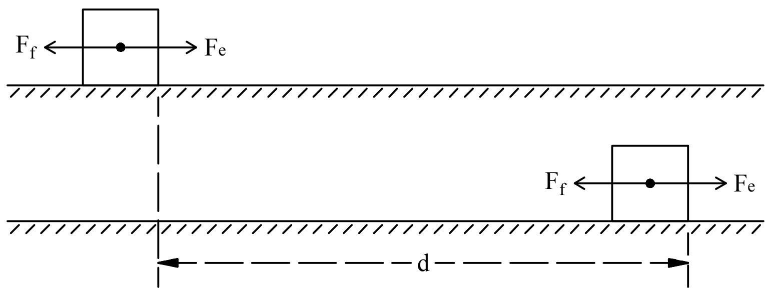

Figure 3.5.2.1

$$\text{for horizontal movement: } F_e = F_f$$

$$\text{subtituting into equation for work: } W_h = F_f d$$

Where
- F_e is the effort force, in N;
- F_f is the force of friction, in N;
- W_h is the work required to achieve the horizontal displacement, in J;
- d is the body's horizontal displacement, in m.

- When work is done on a body to lift it vertically, the force of the body's weight is often the only force that must be overcome by the effort force in order to achieve the body's vertical displacement.

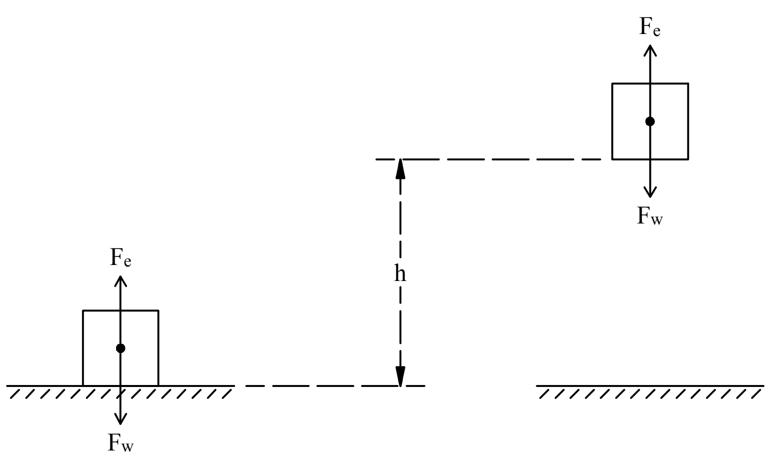

Figure 3.5.2.2

$$\text{for vertical movement: } F_e = F_w$$

$$\text{subtituting into equation for work: } W_v = F_w h$$

Where
- F_e is the effort force, in N;
- F_w is the force of weight, in N;
- W_v is the work required to achieve the vertical displacement, in J;
- h is the body's vertical displacement, or height, in m.

- Once work has been done on a body to lift it vertically, the energy the body has in reference to its original height is referred to as "potential energy".
- **Potential energy**, or **PE**, is the energy possessed by a body by virtue of its height in reference to another.
- Like all energy, potential energy is a scalar whose SI unit is the Joule, abbreviated J.

$$\text{assuming all vertical work is stored in body's potential energy: } PE = W_v$$

$$\text{substituting potential energy into equation for vertical work: } PE = F_w h$$

$$\text{substituting equation for weight: } PE = mgh \tag{3.5.2.1}$$

Where
- PE is the body's potential energy, in J;
- F_w is the body's force of weight, in N;
- h is the body's height above some reference, in m;
- m is the body's mass, in kg;
- g is the acceleration of gravity, in $m\ s^{-2}$.

- NB: This type of potential energy is strictly concerned with linear motion. There are other forms of potential energy, such as electric potential energy and elastic potential energy.
- A body's potential energy is entirely dependent on the reference surface against which its height is considered.

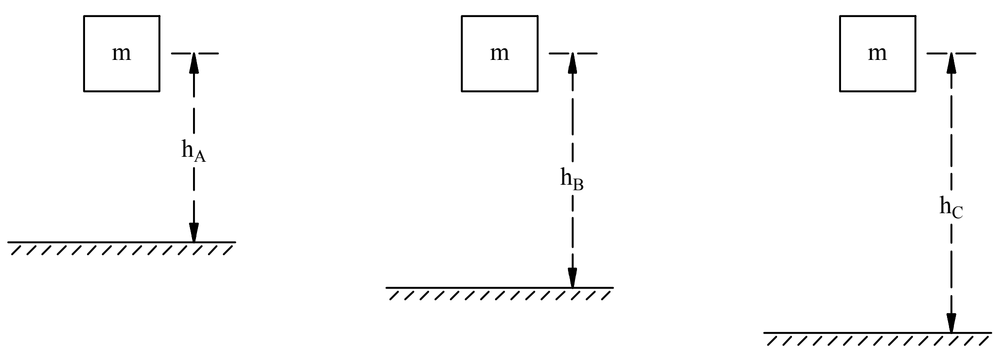

Figure 3.5.2.3

- As shown in figure 3.5.2.3, the same mass at the same position has three different potential energies above three different reference surfaces.

$$\text{comparing heights: } h_C > h_B > h_A$$

$$\text{multiplying through by mass and acceleration of gravity: } mgh_C > mg_B > mg_A$$

$$\text{substituting potential energy: } PE_C > PE_B > PE_A$$

Potential Energy on Inclined Planes

- When a body is pulled up an inclined plane, the effort force F_e must overcome both the force of weight F_w as well as the force of friction F_f.

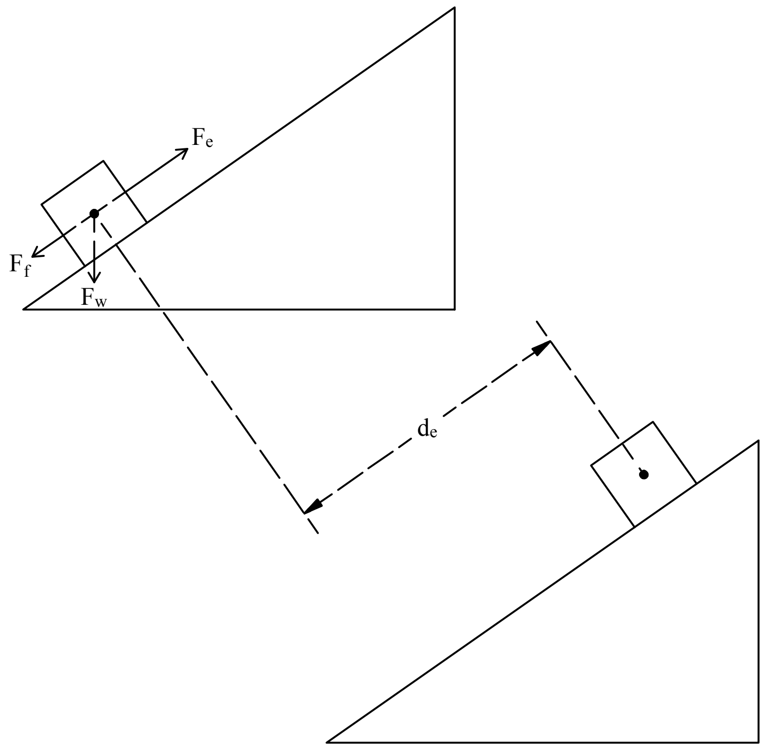

Figure 3.5.2.4

- That is, on an inclined plane, the effort force required is not equal to the force of friction.

$$\text{on an inclined plane: } F_e \neq F_f$$

- Nor is the effort force equal to the force of the body's weight.

$$\text{on an inclined plane: } F_e \neq F_w$$

- Given the angled orientation of the force of weight against the effort force, F_e is not precisely the sum of the force of friction and the force of weight.

$$\text{on an inclined plane: } F_e \neq F_f + F_w$$

- The work done against the force of friction while moving the body up the inclined plane is the product of the friction's force and the inclined distance moved.

$$\text{on an inclined plane: } W_f = F_f d$$

  Where
  - W_f is the work done against friction, in J;
  - F_f is the force of friction, in N;
  - d is the distance the body moves along the inclined plane, parallel to the friction force, in m.

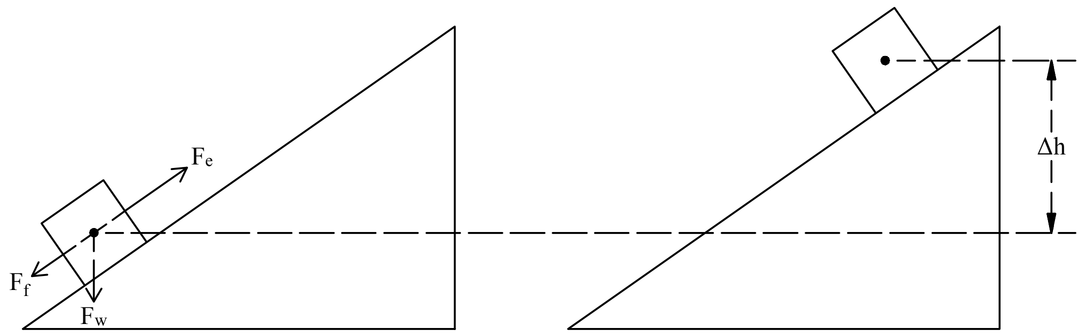

Figure 3.5.2.5

- The vertical work done against the force of weight is the product of the body's force of weight and the difference in its height.

$$\text{on an inclined plane: } W_v = F_w \Delta h$$

  Where
  - W_v is the vertical work done against the body's force of weight, in J;
  - F_w is the body's force of weight, in N;
  - Δh is the difference in the body's height, in m.

- The work done against the body's weight is the body's potential energy at the top of the plane.

$$\text{on an inclined plane: } PE = F_w \Delta h$$

$$\text{substituting equation for weight: } PE = mg\Delta h$$

  Where
  - PE is the body's potential energy at the top of the plane, in J;
  - F_w is the body's force of weight, in N;
  - Δh is the difference in the body's height, in m;
  - m is the body's mass, in kg;
  - g is the acceleration of gravity, in $m\ s^{-2}$.

- The total energy input into the process of lifting the body along the plane is the product of the effort force and the distance moved parallel to the effort force.

$$\text{on an inclined plane: } E_{\text{total}} = F_e d$$

  Where
  - E_{total} is the total energy input into moving the body, in J;
  - F_e is the effort force, in N;
  - d is the distance the body moves along the inclined plane, parallel to the effort force, in m.

- E_{total} is the sum of the work done against friction and the work done against the body's weight.

$$\text{summing all work: } E_{\text{total}} = W_f + W_v$$

$$\text{substituting potential energy: } E_{\text{total}} = W_f + PE$$

$$\text{substituting equation for effort: } F_e d = W_f + PE$$

- If there is no force of friction, all input energy becomes the body's potential energy.

$$\text{on a frictionless, inclined plane: } E_{\text{total}} = PE$$

$$\text{substituting equation for effort, on frictionless, inclined plane: } F_e d = PE$$

GCE Paper 1 Questions

1. A bucket of sand is lifted by pulling a rope passing over a pulley. The energy gained by the bucket of sand is

   A thermal B nuclear C potential D internal

2. A ball of mass 0.5 kg is at a certain height above the ground. If the acceleration due to gravity is 10 $m\ s^{-2}$ and the energy at that height is 20 J, then the height above the ground is

   A 40 m B 10 m C 4 m D 100 m

Questions 3 through 6 refer to figure 3.5.2.6, which shows three body's of masses m_1, m_2 and m_3 at three different heights above a reference surface. Assume $g = 10\ m\ s^{-2}$.

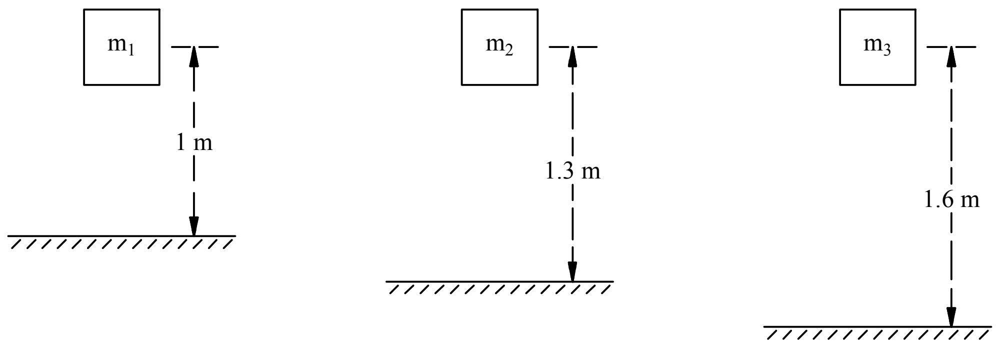

Figure 3.5.2.6

3. If it is given that all three bodies have the same mass, which of the following statements is true?

   A PE_3 is greater than PE_1 and PE_2.
   B PE_2 is greater than PE_1 and PE_3.
   C PE_1 is greater than PE_2 and PE_3.
   D PE_3 is greater than PE_1 but less than PE_2.

4. The following data for the bodies' masses are given:

   $m_1 = 3.7\ kg$ $m_2 = 4.6\ kg$ $m_3 < m_1$

   Which of the following bodies has the greatest potential energy?

   A m_1 B m_2 C m_3 D all have equal PE

5. The following data for the bodies' masses are given:

   $m_1 = 2.8\ kg$ $m_2 = 39\ g$ $m_3 = 600\ mg$

   Which of the following statements is true?

   A $PE_1 = 2.8\ J$, $PE_2 = 0.507\ J$ and $PE_3 = 0.0096\ J$
   B $PE_1 = 28\ J$, $PE_2 = 507\ J$ and $PE_3 = 0.0096\ J$
   C $PE_1 = 28\ J$, $PE_2 = 0.507\ J$ and $PE_3 = 9.6\ J$
   D $PE_1 = 28\ J$, $PE_2 = 0.507\ J$ and $PE_3 = 0.0096\ J$

6. Given the same data in question 5, which of the following statements is true?

   A $PE_1 > PE_2 > PE_3$
   B $PE_1 < PE_2 < PE_3$
   C $PE_2 > PE_1 > PE_3$
   D $PE_2 > PE_1$ and $PE_2 = PE_3$

Questions 7 through 10 refer to figure 3.5.2.7, which shows a body of mass m before it is pulled up a plane at time t_1 and after it is pulled up a plane at time t_2. Assume $g = 10\ m\ s^{-2}$.

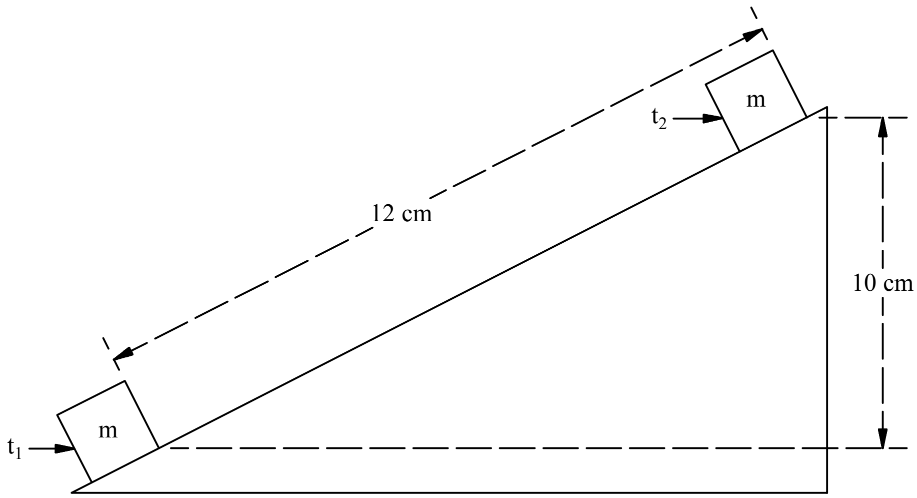

Figure 3.5.2.7

7. If it is given that the body's mass is 2.2 kg, the work done against the force of weight between t_1 and t_2 is

   A 0.022 J B 0.22 J C 2.2 J D 22 J

8. Given the same mass as in question 7, the body's gain in potential energy between t_1 and t_2 is

   A 0.022 J B 0.22 J C 2.2 J D 22 J

9. If it is given that the total energy input is equal to the answer to question 8, the value of F_f is likely

   A 0 N B 0.12 N C 1.36 N D 12 N

10. If it is given that the work done against friction between t_1 and t_2 is 1.2 kJ, the force of friction is

    A 10 N B 100 N C 1000 N D 10000 N

GCE Paper 1 Solutions

1. C 2. C 3. A 4. B 5. D 6. A 7. C 8. C 9. A 10. D

GCE Paper 2 Questions

1. Figure 3.5.2.8 shows a mass 200 kg being pulled up an inclined plane at a constant speed by a force of 1500 N.

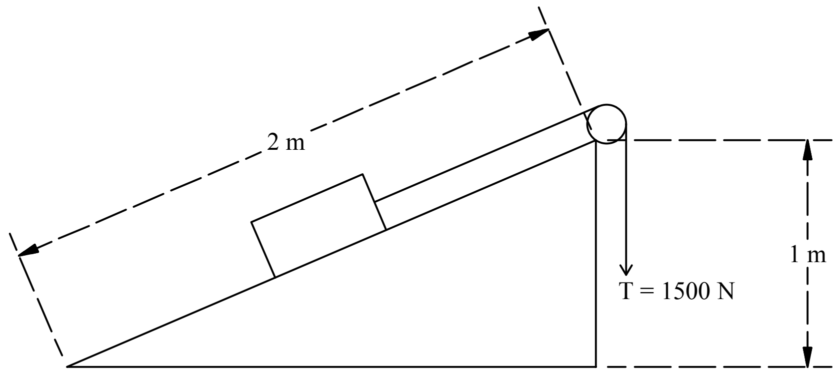

Figure 3.5.2.8

(a) Calculate the gain in the mass' potential energy as it moves up the plane. **(3 mks)**
(b) Express the answer to (a) in kiloJoules. **(2 mks)**
(c) Determine the work done by force (T) in moving the block along the length of the plane. **(2 mks)**
(d) Account for the difference between the values obtained in subsections 1 (a) and 1 (c). **(1 mk)**

. . . .

Solution

(a) *The gain in potential energy is the product of the block's weight and the vertical distance lifted.*

given equation for potential energy on an inclined plane: $PE = mg\Delta h$

substituting known values: $PE = (200\ kg)\left(10\ m\ s^{-2}\right)(1\ m)$

final answer: $\boxed{PE = 2000\ J}$

(b) *The unit equivalence* $1\ kJ = 1000\ J$ *is used.*

given value in Joules: $PE = 2000\ J$

applying conversion factor: $PE = (2000\ J)\left(\frac{1\ kJ}{1000\ J}\right)$

final answer: $\boxed{PE = 2\ kJ}$

(c) *The work done by* T *along the plane is the product of its magnitude and the length of the plane.*

given equation for work $W_T = F \times d$

substituting known values: $W_T = (1500\ N)(2\ m)$

final answer: $\boxed{W_T = 3000\ J}$

(d) Some of the input work from the pull of T across the length of the plane is lost due to the force of friction between the block and the plane.

2. An inclined plane of length 4 m is used to raise a body of mass 20 kg through a vertical height of 1 m. It is found that an effort of 80 N is necessary to move the mass up the plane's slope at a constant speed.

   (a) Calculate the work done by the effort. **(2 mks)**
   (b) Assuming the main purpose of the inclined plane is to lift the body from one lower position to a higher one, calculate the useful work done on the load. **(2 mks)**
   (c) Calculate the ratio of the answer to (b) over the answer to (a). **(2 mks)**
   (d) If the answer to (c) is not equal to 1, explain why. **(2 mks)**

. . . .

Solution

(a) *The work done by the effort is the product of its force's magnitude and the plane's length.*

$$\text{given equation for work } W = Fd$$

$$\text{considering work done by effort: } W_{\text{effort}} = F_e d$$

$$\text{substituting known values: } W_{\text{effort}} = (80\ N)(4\ m)$$

$$\text{final answer: } \boxed{W_{\text{effort}} = 320\ J}$$

(b) *It is assumed the useful work is that done against the force of weight.*

$$\text{given equation for work against weight } W_v = F_w h$$

$$\text{substituting equation for weight: } W_v = mgh$$

$$\text{substituting known values: } W_v = (20\ kg)\left(10\ m\ s^{-2}\right)(1\ m)$$

$$\text{final answer: } \boxed{W_v = 200\ J}$$

(c) *This question is an introduction to efficiency.*

$$\text{requested values: ratio} = \frac{W_v}{W_{\text{effort}}}$$

$$\text{subtituting known values: ratio} = \frac{200\ J}{320\ J}$$

$$\text{final answer: } \boxed{\text{ratio} = 0.625}$$

(d) The answer to (c) is less than 1 because the useful output work done lifting the body, W_v, is less than the input effort work F_e. This is because some of the input effort work, W_{effort}, is lost to the work needed to overcome friction, W_f.

3.5.3 Kinetic Energy

Objectives

By the end of the lesson, students should be able to

1. define kinetic energy and state its unit.
2. solve problems relating a body's kinetic energy to its mass and speed.

Kinetic Energy

- If a body is released from a particular height, its speed increases as its height decreases in time.

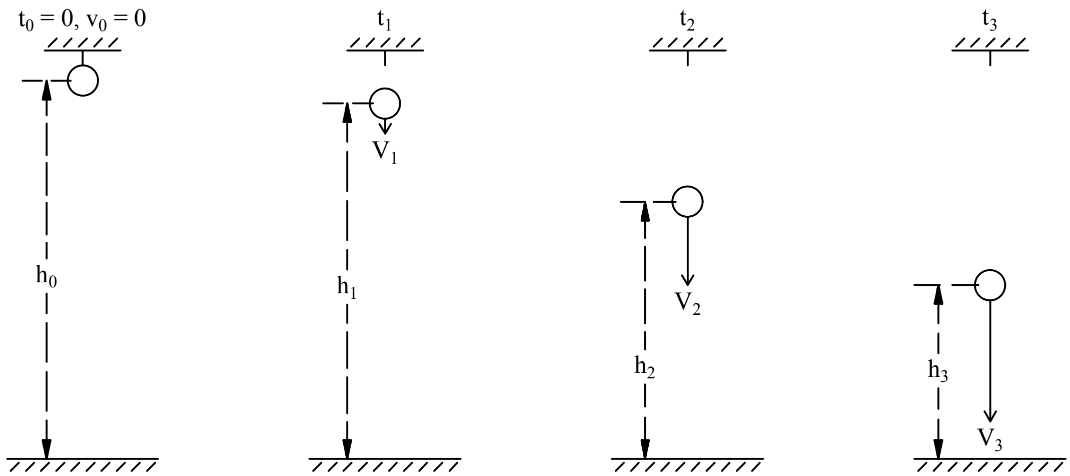

Figure 3.5.3.1

- Assuming no energy is lost to friction, all of its initial potential energy is gradually transformed into the energy associated with its falling motion.

- This energy related to the body's motion is referred to as "kinetic energy".

- **Kinetic energy**, or **KE**, is the energy which a body possesses by virtue of being in motion.

- Like all energy, kinetic energy is a scalar whose SI unit is the Joule, abbreviated J.

- The kinetic energy of a body is directly proportional to its mass.

$$KE \propto m$$

$$KE = km$$

- A body's kinetic energy is also directly proportional to the square of its speed.

$$KE \propto v^2$$

$$KE = kv^2$$

- Therefore, a body's kinetic energy depends on both its mass as well as its speed.

$$KE = \frac{1}{2}mv^2 \tag{3.5.3.1}$$

Where
  - KE is the body's kinetic energy, in J;
  - m is the body's mass, in kg;
  - v is the body's speed, in $m\ s^{-1}$.

- The equation for kinetic energy is dimensionally homogeneous.

$$\text{given equation: } KE = \frac{1}{2}mv^2$$

$$\text{in units: } J = (kg)\left(\frac{m}{s}\right)^2$$

$$J = (kg)\left(\frac{m^2}{s^2}\right)$$

$$J = (kg)\left(\frac{m}{s^2}\right)m$$

$$J = \left(kg\frac{m}{s^2}\right)m$$

$$J = Nm$$

$$J = J$$

- Problems involving kinetic energy can require that one of the following three properties be calculated while the other two are given - energy, mass and speed.

  Example: A 1.5 kg bottle of water is thrown with a speed of 3 $m\ s^{-1}$. Calculate the bottle's kinetic energy.

$$\text{given equation for kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{substituting known values: } KE = \frac{1}{2}(1.5\ kg)\left(3\ m\ s^{-1}\right)^2$$

$$\text{final answer: } \boxed{KE = 6.75\ J}$$

  Example: Calculate the mass of a body that travels at 20 $m\ s^{-1}$ with a kinetic energy of 800 J.

$$\text{given equation for kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{turning mass into subject: } m = \frac{2KE}{v^2}$$

$$\text{substituting known values: } m = \frac{2(800\ J)}{(20\ m\ s^{-1})^2}$$

$$\text{final answer: } \boxed{m = 4\ kg}$$

- **Example:** Calculate the speed of a 3 kg block that travels with a kinetic energy of 13.5 J.

$$\text{given equation for kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{turning speed into subject: } v = \sqrt{\frac{2KE}{m}}$$

$$\text{substituting known values: } v = \sqrt{\frac{2(13.5\ J)}{3\ kg}}$$

$$\text{final answer: } \boxed{v = 3\ m\ s^{-1}}$$

GCE Paper 1 Questions

1. The kinetic energy of a stone of mass 150 g in flight is 216 J. The speed of flight, in $m\ s^{-1}$, is approximately

   A 2880 B 1440 C 54 D 6

2. The kinetic energy of a 2 kg body is 100 J. Its speed, in $m\ s^{-1}$ is

   A 100 B 50 C 0.1 D 10

3. It is desired to double the kinetic energy of a body. This can be done by

   A doubling its speed and keeping its mass the same.
   B doubling its mass and keeping its speed the same.
   C doubling its volume and keeping its mass and speed the same.
   D doubling its speed and quadrupling its mass.

   Questions 4 through 6 refer to figure 3.5.3.2, which shows four bodies of masses m_1, m_2, m_3 and m_4 with four different speeds shown.

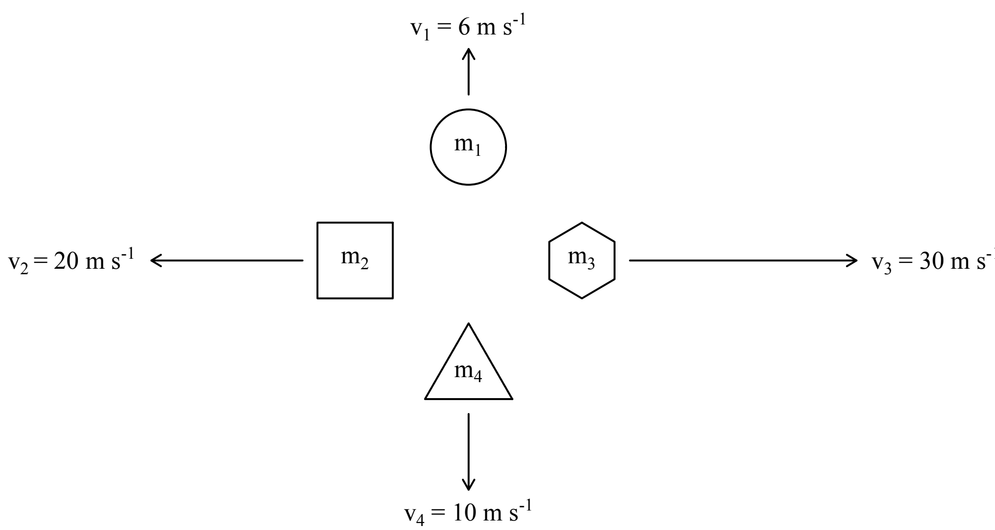

Figure 3.5.3.2

4. Which speed is the greatest?

   A v_1 B v_2 C v_3 D v_4

5. The following data for the bodies' masses are given:

   $m_1 = 6.3\ kg$ $m_2 < \frac{m_1}{15}$ $m_3 < \frac{m_1}{100}$ $m_4 = 63\ g$

   Which body has the greatest kinetic energy?

   A body 1 B body 2 C body 3 D body 4

6. Considering the same given data in question 5, which body has the least kinetic energy?

   A body 1 B body 2 C body 3 D body 4

7. If a ball of mass 0.5 kg has a kinetic energy of 100 J, then its speed is

   A 10 $m\ s^{-1}$ B 20 $m\ s^{-1}$ C 100 $m\ s^{-1}$ D 40 $m\ s^{-1}$

Questions 8 through 10 refer to figure 3.5.3.3, which shows three bodies A, B and C with three given masses in a race. Their speeds are also shown as v_A, v_B, and v_C.

8. If it is given that all three bodies have the same kinetic energy, which body crosses the finish line first?

   A body A
   B body B
   C body C
   D A and B cross as the same time.

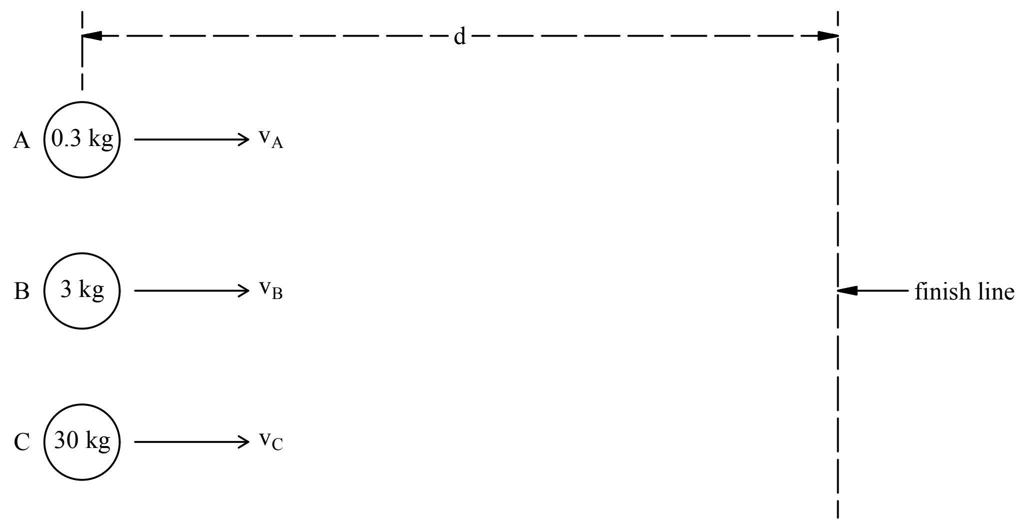

Figure 3.5.3.3

9. If the following data for the bodies' kinetic energies are given:

   $KE_A = 0.6\ J$ $KE_B = 54\ J$ $KE_C = 60\ J$

   Which body crosses the finish line first?

   A body A
   B body B
   C body C
   D Bodies A and C cross together first.

10. Considering the same given data in question 9, which body crosses the finish line last?

   A body A
   B body B
   C body C
   D Bodies A and C cross together last.

GCE Paper 1 Solutions

1. C 2. D 3. B 4. C 5. A 6. D 7. B 8. A 9. B 10. D

GCE Paper 2 Questions

1. A taxi has a mass of 1000 kg and a kinetic energy of 32 kJ. On the same road, a lorry with a weight of 7×10^4 N travels with a speed of 18 $km\ hr^{-1}$.

   (a) Determine the taxi's speed in SI units. **(3 mks)**
   (b) Determine the taxi's speed in $km\ hr^{-1}$. **(2 mks)**
   (c) Determine the lorry's speed, in $m\ s^{-1}$. **(2 mks)**
   (d) Determine the lorry's mass, in kg. **(2 mks)**
   (e) Determine the lorry's kinetic energy, in kJ. **(2 mks)**
   (f) Which vehicle has more kinetic energy? **(1 mk)**
   (g) Which vehicle travels faster? **(1 mk)**
   (h) Account for any differences in (f) and (g). **(2 mks)**

....

Solution

(a) *The unit equivalence* 1 kJ = 1000 J *is used.*

$$\text{given equation for kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{turning speed into subject: } v = \sqrt{\frac{2KE}{m}}$$

$$\text{substituting known values: } v = \sqrt{\frac{2(32\ kJ)}{1000\ kg}}$$

$$\text{applying conversion factor: } v = \sqrt{\frac{2(32\ kJ)\left(\frac{1000\ J}{1\ kJ}\right)}{1000\ kg}}$$

$$\text{final answer: } \boxed{v_{\text{taxi}} = 8\ m\ s^{-1}}$$

(b) *The unit equivalences* 1 km = 1000 m, 1 hr = 60 min *and* 1 min = 60 s *are used.*

$$\text{given value in metres per second: } v = 8\ m\ s^{-1}$$

$$\text{applying conversion factors: } v = \left(8\ \frac{m}{s}\right)\left(\frac{60\ s}{1\ min}\right)\left(\frac{60\ min}{1\ hr}\right)\left(\frac{1\ km}{1000\ m}\right)$$

$$\text{final answer: } \boxed{v_{\text{taxi}} = 28.8\ km\ hr^{-1}}$$

(c) *The same unit equivalences are used as (b), but with the reverse conversion.*

$$\text{given value in kilometres per hour } v_{\text{lorry}} = 18\ km\ hr^{-1}$$

$$\text{applying conversion factor: } v_{\text{lorry}} = \left(18\ \frac{km}{hr}\right)\left(\frac{1000\ m}{1\ km}\right)\left(\frac{1\ hr}{60\ min}\right)\left(\frac{1\ min}{60\ s}\right)$$

$$\text{final answer: } \boxed{v_{\text{lorry}} = 5\ m\ s^{-1}}$$

(d) *It is assumed that* $g = 10\ m\ s^{-2}$.

$$\text{given equation for force of weight: } F_w = mg$$

$$\text{turning mass into subject: } m = \frac{F_w}{g}$$

$$\text{substituting known values: } m = \frac{7 \times 10^4\ N}{10\ m\ s^{-2}}$$

$$\text{final answer: } \boxed{m_{\text{lorry}} = 7 \times 10^3\ kg}$$

(e) *The unit equivalence* $1\ kJ = 1000\ J$ *is used.*

$$\text{given equation kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{subtituting known values: } KE = \frac{1}{2}\left(7 \times 10^3\ kg\right)\left(5\ m\ s^{-1}\right)^2$$

$$\text{final answer in Joules: } KE = 8.75 \times 10^4\ J$$

$$\text{applying conversion factor: } KE = \left(8.75 \times 10^4\ J\right)\left(\frac{1\ kJ}{1000\ J}\right)$$

$$\text{final answer: } \boxed{KE_{\text{lorry}} = 87.5\ kJ}$$

(f) Given that $KE_{\text{taxi}} = 32\ kJ$ and $KE_{\text{lorry}} = 87.5\ kJ$, the lorry has more kinetic energy.

(g) Given that $v_{\text{taxi}} = 28.8\ km\ hr^{-1}$ and $v_{\text{lorry}} = 18\ km\ hr^{-1}$, the taxi travels faster (has a higher speed).

(h) Though the taxi has a greater speed than the lorry, the lorry's significantly greater mass gives it more kinetic energy in this scenario.

3.5.4 Mechanical Energy

Objectives

By the end of the lesson, students should be able to
1. state the law of conservation of energy.
2. solve problems involving the exchange between a body's potential and kinetic energy when in motion.

Mechanical Energy

- Though the energy associated with a body's motion can convert between potential and kinetic, the sum of these energies is always constant, assuming no other energy losses occur.

- **Mechanical energy**, or **ME**, is the sum of a body's kinetic energy and potential energy.

$$ME = PE + KE \tag{3.5.4.1}$$

  Where
  - ME is the body's total mechanical energy, in J;
  - PE is the body's potential energy, in PE;
  - KE is the body's kinetic energy, in KE.

- Like all energy, mechanical energy is a scalar whose SI unit is the Joule, abbreviated J.

  NB: While other fields of physics consider other forms of energy to also be mechanical, this chapter focuses only on the mechanical energy associated with a body's movement.

- A body's mechanical energy can also be expressed in terms of its mass, height above a reference, speed and the acceleration of gravity.

$$\text{given equation for mechanical energy: } ME = PE + KE$$

$$\text{substituting equations for potential and kinetic energy: } ME = mgh + \frac{1}{2}mv^2 \tag{3.5.4.2}$$

  Where
  - ME is the body's total mechanical energy, in J;
  - m is the body's mass, in kg;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - h is the body's height above a reference surface, in m;
  - v is the body's speed, in $m\ s^{-1}$.

Conservation of Energy

- **The law of conservation of energy** states that energy may be transformed from one from to another, but it can neither be created nor destroyed.

- This law can be applied to a body's mechanical energy before and after a certain motion, assuming it doesnt gain or loose any energy from or to other forms, such as thermal or sound.

- This law can be applied to a body's mechanical energy before and after some motion.

$$\text{considering a body's mechanical energy at two points in time: } ME_i = ME_f$$

$$\text{substituting equation for mechanical energy: } PE_i + KE_i = PE_f + KE_f \tag{3.5.4.3}$$

  Where
  - ME_i is the body's initial mechanical energy, in J;
  - ME_f is the body's final mechanical energy, in J;
  - PE_i is the body's initial potential energy, in J;
  - KE_i is the body's initial kinetic energy, in J;

- PE_f is the body's final potential energy, in J;
- KE_f is the body's final kinetic energy, in J.

- As shown in figure 3.5.4.1, a stone being thrown upwards is a demonstration of the conservation of energy as it rises and falls again.

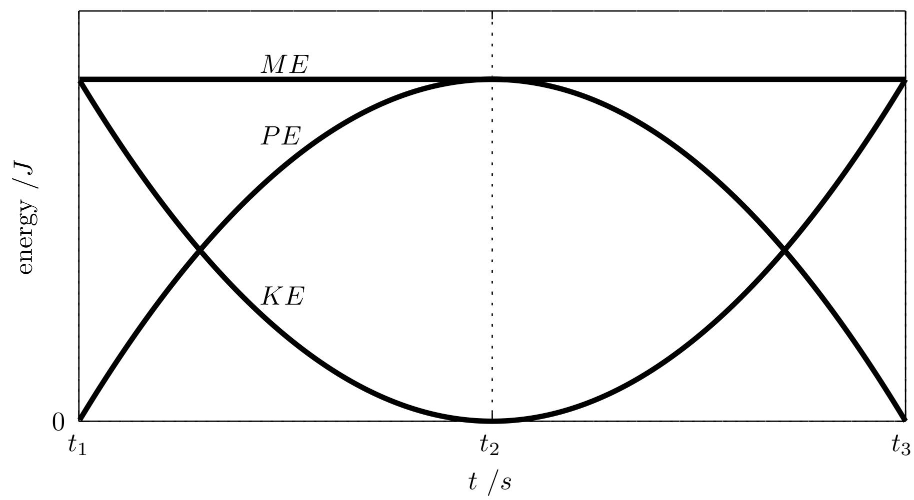

Figure 3.5.4.1

- At time t_1, the stone is first thrown up.
  * Its kinetic energy is maximized given the speed at which it is originally thrown.
  * Its potential energy is minimized given its height above the thrower's hand is zero when first thrown.
  * Its mechanical energy is entirely due to its kinetic energy.

- Between time t_1 and t_2, the stone is still moving up, but slowing down.
  * Its kinetic energy decreases as the stone looses speed.
  * Its potential energy increases as the stone rises above the thrower's hand.
  * Its mechanical energy is a mix of both kinetic and potential energy.

- At time t_2, the stone has stopped, reaching its maximum height.
  * Its kinetic energy is minimized given its speed at that exact moment is zero.
  * Its potential energy is maximized given its highest position above the thrower's hand.
  * Its mechanical energy is entire due to its potential energy.

- Between time t_2 and t_3, the stone is speeding up, but downwards.
  * Its kinetic energy increases as its speed increases downwards.
  * Its potential energy decreases as its height above the thrower's hand reduces.
  * Its mechanical energy is again a mix of both kinetic and potential energy.

- Finally, at time t_3, the stone has reached the original height of the thrower's hand, and it has the same properties of speed and height as it did at t_1.
  * Its kinetic energy is maximized given its downward speed being the same as its initial upward speed.
  * Its potential energy is minimized given that it has reached the same height as the thrower's hand.
  * Its mechanical energy is again entirely due to its kinetic energy.

- As shown in figure 3.5.4.2, a mango falling from a tree is another demonstration of the conservation of energy as it falls from rest and speeds up downwards.

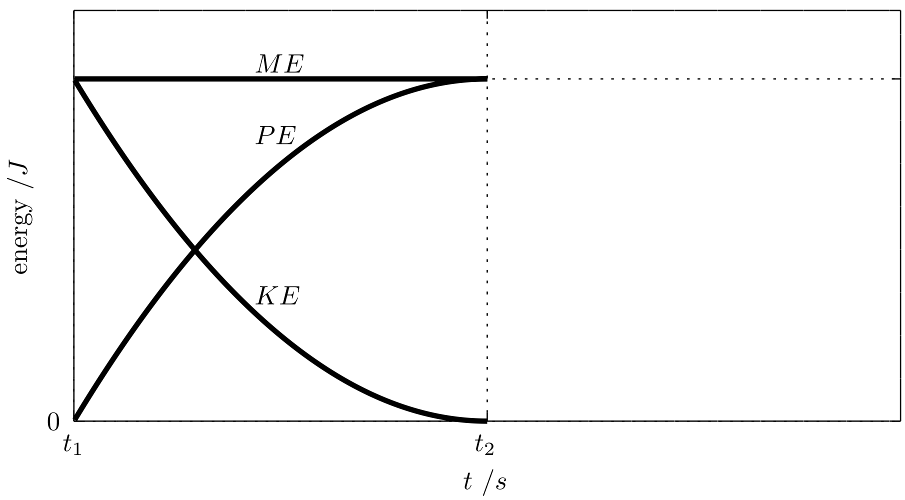

Figure 3.5.4.2

  - At time t_1, the mango first falls.
    - Its kinetic energy is minimized, given its initial speed of zero.
    - Its potential energy is maximized, given its maximum height above the ground.
    - Its mechanical energy is entirely due to its potential energy
  - Between times t_1 and t_2, the mango speeds up downwards.
    - Its kinetic energy increases as its speed increases.
    - Its potential energy decreases as its height above the ground decreases.
    - Its mechanical energy is a mix of both kinetic and potential energy.
  - At time t_2, the mango is just about to hit the ground.
    - Its kinetic energy is maximized as it reaches its maximum speed.
    - Its potential energy is minimized just as it reaches the ground below.
    - Its mechanical energy is entirely due to its kinetic energy.

- Problems involving mechanical energy often concern the complete transformation of a body's energy from one form (kinetic or potential) to the other.

- When a body is dropped from rest, its speed when it reaches the ground is a function of the dropping height.

given equation for mechanical energy: $$PE_i + KE_i = PE_f + KE_f$$

assuming initial kinetic and final potential energies are zero: $$PE_i = KE_f$$

substituting equations for potential and kinetic energies: $$mgh_i = \frac{1}{2}m\left(v_f\right)^2$$

turning final speed into subject: $$v_f = \sqrt{2\frac{mgh_i}{m}}$$

simplifying: $$v_f = \sqrt{2gh_i} \qquad (3.5.4.4)$$

Where
  - v_f is the body's final speed, just before hitting the ground, in $m\ s^{-1}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - h_i is the body's initial dropping height, in m.

- When a body is thrown upwards, its maximum height its a function of its initial throwing speed.

$$\text{given equation for mechanical energy:} \quad PE_i + KE_i = PE_f + KE_f$$

$$\text{assuming initial potential and final kinetic energies are zero:} \quad KE_i = PE_f$$

$$\text{substituting equations for potential and kinetic energies:} \quad \frac{1}{2}m\left(v_i\right)^2 = mgh_f$$

$$\text{turning final height into subject:} \quad h_f = \frac{\frac{1}{2}m\left(v_i\right)^2}{mg}$$

$$\text{simplifying:} \quad h_f = \frac{\left(v_i\right)^2}{2g} \tag{3.5.4.5}$$

  Where
  - v_i is the body's initial speed as it is first thrown up, in $m\ s^{-1}$;
  - g is the acceleration of gravity, in $m\ s^{-2}$;
  - h_f is the maximum height the body reaches before falling back down, in m.

- NB: Both equations 3.5.4.4 and 3.5.4.5 are independent of mass.

  Example: A ball at an initial height of 1.8 m above the ground is dropped from rest. What is its speed just before it hits the ground?

$$\text{given equation for mechanical energy:} \quad PE_i + KE_i = PE_f + KE_f$$

$$\text{assuming initial kinetic and final potential energies are zero:} \quad PE_i = KE_f$$

$$\text{substituting equations for kinetic and potential energy:} \quad mgh_i = \frac{1}{2}m\left(v_f\right)^2$$

$$\text{turning final speed into subject:} \quad v_f = \sqrt{2gh_i}$$

$$\text{substituting known values:} \quad v_f = \sqrt{2\left(10\ m\ s^{-2}\right)^2\left(1.8\ m\right)}$$

$$\text{final answer:} \quad \boxed{v_f = 6\ m\ s^{-1}}$$

  Example: A stone is thrown upwards with an initial speed of 4 $m\ s^{-1}$. Calculate its maximum height.

$$\text{given equation for mechanical energy:} \quad PE_i + KE_i = PE_f + KE_f$$

$$\text{assuming initial potential and final kinetic energies are zero:} \quad KE_i = PE_f$$

$$\text{substituting equations for kinetic and potential energy:} \quad \frac{1}{2}m\left(v_i\right)^2 = mgh_f$$

$$\text{turning final height into subject:} \quad h_f = \frac{\left(v_i\right)^2}{2g}$$

$$\text{substituting known values:} \quad h_f = \frac{\left(4\ m\ s^{-1}\right)^2}{2\left(10\ m\ s^{-2}\right)}$$

$$\text{final answer:} \quad \boxed{h_f = 0.8\ m}$$

Loss of Mechanical Energy to Other Forms

- A ball that is dropped and allowed to bounce is an example of a body whose mechanical energy is undergoing several cycles of exchange between its kinetic and potential forms.
- Ideally, as shown in figure 3.5.4.3, the total mechanical energy is conserved indefinitely.
  - The maximum value of PE repeats at times t_1, t_3, t_5, etc. as the ball reaches its highest rebound height.
  - The maximum value of KE repeats as times t_2, t_4, t_6, etc. as the ball impacts and bounces back from the ground as the same maximum speed.

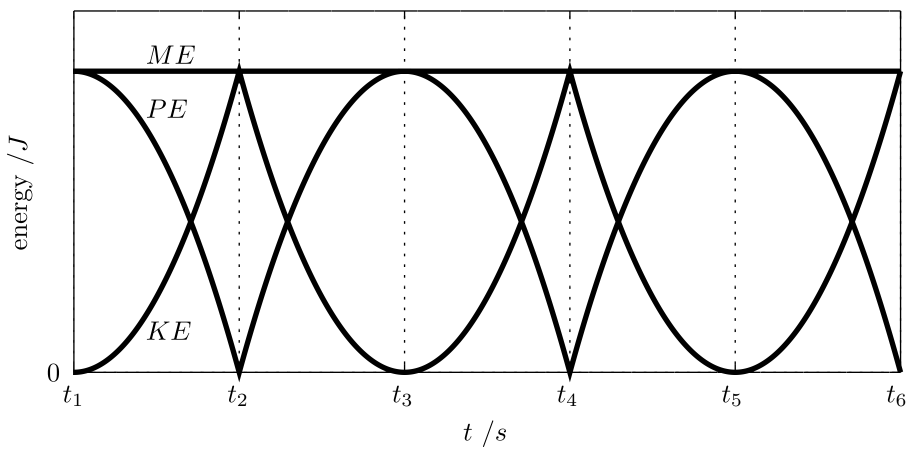

Figure 3.5.4.3

- However, in reality, the ball, like any body, will loose some energy to friction and heat during each bounce.
  - As shown in figure 3.5.4.4, this causes its total mechanical energy to decrease between bounces.
  - The maximum value of PE decreases between bounces as the ball reaches a lower and lower rebound height at times t_3, t_5, etc.
  - The maximum value of KE decreases during each bounce as the ball rises back from the ground at a lower speed than just before it hit the ground at times t_2, t_4, etc.

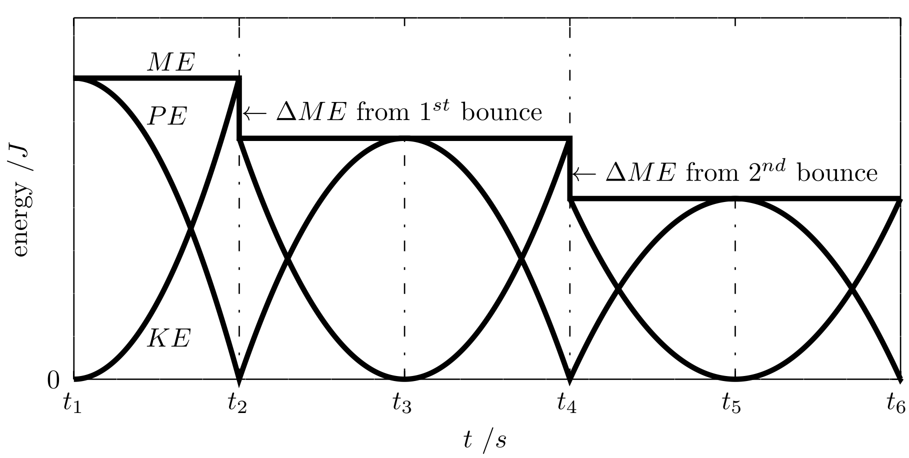

Figure 3.5.4.4

- This successive loss in mechanical energy continues with each bounce until the ball stops bouncing.

GCE Paper 1 Questions

1. A burst water pipe sends a jet of water up to a maximum vertical height of 5 m above the ground. The speed with which the water leaves the pipe in $m\ s^{-1}$ is

   A 200 B 100 C 10 D 5

2. Which of the following is the best order of energy changes taking place in a hydroelectric plant?

   A kinetic - electric - potential
   B kinetic - potential - electrical
   C potential - electrical - kinetic
   D potential - kinetic - electrical

3. A pen is dropped from rest above the ground. If its speed just before impact with the ground is 4.05 $m\ s^{-1}$, the height from which it was released is

   A 0.9 m B 9 m C 81 m D 9 $m\ s^{-1}$

Questions 4 through 7 refer to figure 3.5.4.5, which shows a stone with a mass of 2 kg on top of a mound. It is given that 72 J of work was done to move the stone up the smooth slope of the mound from X to Y.

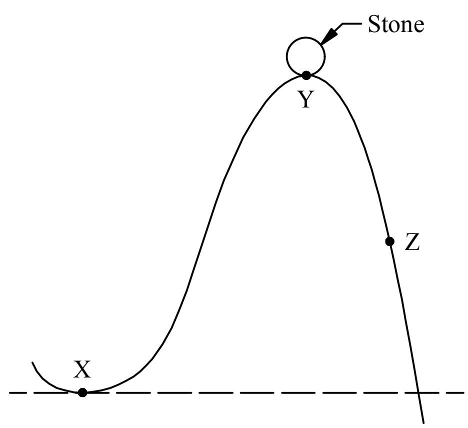

Figure 3.5.4.5

4. If the stone falls to the right, then its KE at Z, which has a height midway between X and Y is

   A 36 J B 54 J C 73 J D 108 J

5. The velocity of the stone as it passes Z is

   A 10.4 $m\ s^{-1}$ B 8.5 $m\ s^{-1}$ C 7.3 $m\ s^{-1}$ D 6 $m\ s^{-1}$

6. The velocity of the stone just before it reaches the ground below, at the same height as X is approximately

   A 10.4 $m\ s^{-1}$ B 8.5 $m\ s^{-1}$ C 7.3 $m\ s^{-1}$ D 6 $m\ s^{-1}$

7. If, instead of falling to the right towards Z, the stone were to slide back to the left, its KE as it reaches X would be

   A 0 J B 72 J C 108 J D 144 J

Questions 8 through 9 refer to figure 3.5.4.6 below, which shows the variation of one quantity, y, with the time, t, of a coconut falling from a tree.

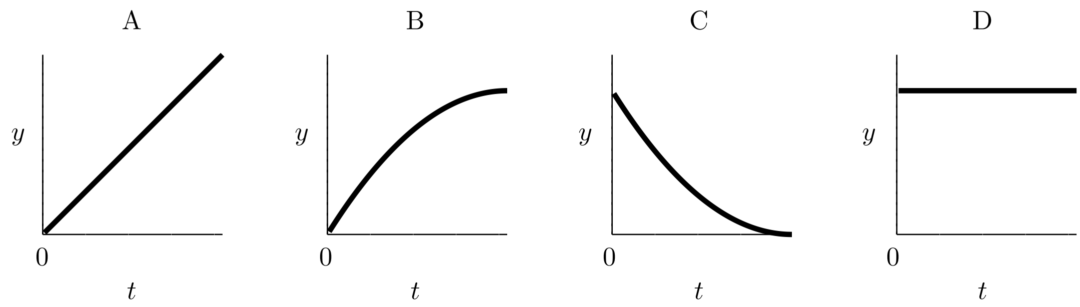

Figure 3.5.4.6

8. Which of the graphs in figure 3.5.4.6 shows how the coconut's kinetic energy changes (as y) with time?

9. Which of the graphs in figure 3.5.4.6 shows how the coconut's total mechanical energy changes with time?

10. Figure 3.5.4.7 shows a mass moving up and down on the end of a spring. X and Y are the highest and lowest points reached by the mass. Which one of the following is true when the mass is at its highest point X?

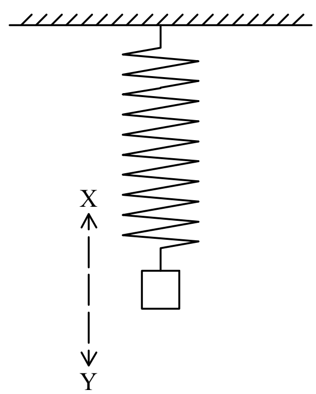

Figure 3.5.4.7

A The KE is maximum and the PE is minimum.
B The KE is maximum and the PE is zero.
C The KE is zero and the PE is maximum.
D The KE is equal to PE.

GCE Paper 1 Solutions

1. C 2. D 3. B 4. A 5. D 6. B 7. B 8. B 9. D 10. C

GCE Paper 2 Questions

1. A ball of mass 0.5 kg falls a distance of 20 m to the ground and rebounds to a height of 6 m.

   (a) Calculate the potential energy of the ball just before it first falls. **(2 mks)**
   (b) Calculate the ball's velocity just before it hits the ground. **(2 mks)**
   (c) Calculate the loss of energy due to collision with the ground. **(1 mk)**

....

Solution

(a) *The initial potential energy is due to the ball's weight and initial height.*

$$\text{given equation for potential energy: } PE = mgh$$

$$\text{substituting known values: } PE_i = (0.5\ kg)\left(10\ m\ s^{-2}\right)(20\ m)$$

$$\text{final answer: } \boxed{PE_i = 100\ J}$$

(b) *The total kinetic energy of the ball just before it hits the ground is assumed to be equal to its total potential energy when it is first dropped.*

$$\text{given equation for mechanical energy: } PE_i + KE_i = PE_f + KE_f$$

$$\text{assuming initial kinetic energy and final potential energy are zero: } PE_i = KE_f$$

$$\text{substituting equations for potential and kinetic energies: } mgh_i = \frac{1}{2}m\left(v_f\right)^2$$

$$\text{turning final speed into subject: } v_f = \sqrt{2gh_i}$$

$$\text{substituting known values: } v_f = \sqrt{2\left(10\ m\ s^{-2}\right)(20\ m)}$$

$$\text{final answer: } \boxed{v_f = 20\ m\ s^{-1}}$$

(c) *The ball's loss of mechanical energy due to its impact with the ground can be calculated as the difference between its mechanical energy before and after it bounces. This difference is most easily calculated by finding the difference in its potential energy at its maximum height before and after the bounce.*

$$\text{considering difference in mechanical energy: } \Delta ME = ME_f - ME_i$$

$$\text{assuming energy of max heights is all PE: } \Delta ME = PE_f - PE_i$$

$$\text{substituting equation for PE: } \Delta ME = mgh_f - mgh_i$$

$$\text{simplifying: } \Delta ME = (mg)\left(h_f - f_i\right)$$

$$\text{substituting known values: } \Delta ME = (0.5\ kg)\left(10\ m\ s^{-2}\right)(20\ m - 6\ m)$$

$$\text{final answer: } \boxed{\Delta ME = 70\ J}$$

2. A student drops a ball from a particular height and the ball continues on until it hits the ground at a vertical speed of 8 $m\ s^{-1}$.

   (a) Describe the energy changes of the ball from when it is first dropped by the student until it is just about to hit the ground. **(3 mks)**
   (b) State the law which controls this energy conversion. **(2 mks)**
   (c) From which height was the ball dropped? **(3 mks)**
   (d) What is the fate of the ball's energy as it impacts the ground? **(2 mks)**

. . . .

Solution

(a)
- Just as it is dropped, the ball's energy is entirely potential.
- That is, it has no kinetic energy initially because it has no speed.
- The ball speeds up downwards, causing its potential energy to decrease as its kinetic energy increases.
- Just before it hits the ground, almost all of its energy is kinetic.
- That is, it has no more potential energy because it has no height above the ground.

(b) The law of conservation of energy controls this conversion of energy.

(c) *It is a assumed that the ball's initial speed is zero.*

given equation for mechanical energy: $PE_i + KE_i = PE_f + KE_f$

assuming initial kinetic and final potential energy are zero: $PE_i = KE_f$

substituting equations for potential and kinetic energies: $mgh_i = \frac{1}{2}m\left(v_f\right)^2$

turning initial height into subject: $h_i = \frac{\left(v_f\right)^2}{2g}$

substituting known values: $h_i = \frac{\left(8\ m\ s^{-1}\right)^2}{2\left(10\ m\ s^{-2}\right)}$

final answer: $\boxed{h_i = 3.2\ m}$

(d)
- It will likely bounce back up vertically and fall back down, creating a bouncing cycle.
- The maximum height reached will reduce between successive cycles.
- This is because the ball looses energy to heat from deformation and sound during each bounce.
- The ball will eventually stop bouncing as all of its initial mechanical energy is lost to heat and sound.

---

3. A large rubber ball which contains no air is dropped from a height, h, onto a hard surface. It bounces back to a height of 4/5 h. Explain all the energy changes that take place while the ball falls and bounces. **(3 mks)**

. . . .

Solution

- When the ball is first dropped, its energy is entirely potential.
- That is, it has no initial speed.
- As it speeds up downward, its kinetic energy increases as its potential energy decreases.
- Just before it hits the ground, its energy is entirely kinetic.
- As it impacts the ground, it looses some of its mechanical energy to sound and to heat as it deforms.
- Just after the bounce, it has less kinetic energy then before the bounce and no potential energy.
- As it rises back up, its kinetic energy transforms back into potential energy as the ball gains height.
- When it reaches the final height of $\frac{4}{5}$ the original dropping height, its energy is entirely potential.
- That is, its total mechanical energy after the bounce is $\frac{4}{5}$ of that before the bounce.

3.5.5 Simple Machines

Objectives

By the end of the lesson, students should be able to

1. define a machine.
2. state the advantages of simple machines.
3. give examples of different machines.
4. explain the use of various machines at building sites.
5. define a machine's mechanical advantage.
6. solve problems involving a machine's mechanical advantage.
7. define a machine's velocity ratio.
8. solve problems involving a machine's velocity ratio.
9. solve problems involving hydraulic jacks as simple machines.

Machine Theory

- A **machine** is a device the reduces the effort required to accomplish a task.
- **Effort**, is the force applied to operate a machine.
- **Load**, is the resistant force overcome by a machine.
- This chapter uses the subscript $_e$ for effort properties and $_l$ for load properties.
- A machine's effort and load can be thought of as its input and output, respectively.
- A **simple machine** uses very few or no moving parts in order to reduce the effort required.
- A hydraulic jack, which depends on Pascal's principle, is a common example of a simple machine.

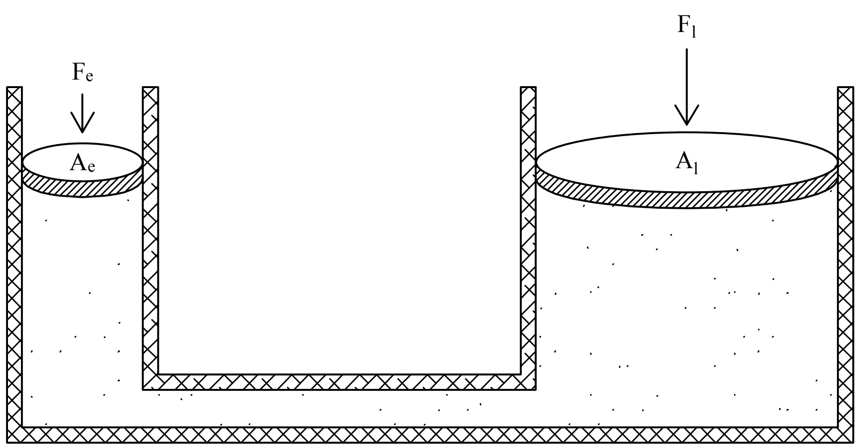

Figure 3.5.5.1

$$\text{assuming pressure is undiminished: } P_e = P_l$$

$$\text{substituting equation for pressure: } \frac{F_e}{A_e} = \frac{F_l}{A_l}$$

$$\text{separating force and area ratios: } \frac{F_l}{F_e} = \frac{A_l}{A_e}$$

- If the effort area is less than the load area ($A_l > A_e$), the effort force is less than the load force ($F_l > F_e$).

Examples of Machines

- **A hydraulic jack** uses liquid contained between an effort and load piston to lift heavy loads like vehicles.
- **An inclined plane** allows a heavy load to be lifted by a small effort force applied over a long distance.
- **A pulley system** allows a heavy load to be lifted with by a small effort force by using different sized wheels.
- **A lever** uses the principle of moments on a rigid bar that turns about a pivot in order to magnify forces.
- **Pliers** uses the principle of moments to magnify the griping force of someone's hand.
- **Gears**, like pulleys, use different sized wheels with interconnected teeth to magnify forces.

Mechanical Advantage

- A machine's **nechanical advantage**, or **MA**, is the ratio of its load force per its effort force.
- Its is a scalar.
- It has no units given that it is the value of one force divided by another.

$$MA = \frac{F_l}{F_e} \tag{3.5.5.1}$$

  Where
  - MA is the mechanical advantage of a machine (unit-less);
  - F_l is the machine's load force, in N;
  - F_e is the machine's effort force, in N.

- NB: The load force that a machine must overcome is often, but not always, the weight of a body being lifted.
- Problems involving a machine's mechanical advantage can require that one of the following three properties be calculated while the other two are given - mechanical advantage, load force, effort force.

  Example: A lever requires 50 N of effort to lift a 250 N load. Calculate its mechanical advantage.

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{substituting known values: } MA = \frac{250\ N}{50\ N}$$

$$\text{final answer: } \boxed{MA = 5}$$

  Example: A pulley system with a mechanical advantage of 7.5 requires what effort force to lift 1500 N?

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning effort force into subject: } F_e = \frac{F_l}{MA}$$

$$\text{substituting known values: } F_e = \frac{1500\ N}{7.5}$$

$$\text{final answer: } \boxed{F_e = 200\ N}$$

Example: A gear system with a mechanical advantage of 3 can lift what load with an effort force of 1000 N?

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning load force into subject: } F_l = (F_e)\,(MA)$$

$$\text{substituting known values: } F_l = (1000\ N)(3)$$

$$\text{final answer: } \boxed{F_l = 3000\ N}$$

Velocity Ratio

- A machine's **velocity ratio**, or **VR**, is the ratio of the distance moved by its effort per that of its load in an interval of time.
- Its is a scalar.
- It has no units given that it is the value of one velocity divided by another.

$$VR = \frac{v_e}{v_l} \tag{3.5.5.2}$$

Where
– VR is the velocity ratio of a machine (unit-less);
– v_e is the speed of the machine's load, in $m\ s^{-1}$;
– v_l is the speed of the machine's effort, in $m\ s^{-1}$.

- A velocity ratio is often simplified as a ratio of distances given that the same amount of time is considered for the speed of both the effort and load.

$$\text{given equation for velocity ratio: } VR = \frac{v_e}{v_l}$$

$$\text{substituting equation for speed: } VR = \frac{\frac{d_e}{\Delta t}}{\frac{d_l}{\Delta t}}$$

$$\text{simplifying: } VR = \frac{d_e}{d_l} \tag{3.5.5.3}$$

Where
– VR is the velocity ratio of a machine (unit-less);
– d_e is the distance moved by the machine's effort, in m;
– d_l is the distance moved by the machine's load, in m;

- Problems involving a machine's velocity ratio can require that one of the following three properties be calculated while the other two are given - velocity ratio, effort distance, load distance.

Example: The effort piston of a hydraulic jack moves a total distance of 7 m while lifting a load a total distance of 0.35 m. What is the jack's velocity ratio?

$$\text{given equation for velocity ratio: } VR = \frac{d_e}{d_l}$$

$$\text{substituting known values: } VR = \frac{7\ m}{0.35\ m}$$

$$\text{final answer: } \boxed{VR = 20}$$

Example: An inclined plane has a velocity ratio of 6. Calculate the vertical distance that a load is lifted if the effort force parallel to the plane is applied over a distance of 0.09 m.

$$\text{given equation for velocity ratio: } VR = \frac{d_e}{d_l}$$

$$\text{turning load distance into subject: } d_l = \frac{d_e}{VR}$$

$$\text{substituting known values: } d_l = \frac{0.09\ m}{6}$$

$$\text{final answer: } \boxed{d_l = 0.015\ m}$$

Example: A lever with a velocity ratio of 15 must be pushed down how far to lift a load of 0.045 m?

$$\text{given equation for velocity ratio: } VR = \frac{d_e}{d_l}$$

$$\text{turning effort distance into subject: } d_e = (VR)(d_l)$$

$$\text{substituting known values: } d_e = (15)(0.045\ m)$$

$$\text{final answer: } \boxed{d_e = 0.675\ m}$$

Hydraulic Jack's as Simple Machines

- The mechanical advantage of a hydraulic jack can be expressed as the ratio of its effort and load piston areas.

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{turning force from pressure equation into subject: } F = PA$$

$$\text{substituting force from pressure equation into equation for mechanical advantage: } MA = \frac{P_l A_l}{P_e A_e}$$

$$\text{assuming pressure is transmitted through liquid undiminished: } P_l = P_e$$

$$\text{simplifying: } MA = \frac{A_l}{A_e} \qquad (3.5.5.4)$$

Where
- MA is the hydraulic jack's mechanical advantage (unit-less);
- A_l is the area of the load piston, in m^2;
- A_e is the area of the effort piston, in m^2.

GCE Paper 1 Questions

1. If an effort of 120 N acts on a machine that lifts a load of 1200 N, then the machine's mechanical advantage is

   A $\frac{1}{10}$ B 1080 C 1320 D 10

2. What are the units a machine's velocity ratio?

   A $m\ s^{-1}$ B $m\ s^{-2}$ C (unit-less) D $s\ m^{-1}$

Questions 3 - 6 refer to figure 3.5.5.2, which shows a simplified hydraulic jack which may be used to lift cars.

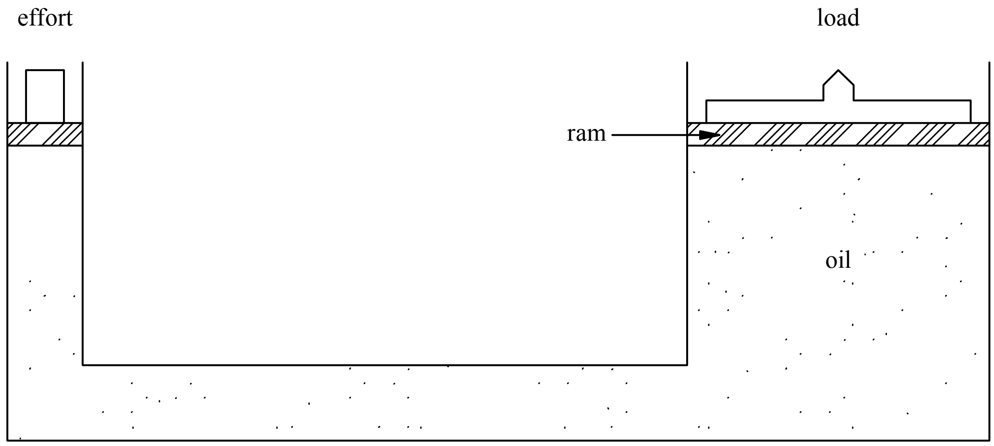

Figure 3.5.5.2

3. Which of the following is not a reason oil is preferred to water as the liquid in the system?

   A Oil is more viscous.
   B Oil does not cause rust.
   C Water is transparent.
   D Air bubbles easily form in water.

4. This system multiplies the

   A energy applied
   B force applied
   C pressure applied
   D power applied

5. The velocity ratio of the system is

   A $\frac{\text{distance moved by the load}}{\text{distance moved by the effort}}$
   B $\frac{\text{effort}}{\text{load}}$
   C $\frac{\text{load}}{\text{effort}}$
   D $\frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$

6. Which of the following statements is true as the jack functions?

   A More oil goes to the load cylinder than leaves the effort cylinder.
   B The pressure generated by the effort is the same pressure everywhere in the oil.
   C The effort force applied is transmitted equally to all parts of the oil.
   D The system works because oil is compressible.

Questions 7 through 10 refer to figure 3.5.5.3, which shows two gears acting as an effort-load machine.

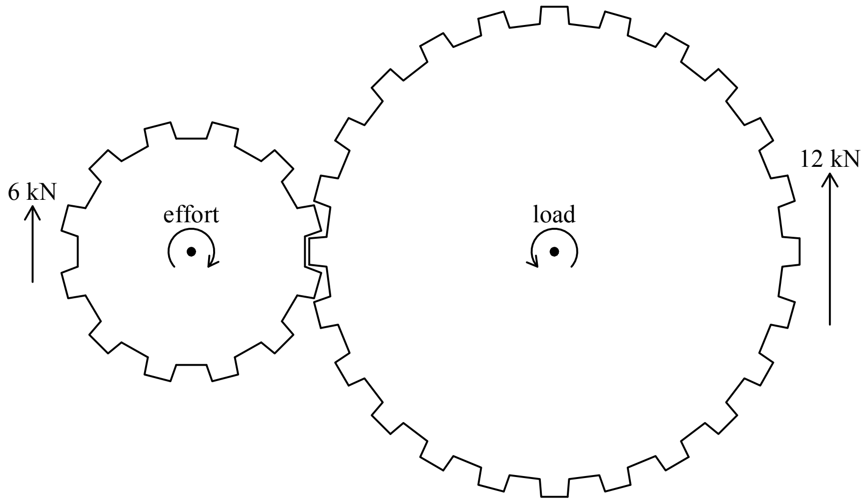

Figure 3.5.5.3

7. The machine's mechanical advantage is

   A 0.25 B 1 C 2 D 3

8. The load force shown is equivalent to

   A 12000 N B 1200 N C 120 N D 12 N

9. Which of the following load forces could be overcome with an effort force of 1500 N?

   A 3000 N B 750 kN C 30 kN D 7.5 kN

10. Which of the following effort forces could be sufficient to overcome the load force required to lift a small vehicle having a mass of 1000 kg?

    A 200 N B 2 kN C 500 N D 5 kN

GCE Paper 1 Solutions

1. D 2. C 3. C 4. B 5. D 6. B 7. C 8. A 9. A 10. D

GCE Paper 2 Questions

1. A hydraulic jack with a mechanical advantage of 3 has an effort piston with a cross-sectional area of 30 cm^2.

   (a) Determine the cross-sectional area of the load piston, in m^2. **(3 mks)**
   (b) Determine the pressure, in kPa, of the jack's liquid if an effort force of 150 N is applied. **(3 mks)**
   (c) Determine the load force created by the effort force in (b). **(2 mks)**

. . . .

Solution

(a) *The unit equivalence* 100 $cm = 1$ m *is used.*

$$\text{given equation for a hydraulic jack's mechanical advantage: } MA = \frac{A_l}{A_e}$$

$$\text{turning load piston's area into subject: } A_l = MA\,(A_e)$$

$$\text{substituting known values: } A_l = 3\left(30\ cm^2\right)$$

$$\text{applying conversion factor: } A_l = 3\left(30\ cm^2\right)\left(\frac{1\ m}{100\ cm}\right)^2$$

$$\text{final answer: } \boxed{A_l = 0.009\ m^2}$$

(b) *Given Pascal's principle, this is a simple pressure problem. The unit equivalences* 100 $cm = 1$ m *and* 1 $kPa = 1000$ Pa *are used.*

$$\text{given equation for pressure: } P = \frac{F}{A}$$

$$\text{substituting known values: } P = \frac{150\ N}{30\ cm^2}$$

$$\text{applying conversion factor: } P = \frac{150\ N}{30\ cm^2}\left(\frac{100\ cm}{1\ m}\right)^2$$

$$\text{final answer in Pascals: } P = 50000\ Pa$$

$$\text{applying conversion factor: } P = (50000\ Pa)\left(\frac{1\ kPa}{1000\ Pa}\right)$$

$$\text{final answer: } \boxed{P = 50\ kPa}$$

(c) *The machine's mechanical advantage is given.*

$$\text{given equation for a machine's mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning load force into subject: } F_l = MA\,(F_e)$$

$$\text{substituting known values: } F_l = (3)(150\ N)$$

$$\text{final answer: } \boxed{F_l = 450\ N}$$

---

2. A pulley system has a mechanical advantage of 4 and a velocity ratio of 5. It is given that an effort force of 100 N must be applied over a distance of 0.5 m to lift the load of a body vertically.

   (a) Calculate the input work that is provided by the effort force. **(3 mks)**
   (b) Calculate the load force overcome by the pulley system. **(2 mks)**
   (c) Calculate the potential energy gained by the body. **(2 mks)**
   (d) Account for any differences is the answers to (a) and (c). **(3 mks)**

....

Solution

(a) *The effort force and distance is given.*

$$\text{given equation for work: } W = Fd$$

$$\text{substituting known values: } W = (100\ N)(0.5\ m)$$

$$\text{final answer: } \boxed{W_e = 50\ J}$$

(b) *The machine's mechanical advantage is given.*

$$\text{given equation for a machine's mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning load force into subject: } F_l = MA\,(F_e)$$

$$\text{substituting known values: } F_l = 4(100\ N)$$

$$\text{final answer: } \boxed{F_l = 400\ N}$$

(c) *The load's gain in potential energy requires that the distance lifted be calculated, which involves the machine's given velocity ratio.*

$$\text{given equation for potential energy: } PE = F_w h$$

$$\text{assuming force of body's weight is only load force: } PE = F_l h$$

$$\text{assuming load distance is height of body lifted: } PE = F_l d_l$$

$$\text{given equation for a machine's velocity ratio: } VR = \frac{d_e}{d_l}$$

$$\text{turning load distance from velocity ratio equation into subject: } d_l = \frac{d_e}{VR}$$

$$\text{substituting load distance into equation for potential energy: } PE = F_l\left(\frac{d_e}{VR}\right)$$

$$\text{substituting known values: } PE = (400\ N)\left(\frac{0.5\ m}{5}\right)$$

$$\text{final answer: } \boxed{PE_l = 40\ J}$$

3. Some energy is lost to heat given friction in the pulley. This causes the useful output potential energy of the load to be less than the work put in at the effort.

3.5.6 Machine Work and Inclined Planes

Objectives

By the end of the lesson, students should be able to
1. define a machine's efficiency.
2. explain why, in reality, an inclined plane's efficiency is always less than 100%.
3. solve problems involving inclined planes as simple machines.

Effort and Load Work

- The **effort work**, or $\mathbf{W_e}$, of a machine is the product of the force applied at the effort and the distance the effort moves parallel to its force's line of action.

$$W_e = F_e d_e$$

  Where
  - W_e is the work input at the effort, in J;
  - F_e is the force applied by the effort, in N;
  - d_e is the distance that the effort must be moved, in m.

- The **load work**, or $\mathbf{W_l}$, for such a system is the product of the force of the load that must be overcome and the distance the load is moved parallel to its force's line of action.

$$W_l = F_l d_l$$

  Where
  - W_l is the load work output by the system, in J;
  - F_l is the force of the load to be overcome, in N;
  - d_l is the distance that the load is moved, in m.

- For an **ideal machine**, all work input at its effort becomes useful work at its output.

$$\text{for an ideal machine: } W_e = W_l$$

- For an ideal hydraulic jack, the increased load force is compensated for by a reduced load distance.

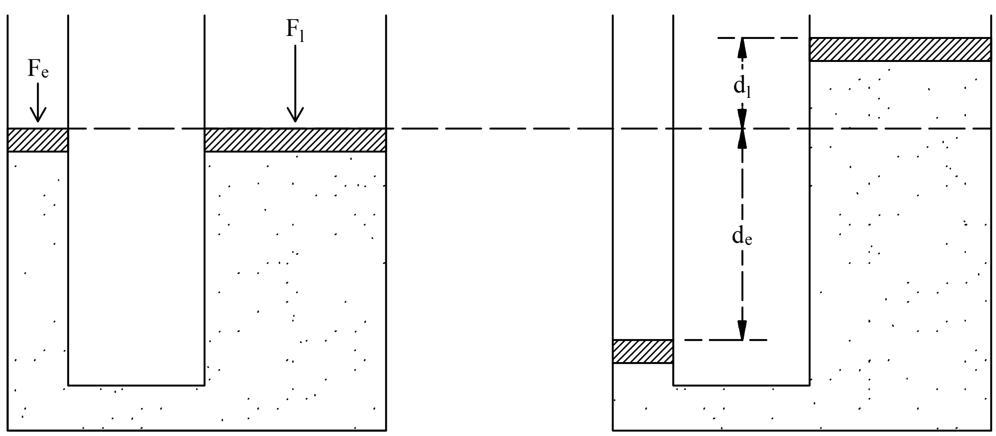

Figure 3.5.6.1

$$\text{assuming machine is ideal: } W_l = W_e$$

$$\text{substituting equation for work: } F_l d_l = F_e d_e$$

$$\text{seperating force and distance ratios: } \frac{F_l}{F_e} = \frac{d_e}{d_l}$$

$$\text{comparing forces and distances: if } F_l > F_e, \text{ then } d_e > d_l$$

Efficiency

- In practice, no machine is ideal.
- In a machine, some amount of input effort work is always lost as heat due to friction and other forms of energy.
- Therefore, not all of the effort work is transformed into load work.

$$\text{for a real machine: } W_e > W_l$$

- A machine that experiences very little loss of W_e to other forms is considered to be efficient.
- The **efficiency**, or η, of a simple machine is the percent ratio of its load work over its effort work.
- Its is a scalar.
- It has no units given that it is the value of one energy divided by another.

$$\eta = \frac{W_l}{W_e} \times 100\% \tag{3.5.6.1}$$

Where
  - η is the machine's efficiency (unit-less);
  - W_l is the machine's load work, in J;
  - W_e is the machine's effort work, in J;

- The efficiency of a machine can be calculated as the ratio of its mechanical advantage to its velocity ratio.

$$\text{given equation for efficiency: } \quad \eta = \frac{W_l}{W_e} \times 100\%$$

$$\text{substituting equation for work: } \quad \eta = \frac{F_l d_l}{F_e d_e} \times 100\%$$

$$\text{substituting equation for mechanical advantage: } \quad \eta = MA\left(\frac{d_l}{d_e}\right) \times 100\%$$

$$\text{inverting distances: } \quad \eta = MA\left(\frac{d_e}{d_l}\right)^{-1} \times 100\%$$

$$\text{substituting equation for velocity ratio: } \quad \eta = MA(VR)^{-1} \times 100\%$$

$$\text{expressing as quotient: } \quad \eta = \frac{MA}{VR} \times 100\% \tag{3.5.6.2}$$

Where
  - η is the machine's efficiency (unit-less);
  - MA is the machine's mechanical advantage (unit-less);
  - VR is the machine's velocity ratio (unit-less).

Concepts of Efficiency

- More generally, the efficiency of any system or mechanism is considered the percent ratio of all energy input over all <u>useful</u> energy output.

$$\eta = \frac{E_{\text{out, useful}}}{E_{\text{in}}} \times 100\%$$

- The usefulness of the energy provided by the machine is emphasized because some output energy like heat or sound is often, but always, useless.

- The efficiency of an ideal machine is 100% while that of real machine is less.

$$\text{for an ideal machine: } \eta = 100\%$$

$$\text{for a real machine: } \eta < 100\%$$

- The mechanical advantage of an ideal machine is equal to its velocity ratio while for a real machine, it is less.

$$\text{for an ideal machine: } MA = VR$$

$$\text{for a real machine: } MA < VR$$

- A machine's gradual decrease in efficiency is reflected in a decrease in its mechanical advantage while its velocity ratio remains constant.

- For example, as a hydraulic jack becomes rusty over time,
  - additional force is required to push the corroded effort piston down.
  - its mechanical advantage reduces as more and more effort force is required to achieve the same load force.
  - its velocity ratio remains unchanged as the same effort distance causes the same load distance.

Inclined Planes as Simple Machines

- An inclined plane acts as a simple machine in which load force being overcome is the weight of some body.

- That is, the load distance being moved is entirely vertical to the load force of weight.

- The effort force opposes the force downhill and parallel to the plane $\left(F_{\|}\right)$.

- The effort distance, which is parallel to the plane, is greater than the load distance.

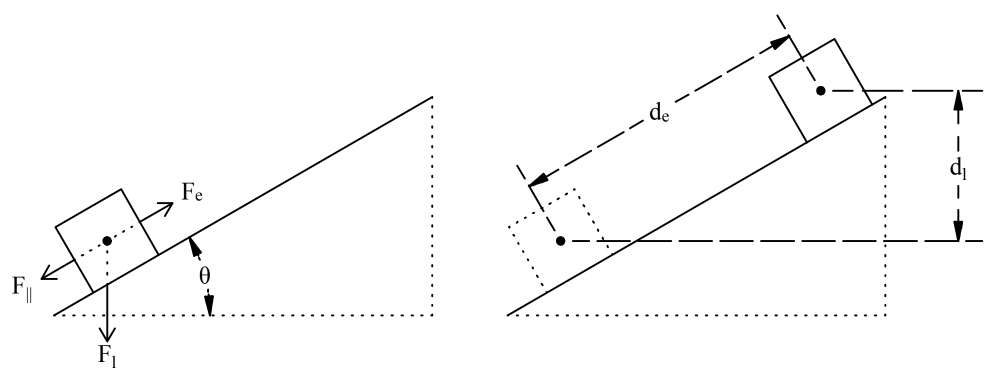

Figure 3.5.6.2

- For an ideal plane with no friction, the force downhill and parallel to the plane is only composed of the component of the weight force that is parallel to the plane.

- For a non-ideal plane, the force downhill and parallel to the plane is the sum of the component of the weight force that is parallel to the plane and the force of friction.

- In either case, the effort force balances the force that is downhill and parallel to the plane.

$$\text{for an ideal plane: } F_e = F_{\|} = F_{w\|p}$$

$$\text{for a non-ideal plane: } F_e = F_{\|} = F_{w\|p} + F_f$$

Where
  - F_e is the effort force, in N;
  - $F_{\|}$ is the force downhill and parallel to the plane, in N;
  - $F_{w\|p}$ is the component of the weight force that is parallel to the plane, in N;
  - F_f is the force of friction, in N.

The following derivation of the effort force as a function of the force of weight, the force of friction and the angle of inclination may involve aspects of trigonometry that have not yet been covered in form 3. It may prove a better use of time to have students simply memorize equations 3.5.6.3 and 3.5.6.4 below.

- The component of the weight force that is parallel to the plane can be calculated using trigonometry.

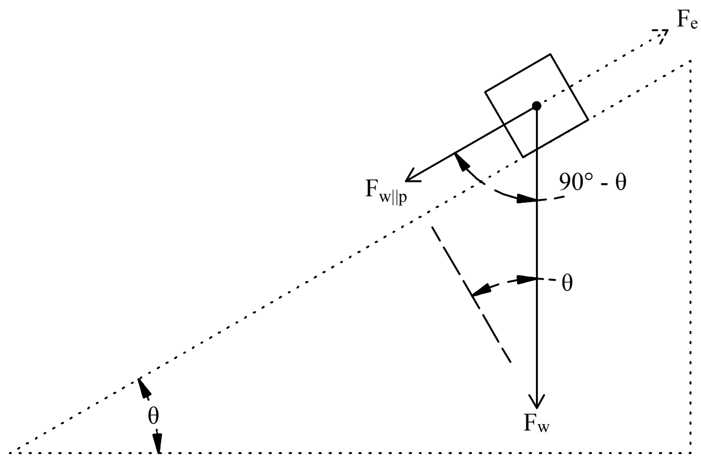

Figure 3.5.6.3

- Given the plane's angle of inclination against the horizontal (θ), the angle between the force of weight and its plane-parallel component can be calculated as $90° - \theta$.

$$\text{applying cosine:} \quad \cos(90^0 - \theta) = \frac{F_{w\|p}}{F_w}$$

$$\text{turning parallel weight component into subject:} \quad F_{w\|p} = \cos(90^0 - \theta)F_w$$

$$\text{substituting sine:} \quad F_{w\|p} = \sin(\theta)F_w$$

Where
- $F_{w\|p}$ is the component of the weight force that is parallel to the plane against horizontal, in N;
- θ is the incline of the plane, in degrees.
- F_w is the force of weight, in N.

- This equation for the parallel weight component can be substituted into the equations for effort force of an ideal and non-ideal plane.

$$\text{for an ideal plane: } F_e = F_w \sin(\theta) \tag{3.5.6.3}$$

$$\text{for a non-ideal plane: } F_e = F_w \sin(\theta) + F_f \tag{3.5.6.4}$$

Where
- F_e is the effort force, in N;
- θ is the incline of the plane, in degrees;
- F_w is the force of weight, in N;
- F_f is the force of friction, in N.

- The mechanical advantage of an ideal inclined plane is a function of its inclination only.

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{substituting equation for ideal inclined plane's effort force: } MA = \frac{F_l}{F_w \sin(\theta)}$$

$$\text{assuming only load force is the body's weight: } MA = \frac{F_w}{F_w \sin(\theta)}$$

$$\text{simplifying: } MA = \frac{1}{\sin(\theta)} \tag{3.5.6.5}$$

  Where
  - MA is the inclined plane's mechanical advantage (unit-less);
  - θ is the plane's angle of inclination, in degrees.

- An ideal inclined plane's MA can also be calculated from the ratio of its total height to its plane length.

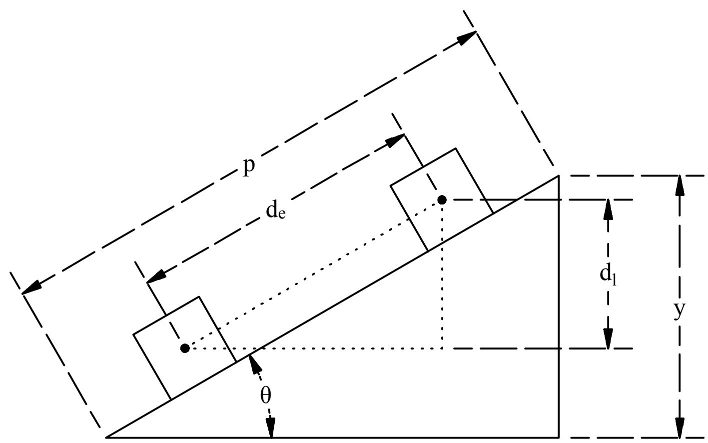

Figure 3.5.6.4

- *It may help to remind students that sine is "opposite over hypotenuse".*

$$\text{given equation for ideal inclined plane's mechanical advantage: } MA = \frac{1}{\sin(\theta)}$$

$$\text{substituting side lengths for sine: } MA = \frac{1}{\left(\frac{y}{p}\right)}$$

$$\text{simplifying: } MA = \frac{p}{y} \tag{3.5.6.6}$$

  Where
  - MA is the ideal inclined plane's mechanical advantage (unit-less);
  - p is the ideal inclined plane's diagonal length, in m;
  - y is the ideal inclined plane's vertical height, in m.

- The velocity ratio of any plane, ideal or non-ideal, is also a function of its inclination only.

$$\text{given equation for velocity ratio: } VR = \frac{d_e}{d_l}$$

$$\text{applying similar triangles: } VR = \frac{p}{y}$$

$$\text{substituting sine: } VR = \frac{1}{\sin(\theta)} \quad (3.5.6.7)$$

- Likewise, the velocity ratio of any inclined plane is the ratio of its total height to its plane length.

$$\text{given equation for velocity ratio: } VR = \frac{1}{\sin(\theta)}$$

$$\text{substituting side lengths for sine: } VR = \frac{1}{\left(\frac{y}{p}\right)}$$

$$\text{simplifying: } VR = \frac{p}{y} \quad (3.5.6.8)$$

- NB: Equations 3.5.6.7 and 3.5.6.8 for velocity ratios apply to all planes, both ideal and non-ideal while equations 3.5.6.5 and 3.5.6.6 apply only to ideal inclined planes.

$$\text{for an ideal plane: } VR = \frac{1}{\sin(\theta)} = \frac{p}{y} = MA$$

$$\text{for a real plane: } VR = \frac{1}{\sin(\theta)} = \frac{p}{y} \neq MA$$

Example: What is the velocity ratio of a plane that is inclined 30° to the horizontal?

$$\text{given equation for inclined plane's velocity ratio: } VR = \frac{1}{\sin(\theta)}$$

$$\text{substituting known values: } VR = \frac{1}{\sin(30°)}$$

$$\text{final answer: } \boxed{VR = 2}$$

- **Example:** An ideal inclined plane has a total height of 0.5 m and a mechanical advantage of 6. What is the diagonal length of its surface?

$$\text{given equation for ideal inclined plane's mechanical advantage: } MA = \frac{p}{y}$$

$$\text{turning surface length into subject: } p = y(MA)$$

$$\text{substituting known values: } p = (0.5\ m)(6)$$

$$\text{final answer: } \boxed{p = 3\ m}$$

GCE Paper 1 Questions

1. Which of the following statements describes the effort and load work of an inefficient (non-ideal) machine?

   A $W_l > W_e$ B $W_l = W_e$ C $W_l < W_e$ D $W_l = 2W_e$

Questions 2 through 6 refer to figure 3.5.6.5, which shows a load on an inclined plane.

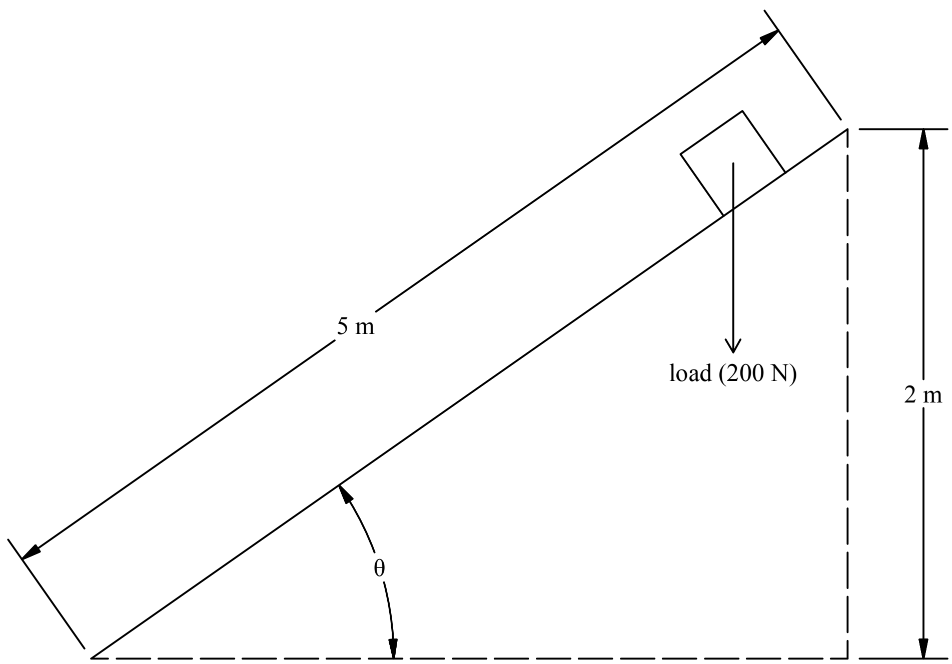

Figure 3.5.6.5

2. Assuming the friction on the surface is negligible, the mechanical advantage of the plane is

   A 7 B 3 C 2.5 D 0.4

3. When the angle θ is decreased, the height to which the load can be raised

   A increases B decreases C remains the same D does not depend on θ

4. Assuming friction is negligible, when the angle θ is decreased, the plane's mechanical advantage

   A increases B decreases C remains the same D does not depend on θ

5. The plane's velocity ratio is

   A $\frac{2}{5}$ B $\frac{5}{2}$ C $\frac{200}{5}$ D 100%

6. As the inclined plane of constant length is made steeper, its mechanical advantage

   A remains the same B does not depend on θ C increases D decreases

Questions 7 through 10 refer to figure 3.5.6.6, which shows a body before it is pulled up an inclined plane at time t_1 and after it has reached the top at time t_2.

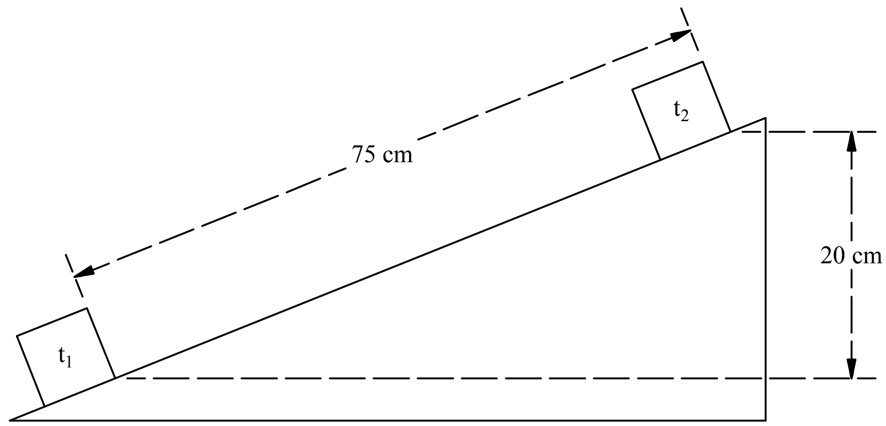

Figure 3.5.6.6

7. If the plane is given to have a mechanical advantage of 3, the plane is

   A ideal B real C perfectly flat D physically impossible

8. If the body is given to have a mass of 6 kg, how much useful work does the ramp do in lifting the load?

   A 12 J B 1200 J C 45 J D 4500 J

9. Considering the mechanical advantage given in question 7 and the load-body's mass given in question 8, the effort force applied is

   A 1 N B 10 N C 2 N D 20 N

10. Considering the effort force calculated in question 9, the work input in pulling the load uphill is

    A 15 J B 150 J C 1500 J D 15000 J

GCE Paper 1 Solutions

1. C 2. C 3. B 4. A 5. B 6. D 7. B 8. A 9. D 10. A

GCE Paper 2 Questions

The solutions to the following problems use "sine equals opposite over hypotenuse of a right triangle". This concept may need to be revised or introduced.

1. A given hill is inclined at 30° to the horizontal. A car of mass 4000 kg is driven from the foot to the top at a steady speed of 5 $m\ s^{-1}$ against a constant frictional force of 800 N. During this trip, the car uses 0.5 litres of petrol. It is given that the car travels a distance of 100 m along the side of the hill and that the total amount of chemical energy available within 1 litre of the petrol used is $6.0 \times 10^6\ J$. Using this information, determine

   the
   (a) total work the car does against friction. **(2 mks)**
   (b) total potential energy gained by the car. **(2 mks)**
   (c) kinetic energy of the car at any moment. **(2 mks)**
   (d) total energy provided by the petrol to the car. **(2 mks)**
   (e) the total energy the car outputs in the process of overcoming all forces against it as well as setting itself into motion. **(1 mk)**
   (f) efficiency of the car in this scenario. **(1 mk)**

....

Solution

Figure 3.5.6.7 is provided to aid in the understanding of the solution. It is however not a required component of the solution. The scenario is represented as an object being moved up an inclined plane by an effort force balanced by a force of friction and the component of the force of weight that is parallel to the plane.

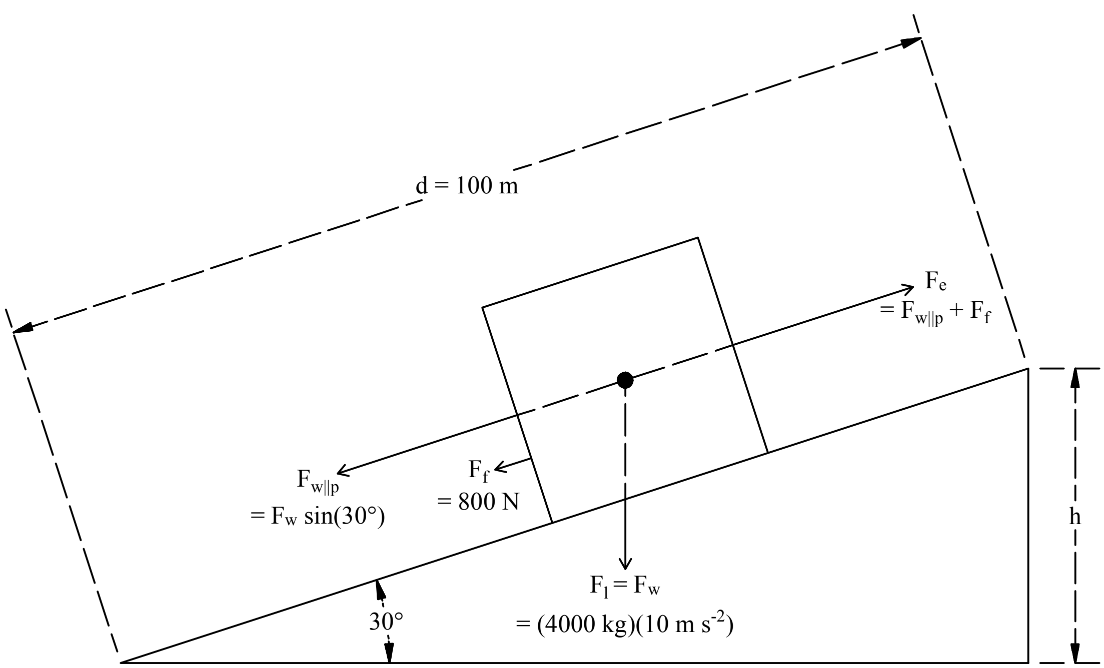

Figure 3.5.6.7

(a) *The total work done by the car against friction is the product of the force of friction and the total distance the car moves.*

given equation for work against friction: $W_f = F_f d$

substituting known values: $W_f = (800\ N)(100\ m)$

final answer: $\boxed{W_f = 8 \times 10^4\ J}$

(b) *This vertical height must be determined using trigonometry.*

$$\text{given equation for potential energy: } PE = mgh$$

$$\text{applying sine to angle of inclination: } \sin(30^\circ) = \frac{h}{d}$$

$$\text{turning height into subject: } h = d\sin(30^\circ)$$

$$\text{substituting height into equation for potential energy: } PE = mg\left[d\sin(30^\circ)\right]$$

$$\text{substituting known values: } PE = (4000\ kg)\left(10\ m\ s^{-2}\right)\left[(100\ m)\sin(30^\circ)\right]$$

$$\text{final answer: } \boxed{PE = 2 \times 10^{6}\ J}$$

(c) *The car's kinetic energy is only dependent on its speed and mass, not height.*

$$\text{given equation for kinetic energy: } KE = \frac{1}{2}mv^2$$

$$\text{substituting known values: } KE = \frac{1}{2}(4000\ kg)\left(5\ m\ s^{-1}\right)^2$$

$$\text{final answer: } \boxed{KE = 5 \times 10^{4}\ J}$$

(d) *The amount of chemical energy available in a 1 litre of petrol is given in the problem.*

$$\text{energy used from petrol: } E_{\text{petrol}} = \frac{\text{chemical energy}}{\text{1 litre of petrol}} \times \Delta V_{\text{petrol}}$$

$$\text{substituting known values: } E_{\text{petrol}} = \left(\frac{6.0 \times 10^{6}\ J}{1L}\right) \times 0.5\ L$$

$$\text{final answer: } \boxed{E_{\text{petrol}} = 3 \times 10^{6}\ J}$$

(e) *The car must output energy to overcome the given force of friction, set itself into motion, and obtain a difference in the position of its height between the top and the bottom of the hill. This output energy therefore is the sum of its work against friction, kinetic energy and potential energy, respectively.*

$$\text{output work is sum of all journey's energies: } E_{\text{output}} = W_f + PE + KE$$

$$\text{substituting known values: } E_{\text{output}} = \left(8 \times 10^{4}\ J\right) + \left(2 \times 10^{6}\ J\right) + \left(5 \times 10^{4}\ J\right)$$

$$\text{final answer: } \boxed{E_{\text{output}} = 2.13 \times 10^{6}\ J}$$

(f) *The efficiency of the car is the ratio of input energy from the petrol to its total useful output energy.*

$$\text{efficiency is ratio of input and output energies: } \eta = \frac{E_{\text{output}}}{E_{\text{input}}} \times 100\%$$

$$\text{all input energy assumed to come from petrol: } \eta = \frac{E_{\text{output}}}{E_{\text{petrol}}} \times 100\%$$

$$\text{substituting known values: } \eta = \frac{2.13 \times 10^{6}\ J}{3 \times 10^{6}\ J} \times 100\%$$

$$\text{final answer: } \boxed{\eta = 71\%}$$

2. Figure 3.5.6.8 shows a 50 kg bag of cement being pulled using a tension-force of 500 N along a plank using a rope passing over a pulley at B as shown. The plank AB is 5 m long and inclined at 60° to the horizontal.

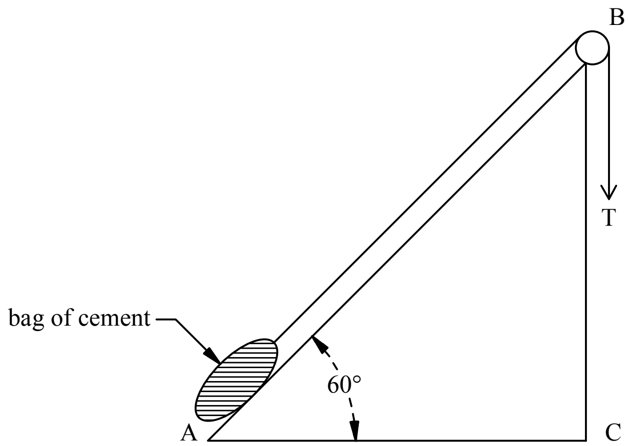

Figure 3.5.6.8

Calculate
(a) the useful energy output of the system. **(2 mks)**
(b) the energy input in taking the cement to the pulley. **(2 mks)**
(c) the efficiency of the system. **(2 mks)**
(d) State a factor that led to the efficiency being less than 100% **(1 mk)**
(e) If, at the pulley, the cement is released and falls vertically, describe the energy changes that occur until it hits the ground. **(3 mks)**

. . . .

Solution

(a) *The useful energy output of the system is the potential energy gained by the cement bag.*

$$\text{given equation for potential energy: } PE = mgh$$

$$\text{considering height as length of plane's vertical side: } PE = mg\left(\overline{BC}\right)$$

$$\text{considering sine of given angle: } \sin(60^\circ) = \frac{\overline{BC}}{\overline{AB}}$$

$$\text{turning vertical side length into subject: } \overline{BC} = \left(\overline{AB}\right)\sin(60^\circ)$$

$$\text{substituting vertical side length into eq. for potential energy: } PE = mg\left[\left(\overline{AB}\right)\sin(60^\circ)\right]$$

$$\text{substituting known values: } PE = (50\ kg)\left(10\ m\ s^{-2}\right)\left[(5\ m)\sin(60^\circ)\right]$$

$$\text{final answer: } \boxed{E_{\text{out, useful}} = PE \approx 2165\ J}$$

(b) *The energy input is the product of the pulling force of tension and the distance pulled.*

$$\text{given equation for work: } W = Fd$$

$$\text{substituting force with tension and distance with plane length: } W = T\left(\overline{AB}\right)$$

$$\text{substituting known values: } W = (500\ N)(5\ m)$$

$$\text{final answer: } \boxed{E_{\text{in}} = W = 2500\ J}$$

(c) *Efficiency is ratio of useful energy output to energy input.*

$$\text{given equation for efficiency: } \eta = \frac{E_{\text{out, useful}}}{E_{\text{in}}} \times 100\%$$

$$\text{substituting known values: } \eta = \frac{2165\ J}{2500\ J} \times 100\%$$

$$\text{final answer: } \boxed{\eta \approx 86.6\%}$$

(d) *The following is a non-exhaustive list of possible causes of inefficiency. One one is required.*

Some input energy is lost to
- friction along the plane as heat energy.
- friction in the pulley as heat energy.
- squeaking in the pulley as sound energy.
- stretching of the tension line as elastic energy.

(e) *The falling bag experiences the same energy conversion as any falling object.*
- Initially at B, the bag's mechanical energy is entirely potential.
- Between B and A, as the bag falls, its potential energy gradually transforms into kinetic energy.
- Just before A, when the bag is about to impact the ground, its mechanical energy is entirely kinetic.
- As the bag impacts the ground, all of the bag's energy is transformed into sound, heat and deformation.

3.5.7 Pulleys

Objectives

By the end of the lesson, students should be able to

1. solve problems involving a pulley system's efficiency, mechanical advantage and velocity ratio.
2. explain why, in reality, a pulley system's efficiency is always less than 100%.

Pulley Systems as Simple Machines

- A pulley system is a type of effort load machine.
- In such a system, an effort force is applied as tension on one thread or rope which induces a load tension in the same or another thread or rope.
- The thread or rope is often referred to as a **tension line** because it acts as a thin line though which the force of tension (and not compression) acts.
- Pulley's can be connected in different ways, resulting in different mechanical advantages and velocity ratios.

Fixed Pulleys

- A single fixed pulley is composed of one pulley attached rigidly to some fixed surface (often underneath).

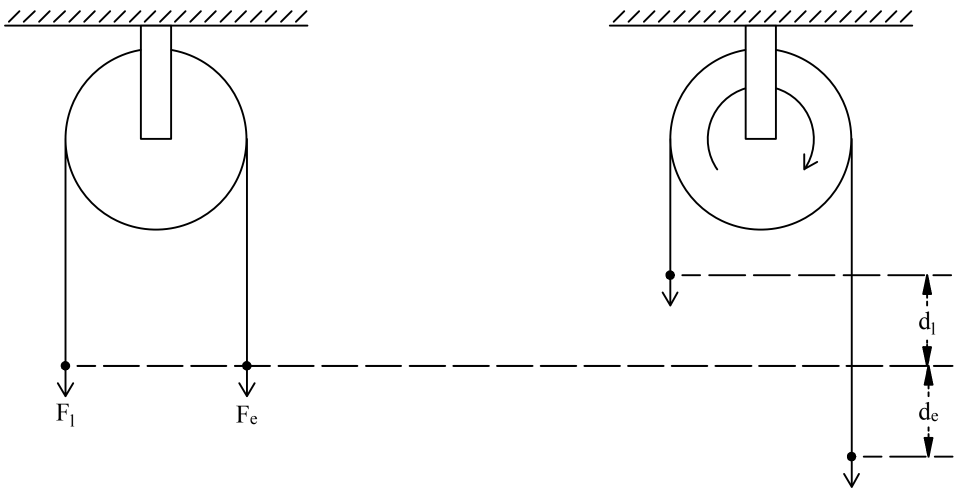

Figure 3.5.7.1

- For such a pulley, the effort and the load move the same distance.

$$\text{for a single fixed pulley: } d_e = d_l$$

- Therefore, assuming ideal conditions, both its mechanical advantage and its velocity ratio are 1.

$$\text{for a single ideal fixed pulley: } MA = VR = 1$$

- Though not always labelled in figures, the line tension is equal on both sides of any ideal pulley at rest.

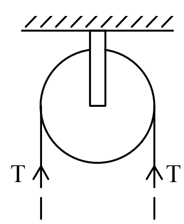

Figure 3.5.7.2

Single Moving Pulleys

- A single moving pulley is composed of a suspended pulley through which a tension line is threaded.
- One end of the tension line is attached underneath some fixed surface.
- The tension line's other end is pulled by the effort force.

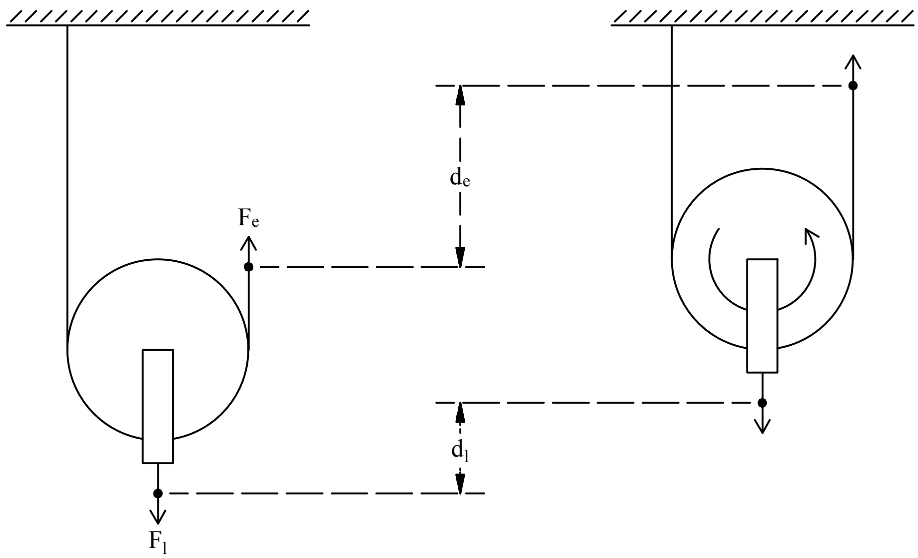

Figure 3.5.7.3

- For such a pulley, twice as much length of tension line is pulled as the effort than is lifted by the load.
- That is, the velocity ratio of a single, moving pulley is 2.

$$\text{for a single moving pulley: } d_e = 2d_l$$

- Therefore, assuming ideal conditions, both its mechanical advantage and its velocity ratio is 2.

$$\text{for a single ideal moving pulley: } MA = VR = 2$$

Example: A single ideal moving pulley can lift what load with an effort force of 100 N?

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning load force into subject: } F_l = F_e(MA)$$

$$\text{substituting known values: } F_l = (100\ N)(2)$$

$$\text{final answer: } \boxed{F_l = 200\ N}$$

Example: A single ideal moving pulley needs what effort force to lift a load of 150 N?

$$\text{given equation for mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning load force into subject: } F_e = \frac{F_l}{MA}$$

$$\text{substituting known values: } F_e = \frac{150\ N}{2}$$

$$\text{final answer: } \boxed{F_e = 75\ N}$$

Multiple Moving Pulleys

- A multiple moving pulley system is composed of multiple, interconnected moving pulleys.

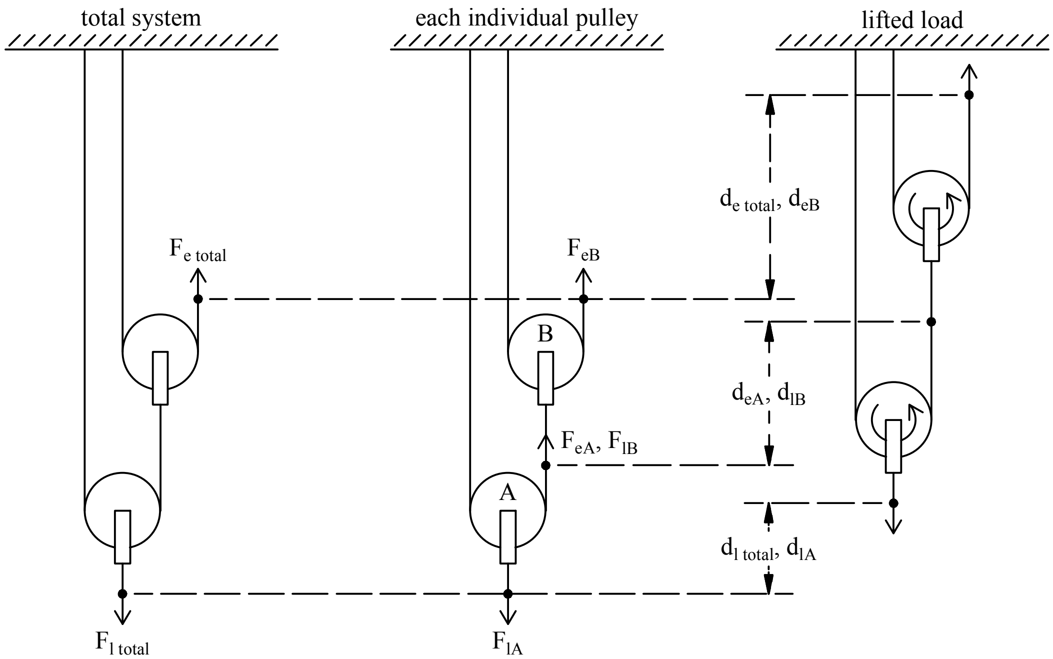

Figure 3.5.7.4

- The velocity ratio of such a system is compounded for each additional moving pulley.

considering velocity ratio of first moving pulley: $VR_A = \dfrac{d_{eA}}{d_{lA}}$

substituting second pulley's load distance for first pulley's effort distance: $VR_A = \dfrac{d_{lB}}{d_{lA}}$

turning second pulley's load distance into subject: $d_{lB} = (VR_A)\,(d_{lA})$

considering velocity ratio of second moving pulley: $VR_B = \dfrac{d_{eB}}{d_{lB}}$

substituting second pulley's load distance: $VR_B = \dfrac{d_{eB}}{(VR_A)\,d_{lA}}$

$d_{e\text{ total}}$ is on second pulley, while $d_{l\text{ total}}$ is on first: $VR_B = \dfrac{d_{e\text{ total}}}{(VR_A)\,d_{l\text{ total}}}$

separating distances from velocity ratios: $\dfrac{d_{e\text{ total}}}{d_{l\text{ total}}} = (VR_A)\,(VR_B)$

subtituting total velocity ratio: $VR_{\text{total}} = (VR_A)\,(VR_B)$

- Given any moving pulley has a velocity ratio of 2, this can be applied to any system of N_m moving pulleys.

for several moving pulleys: $d_e = \left(2^{N_m}\right) d_l$

- This can be applied to such a system's mechanical advantage and velocity ratio, assuming ideal conditions.

for several ideal, moving pulleys: $MA = VR = 2^{N_m}$

- NB: Any additional fixed pulleys are not considered part of N.

Block and Tackle Pulley Systems

- A block and tackle pulley system is composed of one or several rigidly connected, suspended pulleys that act as a compound moving pulley.
- The suspended, movable portion is referred to as the block.

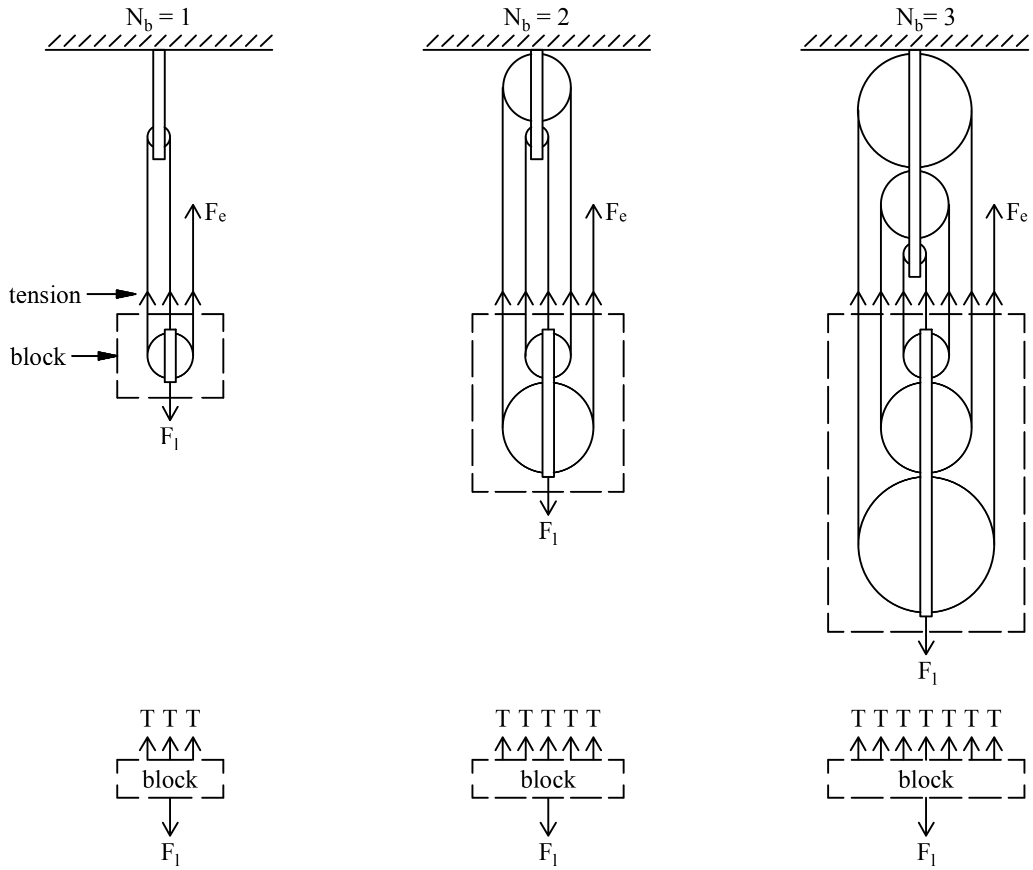

Figure 3.5.7.5

- Assuming ideal conditions, the load/effort ratio can be calculated from the quantity of the blocks pulleys (N_b).

$$\text{summing vertical forces of one-pulley block: } F_l = 3T$$

$$\text{summing vertical forces of two-pulley block: } F_l = 5T$$

$$\text{summing vertical forces of three-pulley block: } F_l = 7T$$

$$\text{inferring pattern: } F_l = (2N_b + 1)\, T$$

$$\text{assuming effort force is equal to tension on block's bottom pulley: } F_l = (2N_b + 1)\, F_e$$

$$\text{turning force quotient into subject: } \frac{F_l}{F_e} = 2N_b + 1$$

- This can be used to calculate the VR and MA of any block and tackle system with N_b block pulleys.

$$\text{for an ideal block and tackle pulley system: } MA = VR = 2N_b + 1$$

$$\text{for any block and tackle pulley system: } VR = 2N_b + 1$$

GCE Paper 1 Questions

Questions 1 through 5 refer to figure 3.5.7.6, which shows a realistic pulley system used to lift a 300 N load.

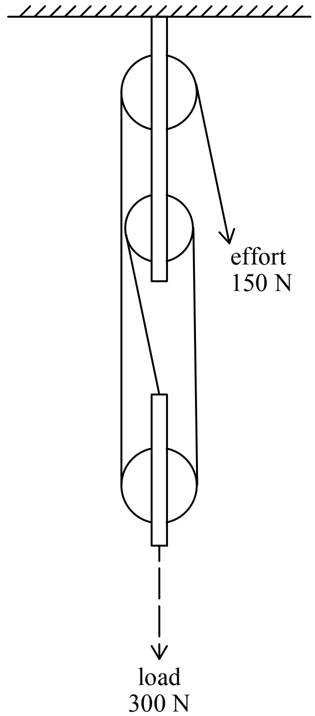

Figure 3.5.7.6

1. The velocity ratio of the system is

   A 1 B 2 C 3 D 4

2. What is the distance moved by the effort when the load is raised through a distance of 1.5 m?

   A 1.5 m B 2.5 m C 3 m D 4.5 m

3. The mechanical advantage of the system is

   A 1 B 2 C 3 D 4

4. Comparing the answer of question 1 to that of question 3 leads to the conclusion that this pulley system is

   A 200% efficient B 100% efficient C not perfectly efficient D 0% efficient

5. Any inefficiency in the system is likely due to

   A a difference in the number of upper pulleys and lower pulleys.
   B a loss of input energy to friction.
   C a velocity ratio that is too high.
   D a velocity ratio that is too low.

Questions 6 through 10 refer to figure 3.5.7.5, which shows four different pulley arrangements.

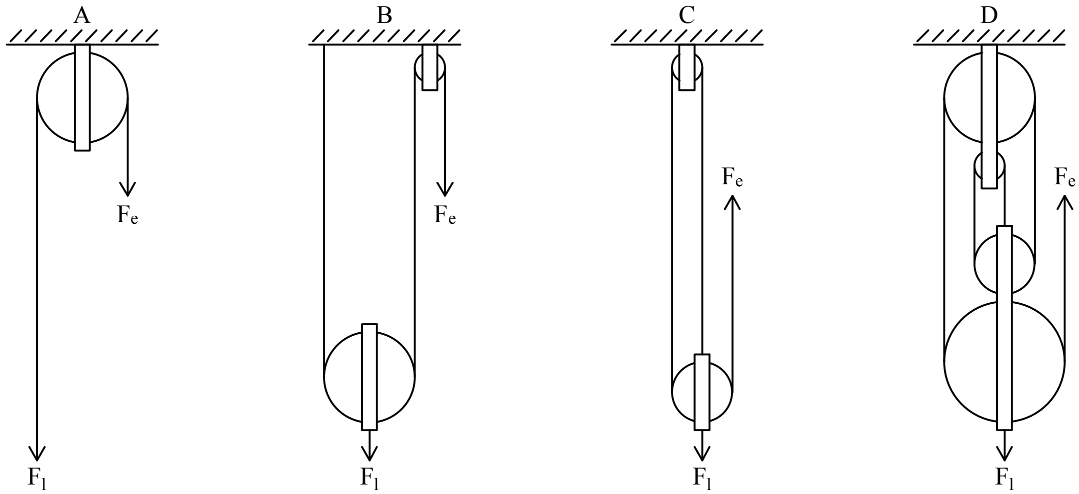

Figure 3.5.7.7

6. Which pulley system has the highest velocity ratio?

7. Assuming ideal conditions, which pulley system has the highest mechanical advantage?

8. Which pulley system has a velocity ratio of 1?

9. If the effort of pulley system B is pulled down a vertical distance of 1 m, the load will rise a total distance of

   A 2 m B 0.5 m C 4 m D 0.2 m

10. If pulley system D is used to lift a car having a mass of 1000 kg, which effort force is required?

   A 2 kN B 2.5 kN C 5 kN D 200 N

GCE Paper 1 Solutions

1. C 2. D 3. B 4. C 5. B 6. D 7. D 8. A 9. B 10. A

GCE Paper 2 Questions

1. Figure 3.5.7.8 shows a pulley system used to draw water from a well 8 m deep. The bucket's volume is 0.001 m^3.

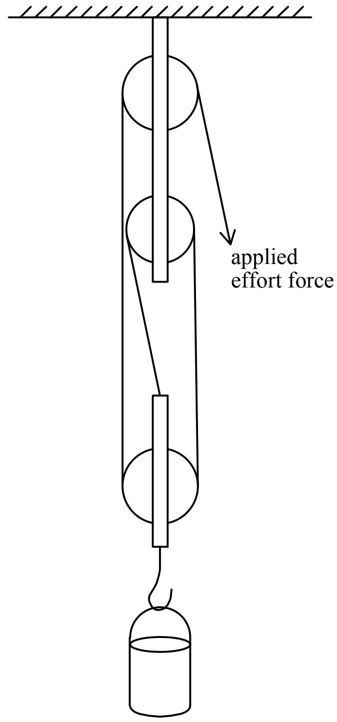

Figure 3.5.7.8

(a) What length of the rope must be pulled in order to lift the bucket out of the well? **(2 mks)**
(b) If the mechanical advantage of the system is given to be 2.4, what effort is needed to lift a full bucket out of the well? Assume the mass of the container is negligible and $\rho_{\text{water}} = 1000\ kg\ m^3$. **(3 mks)**
(c) Determine the system's efficiency. **(3 mks)**
(d) Suggest two possible reasons why the efficiency of the system is less than 100%. **(2 mks)**

. . . .

Solution

(a) *It is assumed that the depth of the well is the total distance through which the bucket must be lifted. It is given in the figure that the system is a block and tackle pulley with a single pulley on its block.*

$$\text{given equation for velocity ratio of a block and tackle pulley: } VR = 2N_b + 1$$

$$\text{substituting effort and load distances: } \frac{d_e}{d_l} = 2N_b + 1$$

$$\text{turning effort distance into subject: } d_e = d_l\,(2N_b + 1)$$

$$\text{substituting known values: } d_e = (8\ m)\,[2(1) + 1]$$

$$\text{final answer: } \boxed{d_e = 24\ m}$$

(b) *The system's mechanical advantage is given.*

$$\text{given equation of mechanical advantage: } MA = \frac{F_l}{F_e}$$

$$\text{turning effort force into subject: } F_e = \frac{F_l}{MA}$$

$$\text{substituting equation for force of weight: } F_e = \frac{mg}{MA}$$

$$\text{substituting liquid density and volume for mass: } F_e = \frac{(\rho V)g}{MA}$$

$$\text{substituting known values: } F_e = \frac{\left[\left(1000\ kg\ m^{-3}\right)\left(0.001\ m^3\right)\right]\left(10\ m\ s^{-2}\right)}{2.4}$$

$$\text{final answer: } \boxed{F_e \approx 4.17\ N}$$

(c) *The efficiency of the system can be calculated directly from its mechanical advantage and its velocity ratio.*

$$\text{given equation for machine's efficiency: } \eta = \frac{MA}{VR} \times 100\%$$

$$\text{substituting equation for velocity ratio of block and tackle pulley: } \eta = \frac{MA}{2N_b + 1} \times 100\%$$

$$\text{substituting known values: } \eta = \frac{2.4}{2(1) + 1} \times 100\%$$

$$\text{final answer: } \boxed{\eta = 80\%}$$

(d) *The following is a non-exhaustive list of possible sources. Only two are required.*

- Input energy is lost as thermal energy due to friction in the pulleys.
- Input energy is lost as sound energy when the pulley's squeak.
- Input energy is lost as elastic energy when the rope stretches.
- The weight of the lower pulley is not considered in the effort force.
- The weight of the bucket itself, without water inside, is not considered in the effort force.

2. (a) Under what conditions is a machine's mechanical advantage and velocity ratio are equal? **(1 mk)**

(b) In an experiment using a frictionless pulley system and a load of known mass, a student recorded the values of effort distance and load distance in the following table.

load distance / m	2.2	3.6	4.1	5.0	5.8	6.2	8.0
effort distance / m	8.8	14.4	16.4	20	23.2	24.8	32

(i) Plot a graph of effort distance along the y-axis against load distance along the x-axis. **(5 mks)**

(ii) Determine the gradient of this graph and state its physical significance. **(2 mks)**

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Solution

(a) A machine's velocity ratio and mechanical advantage are equal only when its efficiency is 100%.

(b) (i) *See figure 3.5.7.9*

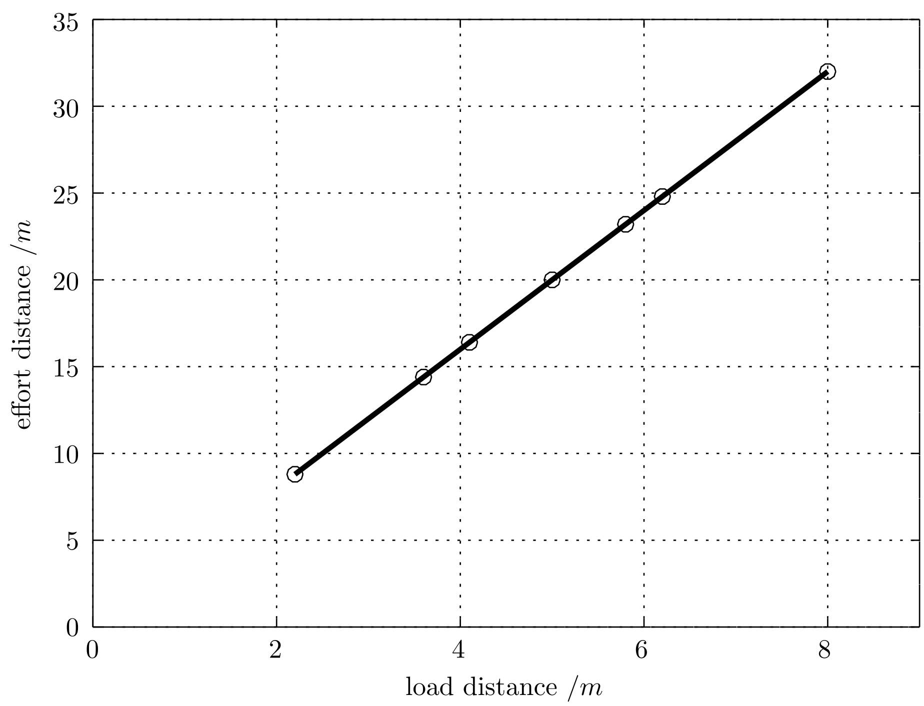

Figure 3.5.7.9

(ii) *The gradient of this graph is most easily determined by considering the the most extreme data points. These can be taken as* $d_l = 2.2\ m,\ d_e = 8.8\ m$ *and* $d_l = 8\ m,\ d_e = 32\ m$.

$$\text{calculating gradient: } k = \frac{\Delta d_e}{\Delta d_l}$$

$$\text{substituting known values: } k = \frac{32\ m - 8.8\ m}{8.0\ m - 2.2\ m}$$

$$\text{final answer: } \boxed{\text{gradient} = k = 4}$$

The gradient of this graph represents the pulley system's velocity ratio.

3.5.8 Mechanical Power

Objectives

By the end of the lesson, students should be able to
1. define power, stating its unit.
2. solve problems involving power, a one-dimensional force and displacement.
3. explain how average power can be measured.
4. solve problems involving average power.

Power

- A device that does a lot of useful work within a short amount of time is called "powerful".
- **Power**, or **P**, is the rate of energy transformed per unit time.
- It is a scalar.
- Its SI unit is the Watt, abbreviated W.

$$P = \frac{E}{\Delta t} \tag{3.5.8.1}$$

  Where
  - P is the power of the device, mechanism, or machine, in W;
  - E is the energy being transformed, in W;
  - Δt is the duration of time, in s.

- The SI unit of work, the Watt, is equivalent to one Joule per second.

$$W = J\ s^{-1}$$

- There are many different types of power.

type	**description**
mechanical power	work done or mechanical energy transformed per time
thermal power	thermal energy flow or heat transfer per time
electric power	transfer of electric energy per time

Table 3.5.8.1

- This chapter focuses entirely on the work of mechanical energy.
- Problems involving mechanical power can require that one of the following three properties be calculated while the other two are given - power, work/energy and time.

  Example: What is a ideal machine's power if it can lift a 10 kg body 1.5 m high in 7.5 seconds?

$$\text{given equation for power: } P = \frac{E}{\Delta t}$$

$$\text{assuming work done is potential energy gained: } P = \frac{PE}{\Delta t}$$

$$\text{substituting equation for potential energy: } P = \frac{mgh}{\Delta t}$$

$$\text{substituting known values: } P = \frac{(10\ kg)\left(10\ m\ s^{-2}\right)(1.5\ m)}{7.5\ s}$$

$$\text{final answer: } \boxed{P = 20\ W}$$

Example: An ideal, 10000 W machine is used to drag a vehicle horizontally against a 2000 N force of friction for a distance of 25 m. How long does this take?

$$\text{given equation for power: } P = \frac{E}{\Delta t}$$

$$\text{assuming energy transformed is work done against friction: } P = \frac{W_f}{\Delta t}$$

$$\text{substituting equation for work done against friction: } P = \frac{F_f d}{\Delta t}$$

$$\text{turning time duration into subject: } \Delta t = \frac{F_f d}{P}$$

$$\text{substituting known values: } \Delta t = \frac{(2000\ N)(25\ m)}{10000\ W}$$

$$\text{final answer: } \boxed{\Delta t = 5\ s}$$

Example: Given 1000 $L = 1\ m^3$, how high can an ideal 5000 W pulley machine lift 250 L of water in 60 s? Assume the density of water is 1000 $kg\ m^{-2}$.

$$\text{given equation for power: } P = \frac{E}{\Delta t}$$

$$\text{assuming work done is potential energy gained: } P = \frac{PE}{\Delta t}$$

$$\text{substituting equation for potential energy: } P = \frac{mgh}{\Delta t}$$

$$\text{substituting liquid density and volume for mass: } P = \frac{(\rho V)gh}{\Delta t}$$

$$\text{turning height into subject: } h = \frac{P\Delta t}{(\rho V)g}$$

$$\text{substituting known values: } h = \frac{(5000\ W)(60\ s)}{[(1000\ kg\ m^{-2})(250\ L)](10\ m\ s^{-2})}$$

$$\text{applying conversion factor: } h = \frac{(5000\ W)(60\ s)}{(1000\ kg\ m^{-2})(250\ L)(10\ m\ s^{-2})}\left(\frac{1000\ L}{1\ m^3}\right)$$

$$\text{final answer: } \boxed{h = 120\ m}$$

Efficiency in Terms of Power

- Only an ideal machine has the same input power as useful output power.
- That is, a machine's input energy per time is equal to its useful output energy per time only if its 100% efficient.

$$\text{given equation for efficiency: } \eta = \frac{E_{\text{out, useful}}}{E_{\text{in}}} \times 100\%$$

$$\text{dividing numerator and denominator by same duration of time: } \eta = \frac{\frac{E_{\text{out, useful}}}{\Delta t}}{\frac{E_{\text{in}}}{\Delta t}} \times 100\%$$

$$\text{substituting power: } \eta = \frac{P_{\text{out, useful}}}{P_{\text{in}}} \times 100\%$$

Where
- η is the efficiency of a machine (unit-less);
- $P_{\text{out, useful}}$ is the machine's useful output power, in W;
- P_{in} is the machine's useful input power, in W.

Average Power

- A student walking up a flight of stairs gains a new height with each step up, but not each step forward.

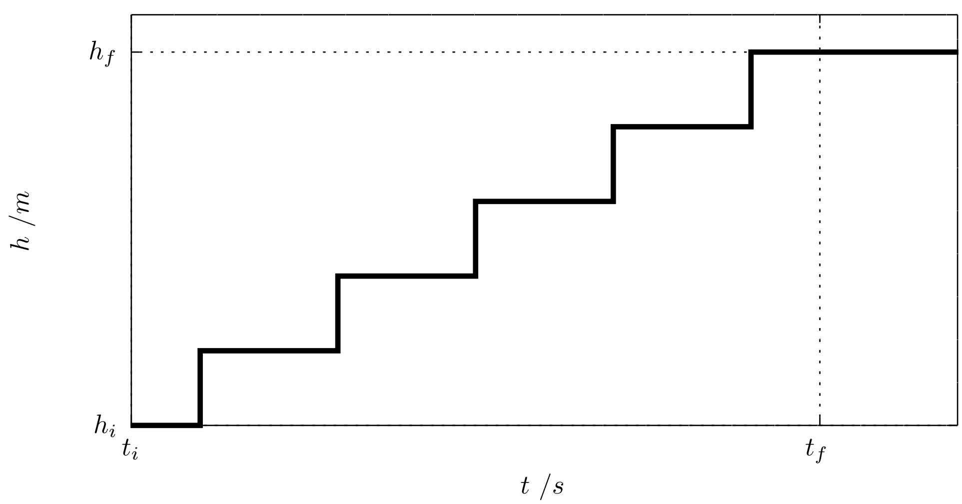

Figure 3.5.8.1

- Therefore, her potential energy is gained with each step up, but not each step forward.
- This in turn means that the power of her work against gravity is not continuous in time.
- In such a scenario, only the student's average power can be calculated.
- **Average power**, or $\overline{\mathbf{P}}$, is the total amount of energy transformed with a total duration of time.

$$\overline{P} = \frac{E_{\text{total}}}{t_{\text{total}}}$$

Where
- $\overline{P}$ is the average power, in W;
- E_{total} is the total energy transformed, in J;
- t_{total} is the total duration of time, in s.

- In the example of the student walking up the stairs, the total energy transformed is taken as the difference between her initial and final potential energy.

$$\text{given equation for average power: } \overline{P} = \frac{E_{\text{total}}}{\Delta t_{\text{total}}}$$

$$\text{considering energy transformation as difference in potential energy: } \overline{P} = \frac{PE_f - PE_i}{\Delta t_{\text{total}}}$$

$$\text{considering duration as difference in initial and final time: } \overline{P} = \frac{PE_f - PE_i}{t_f - t_i}$$

$$\text{substituting equation for potential energy: } \overline{P} = \frac{mg\left(h_f - h_i\right)}{t_f - t_i}$$

- Therefore, the average power can be calculated as the slope or gradient of a line showing the average energy transformed along the y-axis against time along the t-axis.

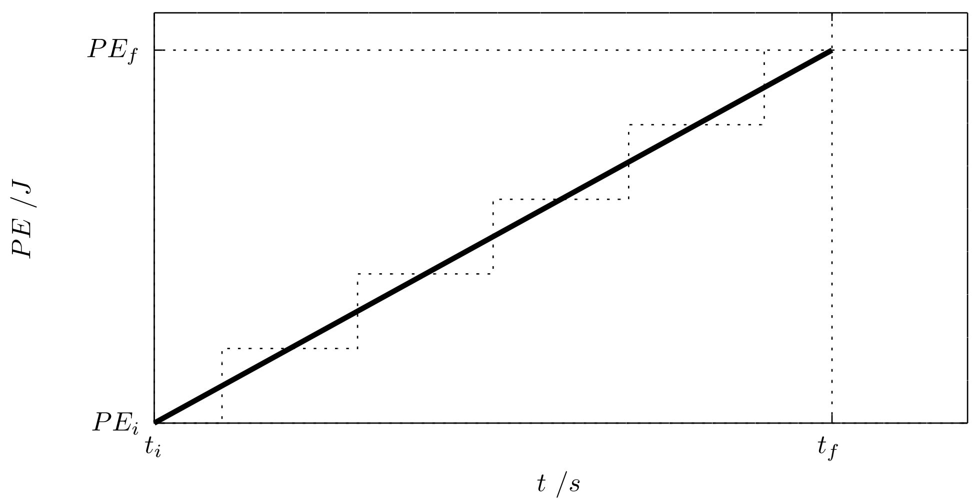

Figure 3.5.8.2

Example: In a single hour, Amadou carries 6 containers of water up a hill having a vertical distance of 30 m. If the mass of each container is 20 kg, calculate the average power of Amadou's work against gravity.

$$\text{given equation for average power: } \overline{P} = \frac{E_{\text{total}}}{t_{\text{total}}}$$

$$\text{assuming all energy transformed is potential: } \overline{P} = \frac{PE_{\text{total}}}{t_{\text{total}}}$$

$$\text{considering PE of all six containers: } \overline{P} = \frac{6\left(mgh\right)}{t_{\text{total}}}$$

$$\text{substituting known values: } \overline{P} = \frac{6\left[\left(20\ kg\right)\left(10\ m\ s^{-2}\right)\left(30\ m\right)\right]}{1\ hr}$$

$$\text{applying conversion factors: } \overline{P} = \frac{6\left[\left(20\ kg\right)\left(10\ m\ s^{-2}\right)\left(30\ m\right)\right]}{1\ hr}\left(\frac{1\ hr}{60\ min}\right)\left(\frac{1\ min}{60\ s}\right)$$

$$\text{final answer: } \boxed{\overline{P} = 10\ W}$$

GCE Paper 1 Questions

1. The energy supplied in one minute to a 3 kW heater is

   A 180 J B 300 J C 3000 J D 180000 J

2. Which of the following is a time rate of working?

   A displacement B velocity C momentum D power

Questions 3 through 7 refer to figure 3.5.8.3, which shows a pulley system before and after a load is lifted.

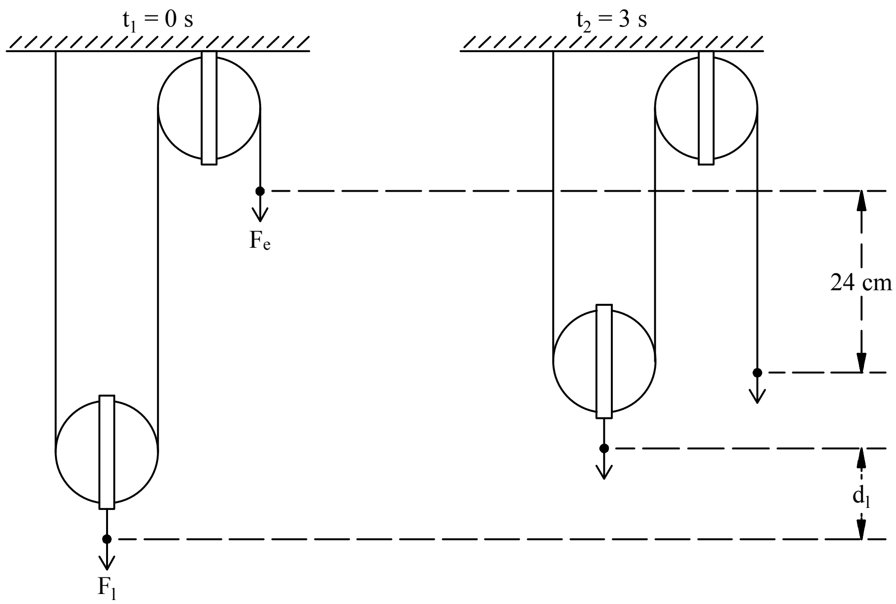

Figure 3.5.8.3

3. The velocity ratio of the system is

   A 0.5 B 1 C 2 D 5

4. The load distance lifted (d_l) is

   A 0.06 m B 0.12 m C 0.24 m D 0.48 m

5. If the load force lifted is given to be 120 N, the useful output power of the system is

   A 4.8 W B 5.6 W C 14.4 W D 16.8 W

6. If the effort force applied is given to be 70 N, the power input into the system is

   A 4.8 W B 5.6 W C 14.4 W D 16.8 W

7. Considering the answers to questions 5 and 6, which of the following conclusions can be made?

   A The system has an efficiency less than 100%.
   B The pulley system has a mechanical advantage of 3.
   C The pulley system is not physically possible.
   D The tension line is made of rubber.

Questions 8 through 10 refer to figure 3.5.8.4, which shows four different vehicles, A, B, C and D having four different speeds v_A, v_B, v_C and v_D at four different times after speeding up from rest at time t_0. The useful energy from each vehicle's engine is transformed completely into the kinetic energy of its forward motion.

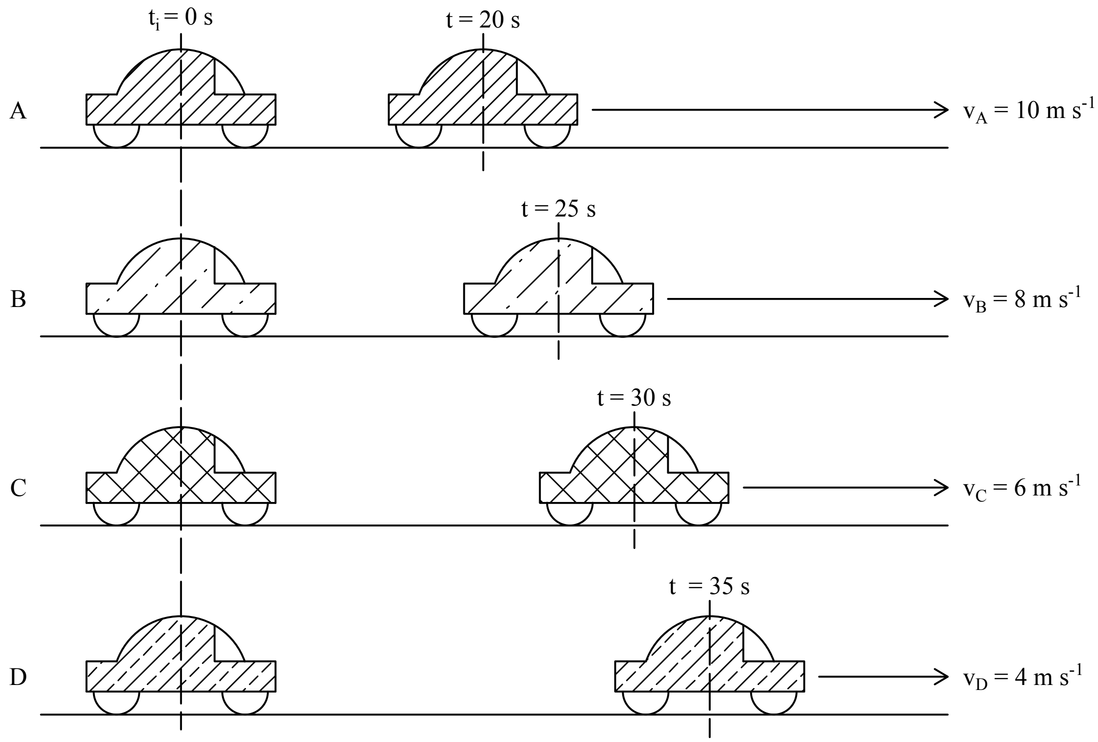

Figure 3.5.8.4

The following data for the mass of each vehicle is given:

$m_A = 900\ kg$ $m_B = 2000\ kg$ $m_C = 1500\ kg$ $m_D = 5000\ kg$

8. Which vehicle has the most powerful engine?
9. Which vehicle has the least powerful engine?
10. Which vehicle's engine has a power greater than 1 kW but less than 2 kW?

GCE Paper 1 Solutions

1. D 2. D 3. C 4. B 5. A 6. B 7. A 8. B 9. C 10. D

GCE Paper 2 Questions

1. A student used several different motors to lift the same mass along the same vertical distance of 1 m. During her experiment, she recorded the following table of information for each motor's power (P) as well as time taken for each lift (Δt).

P /W	120	150	300	600	750
Δt /s	0.125	0.100	0.050	0.025	0.020
$(\Delta t)^{-1}$ /s^{-1}		10		40	

(a) Fill in missing values of $\frac{1}{\Delta t}$ in the table. **(2 mks)**

(b) Plot the values of P along the y-axis against $\frac{1}{\Delta t}$ along the x-axis. **(3 mks)**

(c) Determine the gradient of this graph. **(2 mks)**

(d) Using this gradient, determine the mass lifted in each trial. State any assumptions made. **(3 mks)**

....

Solution

(a) *See table 3.5.8.2. Solutions in bold.*

P /W	120	150	300	600	750
Δt /s	0.125	0.100	0.050	0.025	0.020
$(\Delta t)^{-1}$ /s^{-1}	**8**	10	**20**	40	**50**

Table 3.5.8.2

(b) *See figure 3.5.8.5.*

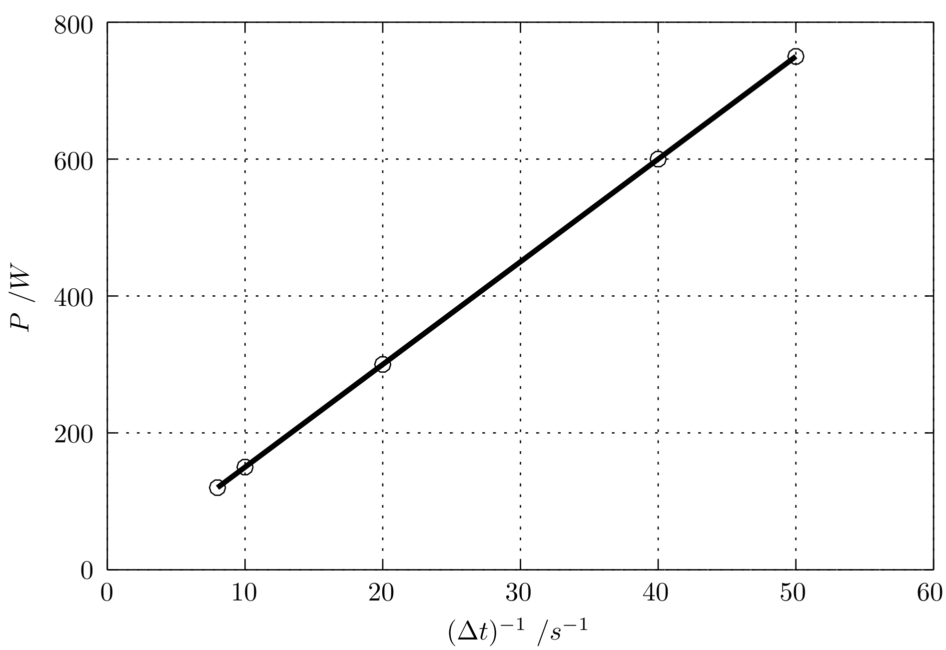

Figure 3.5.8.5

(c) *The gradient, slope, or proportionality is most easily calculated as the difference in the first and last power values per the difference in the first and last inverse-time values.*

$$\text{calculating gradient:} \quad k = \frac{\Delta P}{(\Delta t)^{-1}}$$

$$\text{substituting first and last data points:} \quad k = \frac{750\ W - 120\ W}{(50\ s^{-1}) - (8\ s^{-1})}$$

$$\text{final answer:} \quad \boxed{\text{gradient} = k = 15\ W\ s}$$

(d) *It is assumed that this gradient is the potential energy gained by the mass during each trial.*

$$\text{given equation for power:}\ P = \frac{E}{\Delta t}$$

$$\text{expressing right side as product:}\ P = E\left(\frac{1}{\Delta t}\right)$$

$$\text{substituting k according to graph:}\ P = k\left(\frac{1}{\Delta t}\right)$$

$$\text{inferring equivalence:}\ k = E$$

$$\text{assuming energy gained during each trial is completely potential:}\ k = PE$$

$$\text{substituting equation for potential energy:}\ k = mgh$$

$$\text{turning mass into subject:}\ m = \frac{k}{gh}$$

$$\text{substituting known values:}\ m = \frac{15\ W\ s}{(10\ m\ s^{-2})\,(1\ m)}$$

$$\text{final answer:}\ \boxed{m = 1.5\ kg}$$

Assumptions in this calculation include

- the acceleration of gravity is $10\ m\ s^{-2}$.
- all motors used are 100% efficient.
- all energy provided by the motor is transformed only into potential energy.

3.6 Optics

3.6.1 Optics in Space

Objectives

By the end of the lesson, students should be able to

1. state the basic properties of light.
2. describe how shadows are formed.
3. describe lunar and solar eclipses.

Introduction to Optics

- Optics is the study of the production, propagation and measurement of light and its properties.
- Light is an electromagnetic wave that travels in a straight line.
- Light is often represented as straight lines emitted from some central source.
- There are many sources of light.

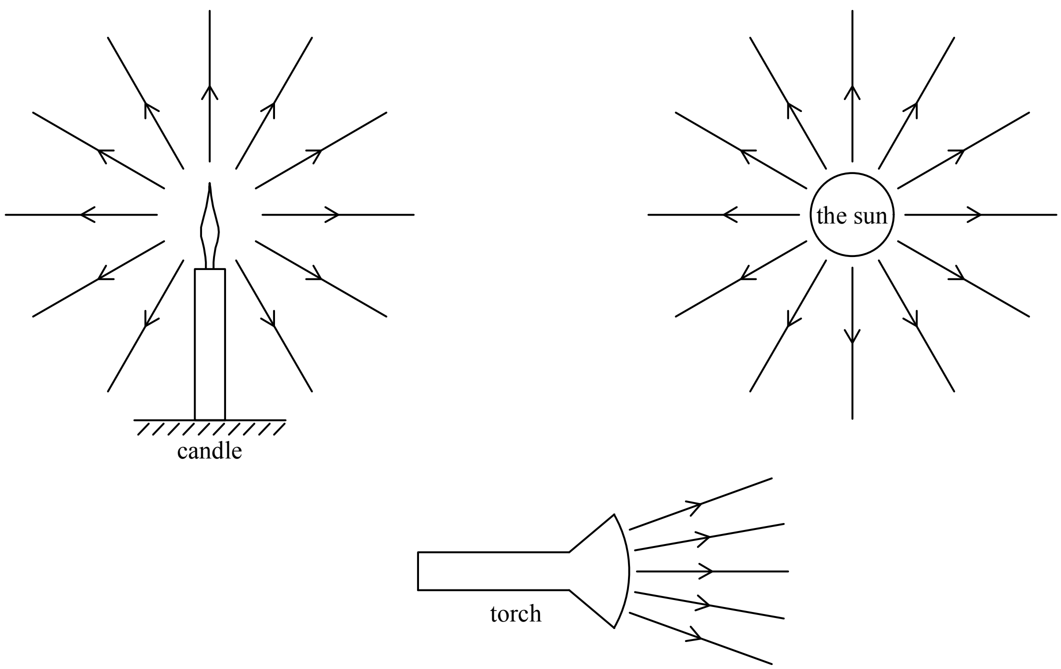

Figure 3.6.1.1

Basic Properties of Light

- Light is a from of wave motion.
  - Light is a type of electromagnetic wave.
  - Electromagnetic waves behave similarly, but not identically to mechanical waves.
  - Electromagnetic and mechanical waves are distinguished mostly by their speed and ability to propagate.
    - Electromagnetic waves travel much faster than mechanical waves.
    - The speed of a typical light wave is around $3 \times 10^8\ m\ s^{-1}$.
    - The typical speed of sound, one of the fastest mechanical waves, is $3 \times 10^2\ m\ s^{-1}$.
    - That is, light travels 10^6, or one million times faster than sound.
    - Electromagnetic waves can travel trough space in the absence of any matter.
    - Mechanical waves require some type of matter through which to propagate.

- Light transfers energy from one place to another.
  - A body that emits light looses energy.
  - A body that absorbs light gains energy.
  - A body that reflects light has no net gain or loss in energy.

- Light is a form of radiation.
  - **Radiation** is a general term used to describe anything that travels outward (radiates) from a source but cannot be identified as a solid, liquid or gas.
  - Light is made of photons which do not behave like normal matter.
  - Photons have no mass, volume or density.
  - Photons can travel through the vacuum of space.
  - The nature of photons is a current subject of investigation in physics; there is still much unknown.

- Light is only the type of electromagnetic wave that can be detected by the human eye.
  - **Visible light** is a term used to distinguish the light humans can see from other electromagnetic waves.

Optical Properties

- A **medium** is the material or region through which a wave travels.

- Both “mediums” and “media” are acceptable plural forms of “medium”.

- The optical properties of a medium concern how well it allows light to pass through it.

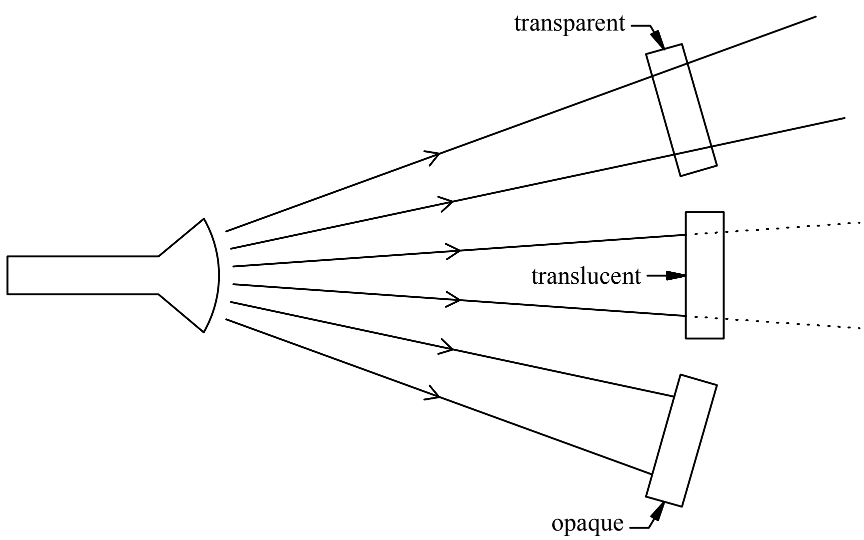

Figure 3.6.1.2

- **Transparent** media, which pose limited or no obstruction to light, include
  - air
  - water
  - glass
  - clear plastic
  - empty space

- **Translucent** media, which obstruct some, but not all light, include
  - wet paper
  - semi-coloured plastics
  - human skin

- **Opaque** media, which completely block light, include
  - wood
  - meat
  - cement
  - dirt
  - thick fabric

Formation of Shadows

- Sources of light are often modelled as point sources or extended sources.
- **Point sources** act as single points from which light rays radiate.
- **Extended sources** act as surfaces covered with point sources.
- That is, light from an extended source radiates from all points of its surface, not just one.

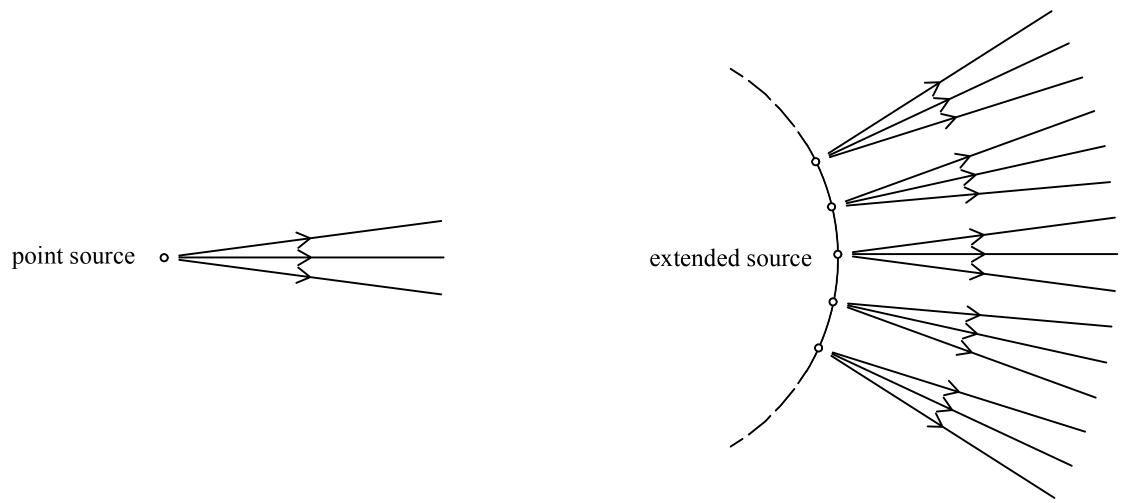

Figure 3.6.1.3

- **Shadows** are formed when an opaque object is placed between a light source and a surface such as a screen.
  - The opaque object blocks some of the light rays from reaching the surface.
  - The light rays that are not blocked do reach the screen.
  - The region of the surface accessible by light rays is well lit.
  - The region of the surface where light rays have been blocked is dark.
  - This dark region, as distinguished from the well-lit region, is called a shadow.
- Point sources create shadows that are full and sharp.

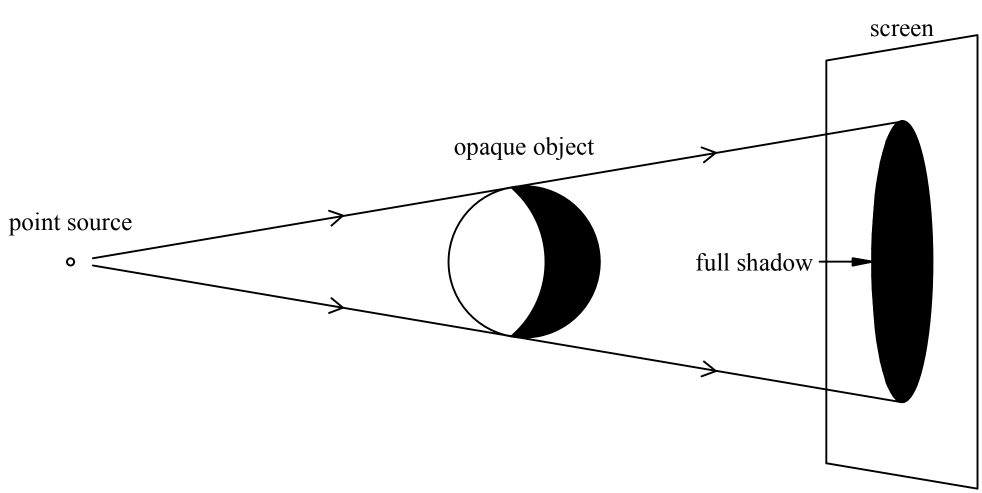

Figure 3.6.1.4

- Extended sources cause two shadows on the surface.
  - The region of the surface were all light is blocked is a full shadow.
  - The region of the surface were some light is able to come from the bottom of the extended source across the top of the opaque object, or vice-versa, is a partial shadow.
  - This lack of sharp distinction between full, partial and no shadow on the surface causes the shadow to be blurry or indistinct.

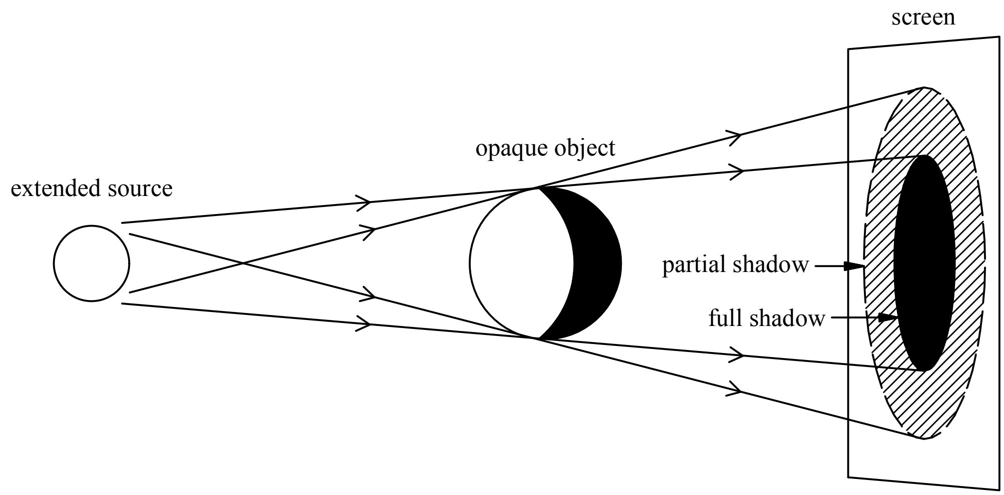

Figure 3.6.1.5

Eclipses

- Eclipses occur when one the moon block's the sun's light from reaching the earth, or vice versa.

- A **solar eclipse**, or **eclipse of the sun** occurs when the moon passes directly between the sun and the earth causing the moon's shadow to fall on a specific part of the earth's surface.

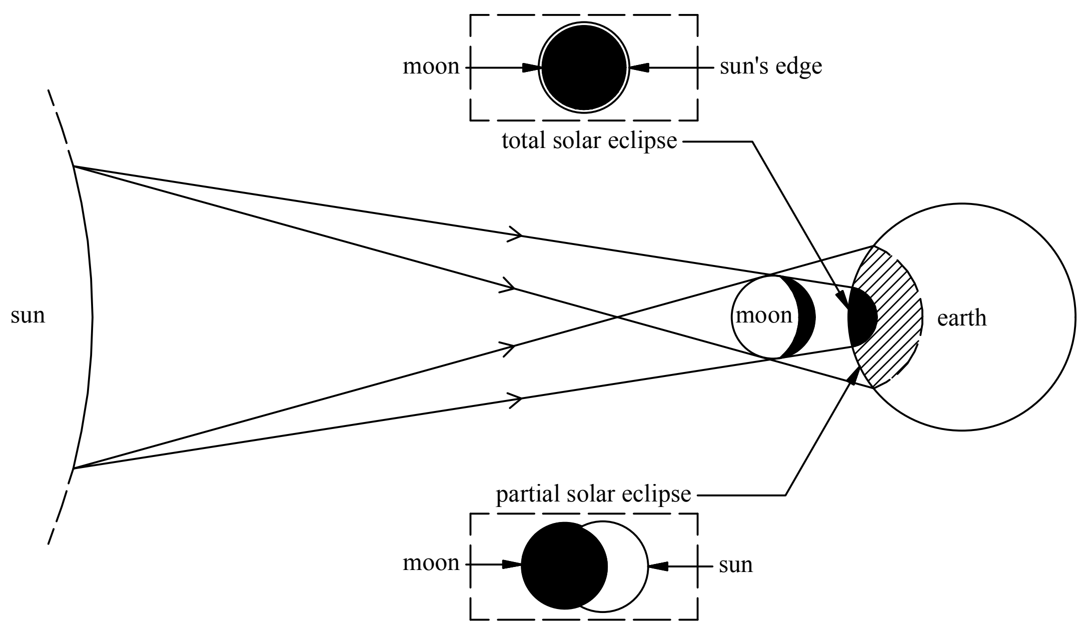

Figure 3.6.1.6

  - Only the region where the moon's shadow falls observes any effect of a solar eclipse.
  - Of this region, a smaller portion observes a full eclipse while another observes only a partial eclipse.

- A **lunar eclipse**, or **eclipse of the moon** occurs when the directly passes directly between the sun and the moon causing the earth's shadow to fall on the moon.

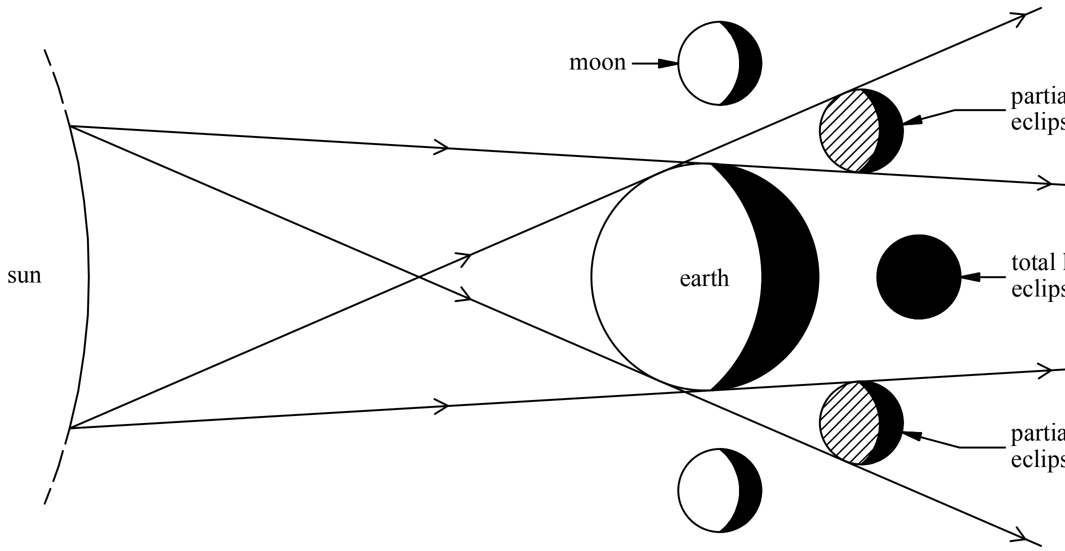

Figure 3.6.1.7

  - A lunar eclipse is seen as the same from all parts of the earth capable of observing the moon.
  - The completeness of a lunar eclipse however depends on the location of the moon.
  - A lunar eclipse can be total or partial, depending on where in the earth's shadow it is located.
  - NB: A lunar eclipse is not part of the gradual, 28-day lunar cycle.

Pinhole Cameras

- If the light of some object is made to pass through a very small opening on the side of an enclosure, an upside-down image is created on the interior surface of the far wall.

- If film is placed on this far wall, an image is captured.

- Such a camera functions because light travels in straight lines.

- The outline of the image formed depends on the shape of the hole as well as the intensity of light allowed inside.

- The hole must be small; increasing increasing its size causes a brighter but blurrier image.

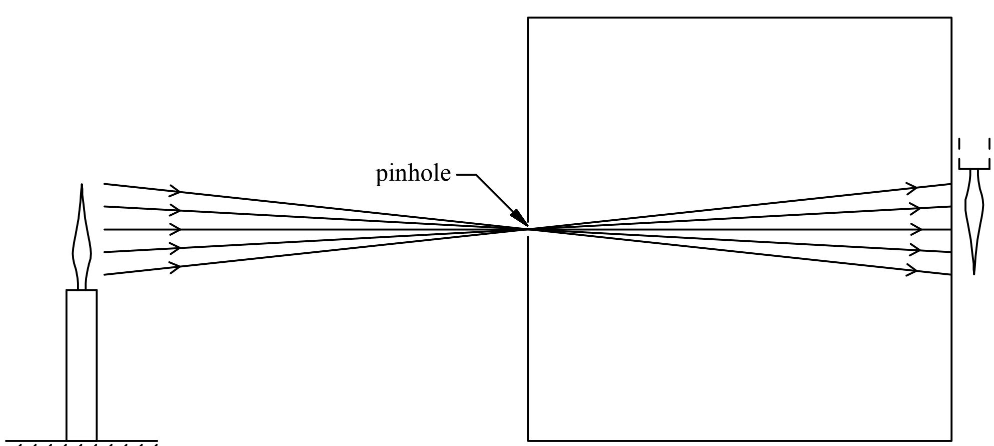

Figure 3.6.1.8

GCE Paper 1 Questions

1. An image formed in on the interior surface of a pinhole camera is

   A upside down B right side up C invisible D a different color

2. Shadows are formed because light

   A travels faster in a vacuum.
   B travels faster than sound.
   C travels both through a vacuum and matter.
   D travels in a straight line.

3. During a lunar eclipse,

   A the moon stops sunlight from reaching the earth.
   B the earth stops sunlight from reaching the moon.
   C the moon stops earthlight from reaching the sun.
   D the sun stops earthlings from mooning the sun.

4. During a solar eclipse,

   A the moon stops sunlight from reaching the earth.
   B the earth stops sunlight from reaching the moon.
   C the moon stops earthlight from reaching the sun.
   D the moon kidnaps every first-born son.

5. The effects of a solar eclipse are visible

   A everywhere on earth.
   B only in the moon's partial shadow.
   C only in the moon's full shadow.
   D only in certain regions of the earth.

6. Light waves, in general, travel

   A faster than sound. B slower than sound. C infinitely fast. D through glass only.

7. Light passes most easily through which of the following media?

   A wood B cement C wet paper D the vacuum of space

8. The speed of a typical light wave is

   A $3 \times 10^6\ m\ s^{-1}$ B $3 \times 10^6\ m\ s^{-2}$ C $3 \times 10^8\ m\ s^{-1}$ D $3 \times 10^8\ m\ s^{-2}$

9. Which of the following has no mass?

   A 1 L of water B air C wood D a photon

10. Which of the following celestial bodies is <u>not</u> directly involved in a solar eclipse?

    A the sun B the earth C the moon D mars

GCE Paper 1 Solutions

1. A 2. D 3. B 4. A 5. D 6. A 7. D 8. C 9. D 10. D

GCE Paper 2 Questions

1. On December 2^{nd}, 2002, there was a solar eclipse. It was observed in several southern African countries at about noon. It was however not observed in Cameroon.

   (a) Explain what was observed in those southern African countries at about noon. **(2 mks)**
   (b) Explain how a solar eclipse occurs. **(3 mks)**
   (c) Explain why it was not observed worldwide. **(2 mks)**
   (d) State the property of light for which a solar eclipse is evidence. **(2 mks)**

   ....

   Solution

   (a) In those southern African countries, the sun was observed to be blocked by a large, dark circle. Only a thin ring of light could be seen around the obstruction.

   (b) During a solar eclipse,
   - the moon's orbit passes directly between the sun and the earth.
   - the moon forms a full shadow on certain portions of the earth's surface.
   - the moon also forms a partial shadow that surround the full one.
   - those within the region of the full shadow see the sun almost completely obstructed.
   - those within the region of the partial shadow see the sun partially obstructed.

   (c) Such an eclipse is not observed worldwide because the moon's shadow is only formed on a portion of the earth. It is only in this portion that the eclipse is observed.

   (d) Light travels in a straight line.

3.6.2 Plane Mirrors

Objectives

By the end of the lesson, students should be able to

1. recall the laws of reflection.
2. construct ray diagrams to show the formation of images by plane mirrors.
3. state characteristics of images formed by plane mirrors.
4. describe an experiment verify the laws of reflection.
5. state some applications of plane mirrors.
6. describe how light reaches the earth from the sun and the moon.

Basics of Reflection

- **Reflection** is the striking and returning of a light wave when it runs into a reflective surface.
- The **incident ray** is the light wave approaching the surface.
- The **reflected ray** is the light wave leaving the surface.
- The **normal line**, or **N**, is an imaginary line perpendicular, or at a right angle to the reflective surface.
- The **optical origin**, or **O**, the point at which the imaginary normal line intersects the reflective surface.
- The **angle of incidence**, or **i**, is the angle the incident ray makes against the normal line.
- The **angle of reflection**, or **i′**, is the angle the reflected ray makes against the normal line.

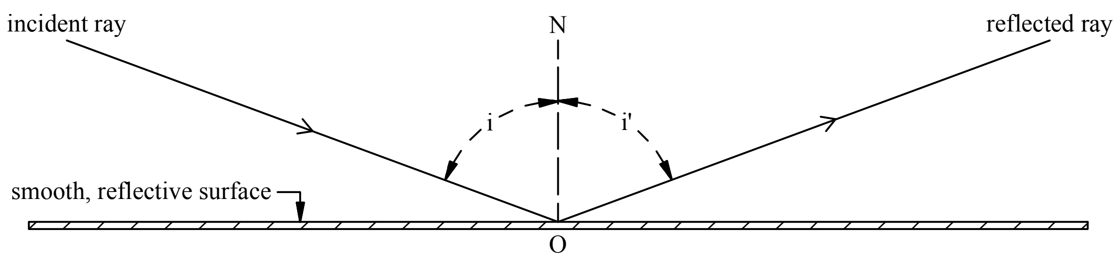

Figure 3.6.2.1

- The **laws of reflection** concern the relationship between these two rays and the normal line.
    1. The angle of incidence is equal to the angle of reflection.

$$\text{during reflection: } i = i' \qquad (3.6.2.1)$$

    Where
    - i is the angle of incidence, in degrees;
    - i' is the angle of reflection, in degrees.

    2. The incident ray, the reflected ray, and the normal line all lay in the same plane.
        - For example, considering the reflection diagram shown in figure 3.6.2.1, if the incident ray and the normal line are drawn on the paper or board, the reflected ray could not be shown coming out of the paper or board while still maintaining the equivalence $i = i'$.
        - NB: The term "coplanar" is often used to describe features that are all located on the same plane.

- **Mirrors** are common reflective surfaces.

The Formation of Images in Plane Mirrors

- **Real objects** that actually exist create imaginary images in plane mirrors that appear to be behind the mirror.
- **Virtual images** are any images that appear behind a mirror or optical lens.

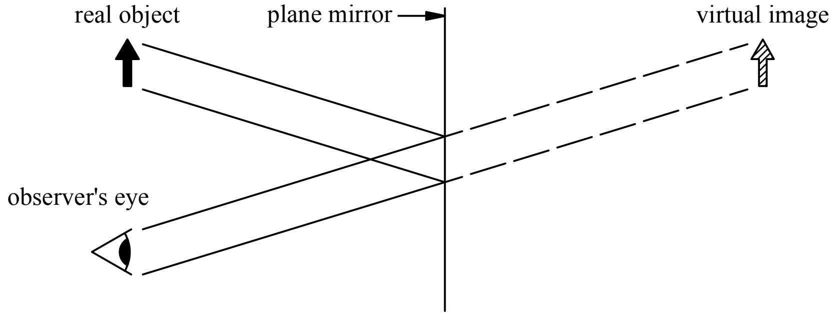

Figure 3.6.2.2

- There are several laws which govern characteristics of images formed in plane mirrors.

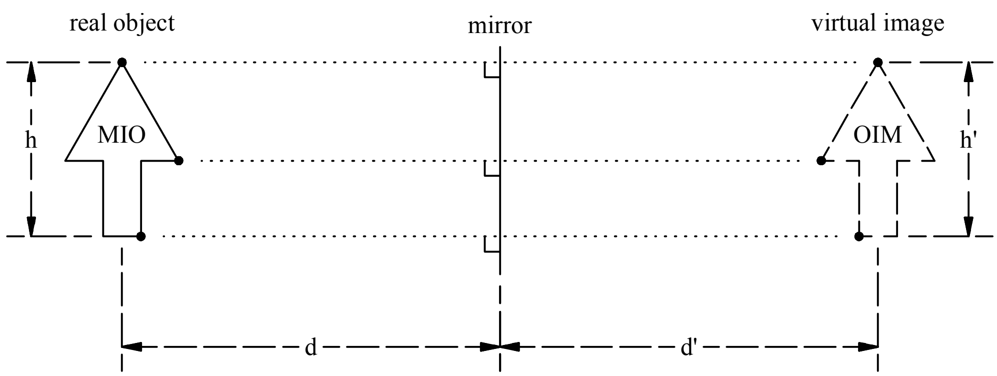

Figure 3.6.2.3

1. The image is virtual, behind the mirror.
2. The image is upright, having the same vertical orientation as the real object.
3. The image has the same size as the real object.

$$h = h'$$

   Where
   - h is the height of the real object, in m;
   - h' is the apparent height of the virtual image, in m.

4. The image's distance from the mirror is the same as that from the mirror to the real object.

$$d = d'$$

   Where
   - d is the distance between the real object and the mirror's surface, in m;
   - d' is the apparent distance between the mirror's surface and the virtual image, in m.

5. Any line connecting a point on the image to the corresponding point on the real object passes through the mirror perpendicularly.
6. The image is inverted laterally.
   - This is shown as the letters "MIO" on the real object appearing as "OIM" on the virtual image.

Experiment to Verify the Laws of Reflection

I Procedure

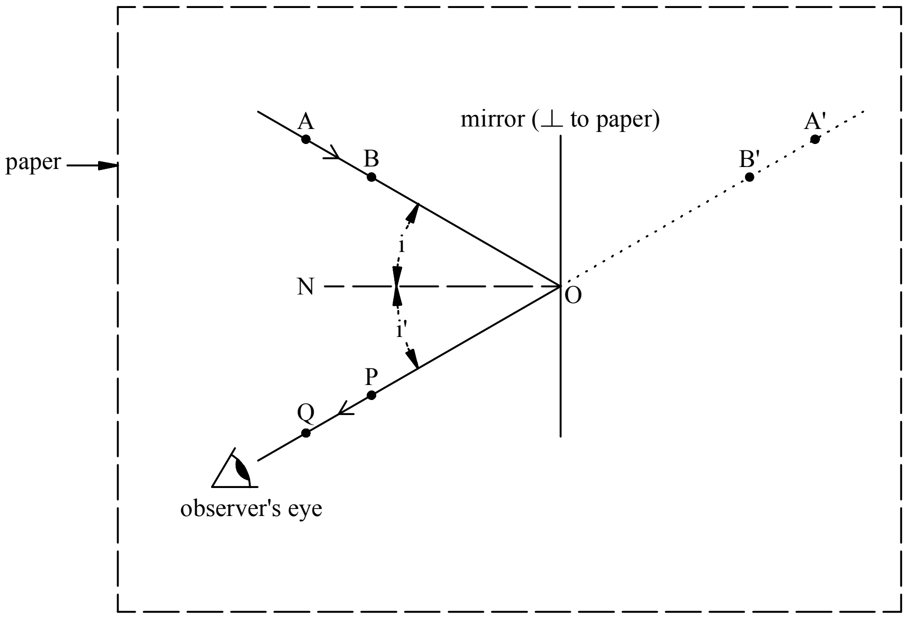

Figure 3.6.2.4

1 A piece of white paper is placed on a flat, horizontal surface like a desk or lab table.
2 A plane mirror is placed vertically perpendicular to the paper.
3 Pins labelled A and B are placed in front of the mirror to indicate points.
4 The incident ray is traced as a line that connects these points and that touches the mirror.
5 The normal line N is drawn perpendicularly to the mirror touching at the same point as the incident ray.
6 The optical origin O is labelled as intersection between the incident ray and normal line.
7 The observer places their eye so that the virtual image of pin B appears directly in front of that of pin A.
8 Pins labelled P and Q are placed so that they block the virtual image of pin B from the observer.
9 The observer removes their eye.
10 The reflected ray is traced as a line connecting O as well as pins P and Q.
11 The angle of incidence, i, is measured with a protractor as the angle between line ABO and N.
12 The angle of reflection, i', is measured with the same protractor as the angle between line OPQ and N.

II Observations

- The angle of incidence is equal to the angle of reflection.

$$i = i'$$

III Conclusion

- Given that the angles of incidence and reflection are equal and that the incident and reflected rays are coplanar, the laws of refection are verified.

IV Precautions

- The experiment is repeated with other pin locations to achieve different angles of incidence and reflection.
- A carpenter's square is used if available to make sure N is truly perpendicular to the mirror.
- The observer removes their eye before tracing the reflected ray to avoid injury.

Applications of Plane Mirrors

- Periscopes
  - Two plane mirrors are installed in a pipe allow light to be rerouted.
  - This allows objects in a dangerous area to be viewed by an observer in a safe area.
  - Periscopes are commonly used in submarines to allow sailors to peak above the water's surface.

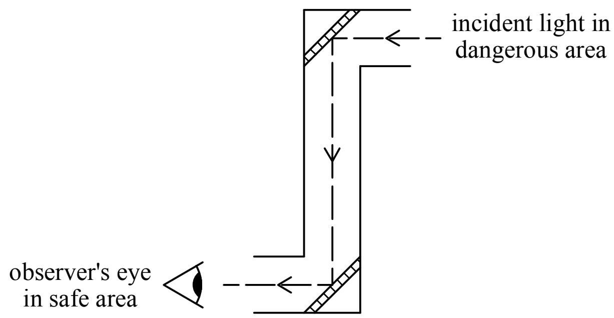

Figure 3.6.2.5

- Experiments involving the re-routing of laser beams.
- Kaleidoscopes
- Navigational sextants

Moonlight as Reflected Light

- The sun is so far from the earth and the moon that its rays reach the earth almost perfectly parallel.
- While sunlight creates day on one side of the earth, the other side experiences night.
- The moon acts like a plane mirror because its size is so large and far away.
- Therefore, sunlight is reflected off the moon like a natural mirror.
- Light from the moon also reaches the earth as almost completely parallel rays.
- This allows those on the dark side of the earth to see the sun's light at night as moonlight.

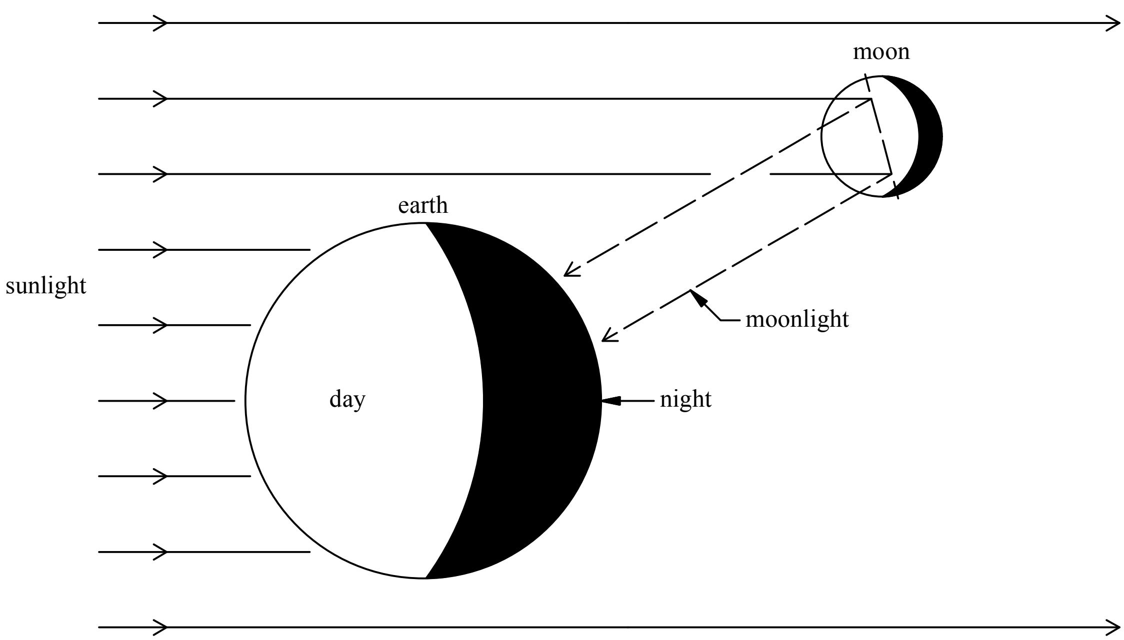

Figure 3.6.2.6

- However, much of the sun's light passes the earth as well as the moon and continues off into space.

GCE Paper 1 Questions

1. A boy stands 2 m in front of a plane mirror. If the mirror is moved 1 m closer to the boy, the distance from him to his final image is

   A $2m$ B $3m$ C $5m$ D $1m$

2. A ray of light strikes a mirror at an angle of 60° to the mirror's surface. The angle of reflection is

   A 60° B 90° C 30° D 120°

3. An incident ray approaches from the top-left, touches a plane mirror and is reflected to the top-right. The angle between the incident and the reflected rays is 100°. The mirror is rotated clockwise through an angle of 20°. If the incident ray is kept constant, then the angle between the two rays is

   A 120° B 60° C 80° D 140°

4. Light from the sun reaches the surface of the earth as

   A a parallel beam. B a convergent beam. C a divergent beam. D a spectrum.

5. Light rays from the moon reach human eyes on earth as a

   A convergent beam. B divergent beam. C parallel beam. D spectrum.

6. A woman who is 152 *cm* tall stands in front of a plane mirror attached to a vertical wall. What is the minimum length that the mirror must have for the woman to see her entire height?

   A 76 *cm* B 100 *cm* C 38 *cm* D 152 *cm*

7. A student stands 10 *m* in front of a large plane mirror. How far must he walk before he is 5 *m* from his image?

   A 5 *m* towards the mirror
   B 7.5 *m* towards the mirror
   C 5 *m* away from the mirror
   D 7.5 *m* away from the mirror

8. If an incident ray makes an angle of 40° with a mirror's surface, its angle of reflection is

   A 40° B 50° C 90° D 130°

9. If a reflected ray makes an angle of 17° with the normal line, its incident ray has an angle of incidence of

   A 107° B 90° C 73° D 17°

10. The angle between a normal line and its mirror's surface is

    A 0° B 30° C 90° D 180°

GCE Paper 1 Solutions

1. A 2. C 3. D 4. A 5. C 6. D 7. B 8. B 9. D 10. C

GCE Paper 2 Questions

1. A ray of light strikes a mirror at 60° to the mirror surface.

   (a) State two laws of reflection of light. **(2 mks)**
   (b) Draw a ray diagram showing the normal lines as well as the reflected ray. **(2 mks)**
   (c) What would be the angle of reflection if the light were incident normally (perpendicularly) to the mirror surface? **(1 mk)**

. . . .

Solution

(a) – The angle of incidence of a reflected ray is equal to its angle of reflection.
– The incident ray, normal line and reflected ray are all on the same plane.

(b) *See figure 3.6.2.7. The angle between the incident ray and the mirror's surface is given, not the angle of incidence. Therefore, the angle of incidence must be calculated.*

given equation for reflected angles: $i' = i$

considering angle of incidence as complimentary with given angle: $i + 60^\circ = 90^\circ$

turning angle of incidence into subject: $i = 90^\circ - 60^\circ$

simplifying: $i = 30^\circ$

substituting into equation for reflected angles: $i' = i = 30^\circ$

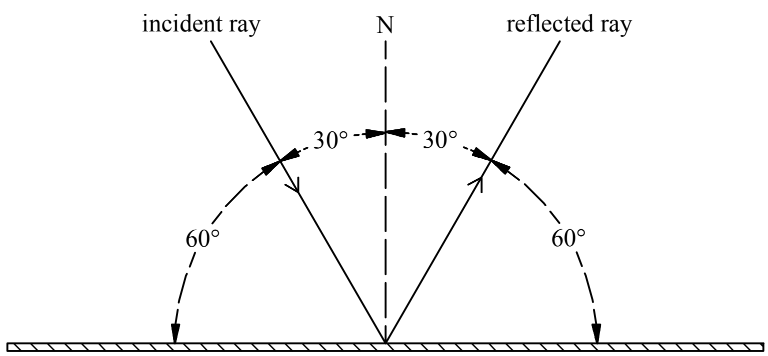

Figure 3.6.2.7

(c) – If the incident ray is perpendicular to the mirror, the angle of incidence is 90°.
– Therefore, the given the laws of reflection, the angle of reflection is also 90°.
– The reflected ray returns along the same direction as the incident ray.

2. While experimenting with a plane mirror, Hassana draws the following figure to represent her work.

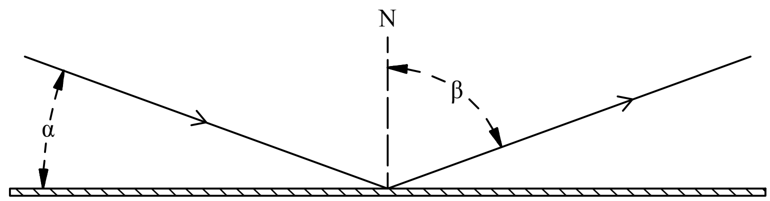

Figure 3.6.2.8

While varying the incident ray, she records the following data for the angles α and β shown in the figure.

α /°	10	15	45	60	70	73
β /°	80	75	45	30	20	17

(a) Plot a graph with the values of β along the y-axis and α along the x-axis. **(3 mks)**
(b) Determine the gradient of this graph. **(2 mks)**
(c) Explain the gradient's physical significance and account for its negative value. **(2 mks)**

. . . .

Solution

(a) *See figure 3.6.2.9*

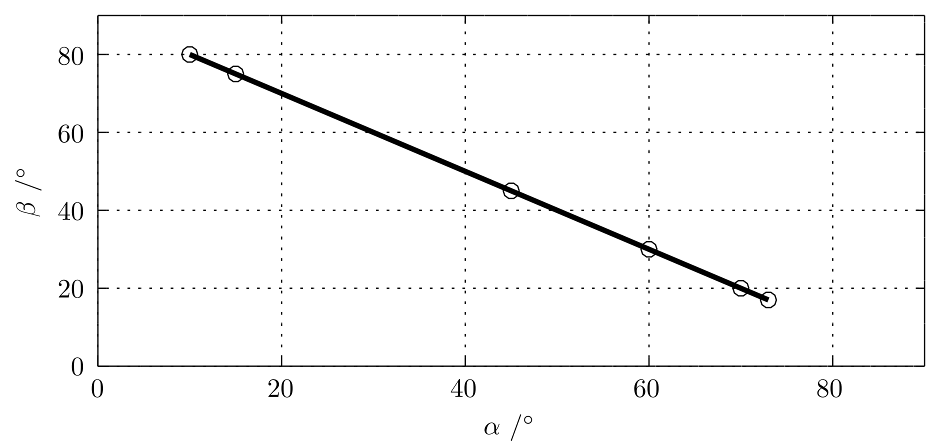

Figure 3.6.2.9

(b) *The gradient, slope, or proportionality is most easily calculated as the difference in the first and last values of β per the first and last values of α.*

$$\text{calculating gradient:} \quad k = \frac{\Delta\beta}{\Delta\alpha}$$

$$\text{substituting first and last data points:} \quad k = \frac{17^\circ - 80^\circ}{73^\circ - 10^\circ}$$

$$\text{final answer:} \quad \boxed{\text{gradient} = k = -1}$$

(c) The slope is verification of the law of reflection. This is because β is the angle of reflection and α is complimentary with the angle of incidence. That is, $i = 90^\circ - \alpha$ and $i' = \beta$. Therefore, an increase in α causes the same magnitude decrease in the angle of incidence as well as the angle of reflection.

3.6.3 Refraction

Objectives

By the end of the lesson, students should be able to

1. define refraction.
2. define and object's absolute refractive index.
3. state the laws of refraction.
4. draw ray diagrams illustrating refraction.
5. describe several everyday examples of light refraction.
6. understand how an object's apparent depth in a medium relates to the mediums absolute refractive index.

Introduction to Refraction

- **Refraction** is the change in direction of a wave as is it crosses the boundary between two media of propagation.
- **Light refraction** is the change a light wave's direction as it leaves one transparent medium and enters another.
- Like reflection, the incoming ray is called the incident ray.
- However, instead of returning back, the ray passes through the boundary and emerges as a refracted ray.

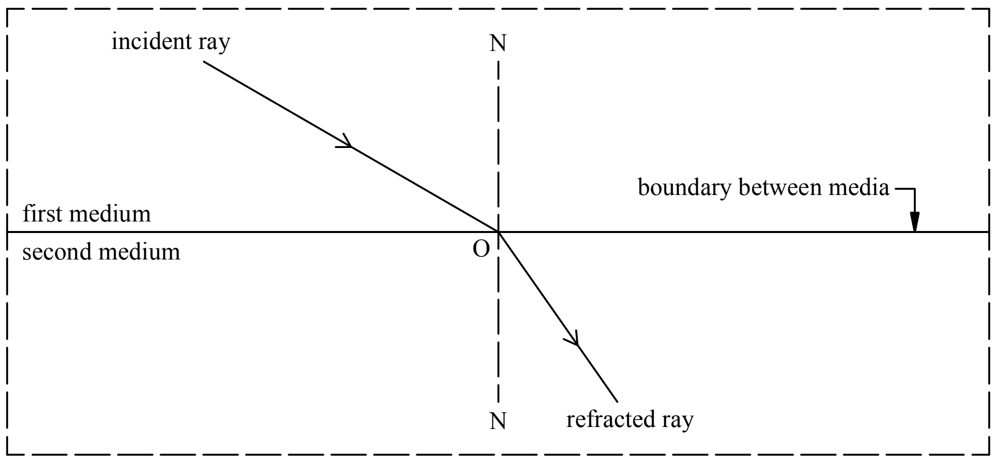

Figure 3.6.3.1

- A light wave is refracted when its speed increases or decreases.
- The degree to which a material refracts light by changing its speed is referred to as its "refractive index".
- The **absolute refractive index**, or **n**, of a material is the ratio of the speed of light through the material per the speed of light in an absolute vacuum.
- It is a scalar.
- It has no units given that it is the value of one speed divided by another.

$$n = \frac{c}{v} \tag{3.6.3.1}$$

$$\text{considering speed of light in a vacuum: } n = \frac{3 \times 10^8 \ m \ s^{-1}}{v} \tag{3.6.3.2}$$

Where
- v is the speed of light through the material, in $m \ s^{-1}$;
- n is the material's absolute refractive index (unit-less);
- c is the speed of light in an absolute vacuum, in $m \ s^{-1}$.

Laws of Refraction

- Like reflection, the angle between the incident ray and the normal line is called the angle of incidence (i).
- The **angle of refraction**, or **r**, is the angle the refracted ray makes against the normal line.

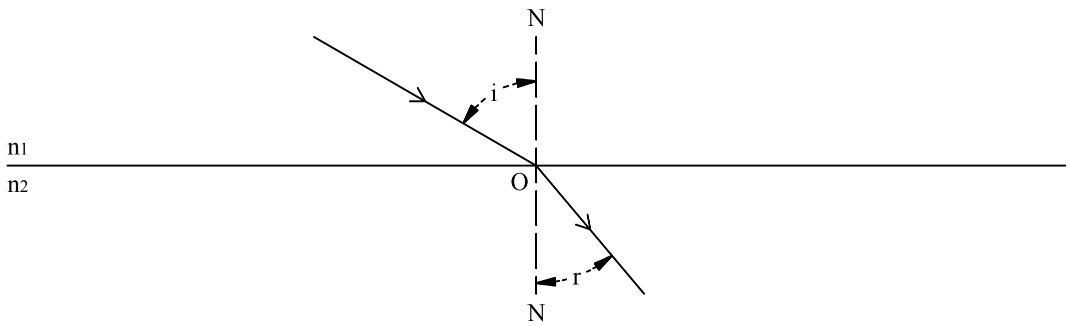

Figure 3.6.3.2

- The **laws of reflection** concern the relationship between these two rays and the normal line.
  1. The incident and refracted rays are coplanar with the normal line and on opposite sides of the boundary.
  2. The quotient of the sine of the incident angle and that of the refracted angle is constant between mediums.
  - NB: This second rule of refraction is often known as **Snell's Law**.

$$\frac{\sin(i)}{\sin(r)} = k$$

- The angle of refraction can be calculated using the ratio of the first and second material's refractive indices.

$$\text{considering proportionality of indices:} \quad \frac{\sin(i)}{\sin(r)} = \frac{n_2}{n_1}$$

$$\text{separating properties of each material:} \quad n_1 \sin(i) = n_2 \sin(r) \tag{3.6.3.3}$$

  Where
  - n_1 is the first material's absolute refractive index (unit-less);
  - n_2 is the second material's absolute refractive index (unit-less);
  - i is the angle of incidence, in degrees;
  - r is the angle of refraction, in degrees.

- If the ray crosses into a higher refractive index, ($n_2 > n_1$), the refracted ray approaches the normal line ($r > i$).
- If the ray crosses into a lower refractive index, ($n_2 < n_1$), the refracted ray departs from the normal line ($r < i$).

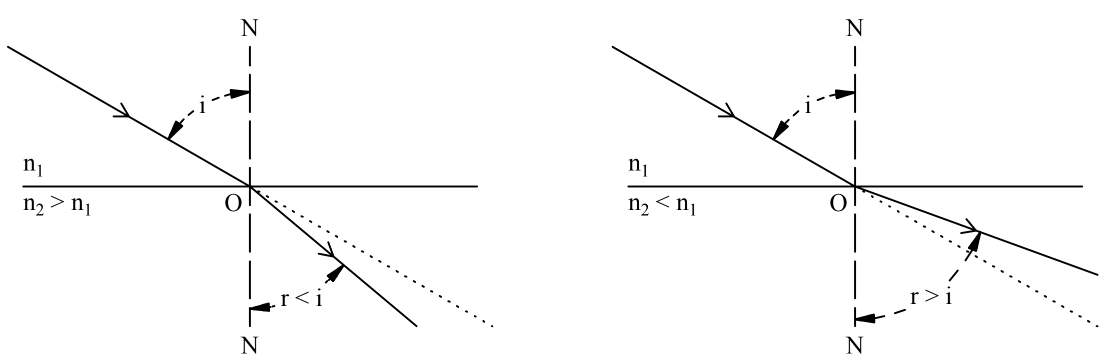

Figure 3.6.3.3

Examples of Light Refraction

- The bottom of a clear pond appears more shallow than it really is.
- A straight stick or spoon appears bent when its partially submerged in water.
- Objects viewed through a glass prism appear closer to the glass' surface than they really are.
- Air above hot surfaces like a tarred road on a hot day appear distorted and blurry.
  - The refractive index of air varies slightly with temperature.
  - This causes light to refract slightly when moving between regions of different-temperature air.

Refraction Incident from Air or Vacuum

- Typical air has optical properties very similar to that of an absolute vacuum.
- That is, for incident rays in air, the approximation $n_1 \approx 1$ is made.
- Therefore, if incident from air, a refracted ray's angle against the normal line can be calculated from the second material's absolute refractive index.

$$\text{given equation for refraction:} \quad \frac{\sin(i)}{\sin(r)} = \frac{n_2}{n_1}$$

$$\text{approximating value of 1 for first refractive index:} \quad \frac{\sin(i)}{\sin(r)} = \frac{n_2}{1}$$

$$\text{turning second material's refractive index into subject:} \quad n_2 = \frac{\sin(i)}{\sin(r)}$$

  Where
  - n_2 is the absolute refractive index of the second material while that of the first is assumed 1 (unit-less);
  - i is the angle of incidence, assumed to be in air, in degrees;
  - r is the angle of refraction in the second material, in degrees.

- This allows the approximate refractive indices of different materials to be measured by measuring the angle of refraction they produce from a known angle of incidence in air.

	material	**r with i = 45°**	**n (decimal)**	**n (fractional)**
liquids	water	32.1°	1.33	$\frac{4}{3}$
	ethyl alcohol	31.3°	1.36	-
	paraffin oil	29.4°	1.44	-
solids	perspex	28.3°	1.49	-
	crown glass	28.1°	1.50	$\frac{3}{2}$
	diamond	17.0°	2.42	-

Table 3.6.3.1

- NB: The following approximations are used quite frequently.

$$n_{\text{air}} = 1 \qquad n_{\text{water}} = \frac{4}{3} \qquad n_{\text{glass}} = \frac{3}{2}$$

The Speed of Light in Transparent Media

- The speed at which light propagates through a particular transparent material can be calculated from its absolute refractive index.

$$\text{given equation for a material's absolute refractive index: } n = \frac{c}{v}$$

$$\text{turning speed in material into subject: } v = \frac{c}{n}$$

$$\text{considering speed of light in a vacuum: } v = \frac{3 \times 10^8 \ m \ s^{-1}}{n}$$

Where
  - v is the speed of light in the material, in $m \ s^{-1}$;
  - n is the material's absolute refractive index (unit-less);
  - c is the speed of light in a vacuum, in $m \ s^{-1}$.

- This allows the speed of light to be calculated as it propagates through different materials.

	material	**n**	$\mathbf{v_{light}}$ **in medium** $(\mathbf{10^7 \ m \ s^{-1}})$
liquids	water	1.33	2.26
	ethyl alcohol	1.36	2.21
	paraffin oil	1.44	2.08
solids	perspex	1.49	2.01
	crown glass	1.50	2.00
	diamond	2.42	1.24

Table 3.6.3.2

Real and Apparent Depth

- When an object surrounded in a refracting medium is viewed at a slight angle from the outside air, its apparent depth can be calculated from its actual depth as well as the medium's absolute refractive index.

$$d' = \frac{d}{n} \tag{3.6.3.4}$$

Where
  - d' is the object's virtual or apparent depth viewed from air, in m;
  - d is the object's actual depth, in m;
  - n is the absolute refractive index of the medium surrounding the object (unit-less).

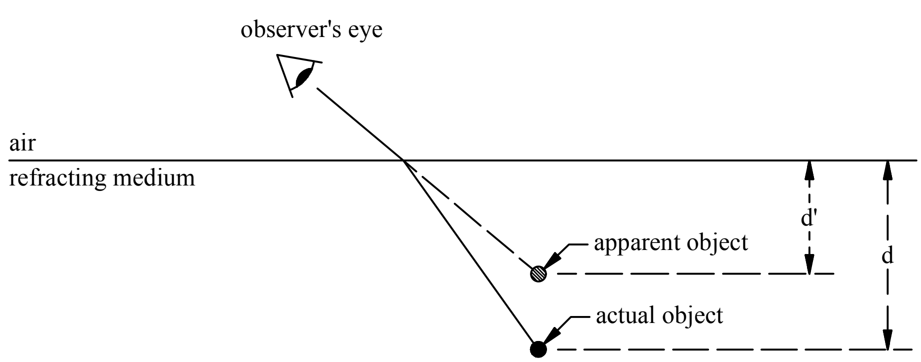

Figure 3.6.3.4

- When viewed from directly above, there is no refraction and the object appears at its actual depth.

GCE Paper 1 Questions

1. A pond of water having a refractive index of $\frac{4}{3}$ is only 4 m deep. A man at the edge of the pond looks directly vertically downward from above. A fish at the bottom of the pond appears to be

   A 4 m deep.
   B 3 m above the bottom.
   C 1 m above the bottom.
   D invisible.

2. The bending of light as it leaves one medium and enters another is known as

   A subtraction B refraction C reflection D transmission

3. If water has a refractive index of $\frac{4}{3}$, what is the speed of light in water if the speed of light in air is 300000 $km\ s^{-1}$?

   A 100000 $km\ s^{-1}$ B 225000 $km\ s^{-1}$ C 300000 $km\ s^{-1}$ D 400000 $km\ s^{-1}$

4. If a light ray travels from air into glass, the refracted ray most likely bends

   A towards the normal line.
   B away from the normal line.
   C along the normal line.
   D perpendicular to the normal line.

5. If a light ray travels from glass into diamond, the refracted ray most likely bends

   A towards the normal line.
   B away from the normal line.
   C along the normal line.
   D perpendicular to the normal line.

6. If a light ray travels from water into air, the refracted ray most likely bends

   A towards the normal line.
   B away from the normal line.
   C along the normal line.
   D perpendicular to the normal line.

7. When viewed from the outside air at the edge of a lake, a fish in the water appears

   A at its actual depth.
   B deeper than its actual depth.
   C more shallow than its actual depth.
   D a different color.

8. If the absolute refractive index of some transparent material is 3, light travels through it as a speed of

   A $3 \times 10^8\ m\ s^{-1}$ B $9 \times 10^8\ m\ s^{-1}$ C $6 \times 10^8\ m\ s^{-1}$ D $1 \times 10^8\ m\ s^{-1}$

9. Light travels fastest through which of the following media?

   A diamond B water C glass D air

10. A light ray in air strikes a block of glass of $n = 1.33$ at an angle of 70° to its surface. The ray emerging into the block has an angle of refraction of

    A 44.9° B 48.8° C 14.9° D 75.1°

GCE Paper 1 Solutions

1. A 2. B 3. B 4. A 5. A 6. B 7. C 8. D 9. D 10. C

GCE Paper 2 Questions

1. Figure 3.6.3.5 shows a ray of light travelling from air into a transparent medium.

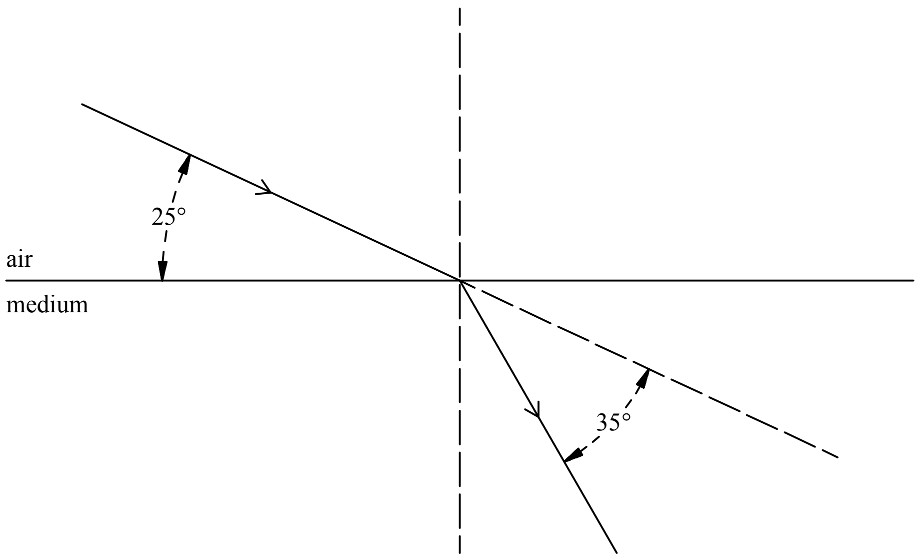

Figure 3.6.3.5

(a) Calculate the angle of incidence. **(2 mks)**
(b) Calculate the angle of refraction. **(3 mks)**
(c) Hence calculate the refractive index of the medium. **(3 mks)**

. . . .

Solution

(a) *The top given angle is complimentary with the angle of incidence.*

$$\text{assuming given angle in air is complimentary with angle of incidence: } 25^\circ + i = 90^\circ$$

$$\text{turning angle of incidence into subject: } i = 90^\circ - 25^\circ$$

$$\text{final answer: } \boxed{i = 65^\circ}$$

(b) *The given top angle is opposite the angle that is complimentary with the given bottom angle and the angle of refraction.*

$$\text{summing bottom angles: } 25^\circ + 35^\circ + r = 90^\circ$$

$$\text{turning angle of refraction into subject: } r = 90^\circ - 25^\circ - 35^\circ$$

$$\text{final answer: } \boxed{r = 30^\circ}$$

(c) *The refractive index can be calculated from the angle of incidence and the angle of refraction.*

$$\text{given equation for refraction of light incident from air: } n_2 = \frac{\sin(i)}{\sin(r)}$$

$$\text{substituting known values: } n_2 = \frac{\sin(65^\circ)}{\sin(30^\circ)}$$

$$\text{final answer: } \boxed{n_2 \approx 1.81}$$

2. Figure 3.6.3.6 shows an object at O seen at the bottom of a 1 m deep pond of water. It is given that the refractive index of the water is 1.33.

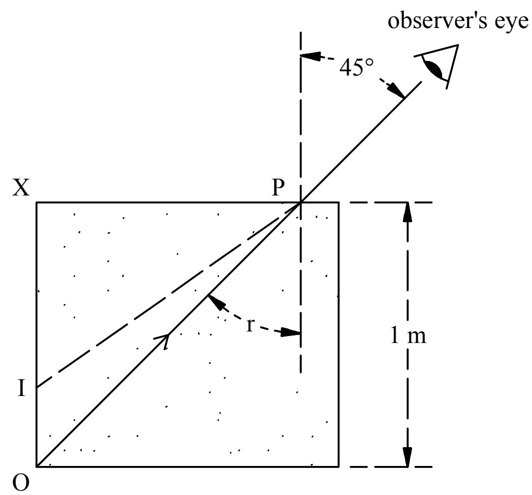

Figure 3.6.3.6

(a) Calculate the angle marked r. **(3 mks)**
(b) Calculate the distance marked IO. **(3 mks)**

....

Solution

(a) *The angle marked r is actually the angle of incidence with the light ray moving up and out of the pond. However, the angle convention of i above and r below can be maintained as long as the corresponding absolute refractive indices are applied properly. It is assumed that the object O is viewed from air.*

$$\text{given equation for refraction: } n_1 \sin(i) = n_2 \sin(r)$$

$$\text{assuming outside medium is air: } (1)\sin(i) = n_2 \sin(r)$$

$$\text{turning angle r into subject: } r = \sin^{-1}\left(\frac{\sin(i)}{n_2}\right)$$

$$\text{substituting known values: } r = \sin^{-1}\left(\frac{\sin(45°)}{1.33}\right)$$

$$\text{final answer: } \boxed{r \approx 32.12°}$$

(b) *The distance marker IO is the difference between the actual and apparent depths.*

$$\text{considering distance as difference is apparent and actual depths: } \overline{IO} = d - d'$$

$$\text{substituting equation for objects apparent depth: } \overline{IO} = d - \frac{d}{n}$$

$$\text{substituting known values: } \overline{IO} = 1.0\ m - \frac{1.0\ m}{1.33}$$

$$\text{final answer: } \boxed{\overline{IO} \approx 0.248\ m}$$

3. A boy standing at the edge of a swimming pool sees the bottom of the pool and concludes that it is exactly 3 m below the water surface.

   (a) Draw a labelled ray diagram showing that the pool is in fact deeper than 3 m. **(4 mks)**
   (b) Which wave behaviour accounts for the diagram produced for subsection 3 (a)? **(2 mks)**
   (c) Name another natural phenomenon that can be explained using the behaviour provided for the response to subsection 3 (b). **(2 mks)**
   (d) If the refractive index of the water in the pool is 1.33, calculate the pool's true depth. **(3 mks)**

. . . .

Solution

(a) *See figure 3.6.3.7. It is assumed that the boy views the pool's bottom from air.*

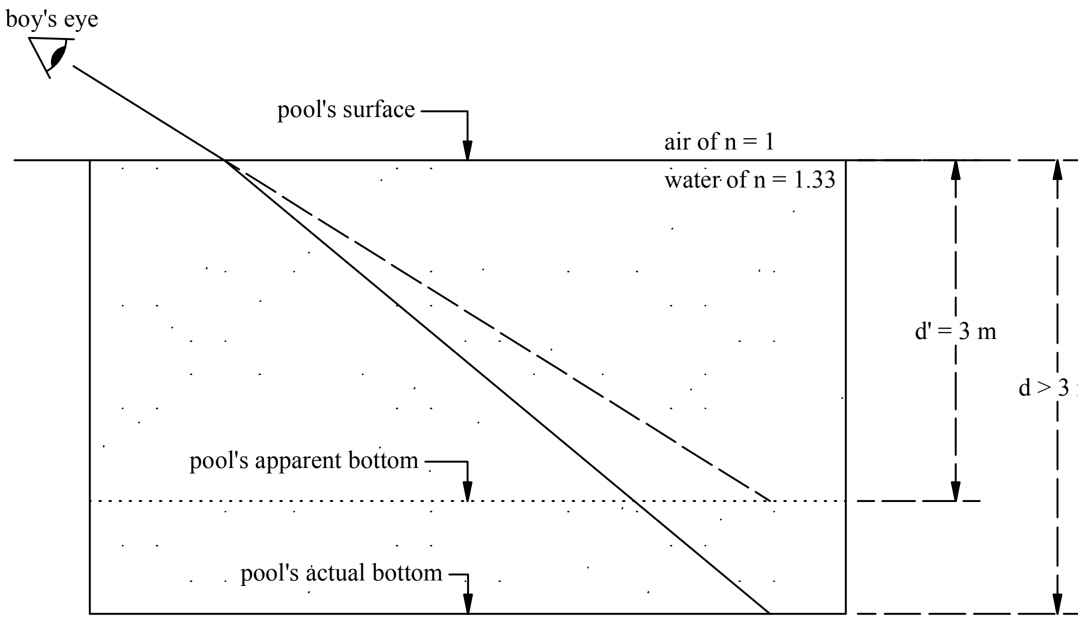

Figure 3.6.3.7

(b) The refraction of light accounts for this diagram.

(c) *The following is a non-exhaustive list of phenomena. Only one is required.*
   – A straight stick appears bent in water.
   – An object appears closer to a prism's surface when viewed from the far side.
   – Air above a hot surface looks blurry.

(d) *It is assumed that the boy is viewing the pool's bottom at an angle and not directly from above.*

$$\text{given equation for apparent depth of object: } d' = \frac{d}{n}$$

$$\text{turning actual depth into subject: } d = d'n$$

$$\text{substituting known values: } d = (3\ m)(1.33)$$

$$\text{final answer: } \boxed{d = 3.99\ m}$$

3.6.4 Refraction Experiments

Objectives

By the end of the lesson, students should be able to

1. describe the path of a light ray entering and exiting a glass block.
2. describe an experiment to determine the absolute refractive index of a glass block.
3. describe the path of a light ray entering and exiting a non-rectangular transparent medium.

Light Entering and Exiting a Rectangular Refracting Medium

- When a light ray in air enters and exits a refracting medium, it exits back into air.
- If the exiting surface is parallel to the entering surface, the exiting ray is parallel to the entering ray.
- This is because any refraction that occurs upon entering is undone by the inverse refraction upon exiting.
- The exiting ray leaves from E, the emergence point.
- This exiting point is offset from E', where the ray would exit with no refraction.

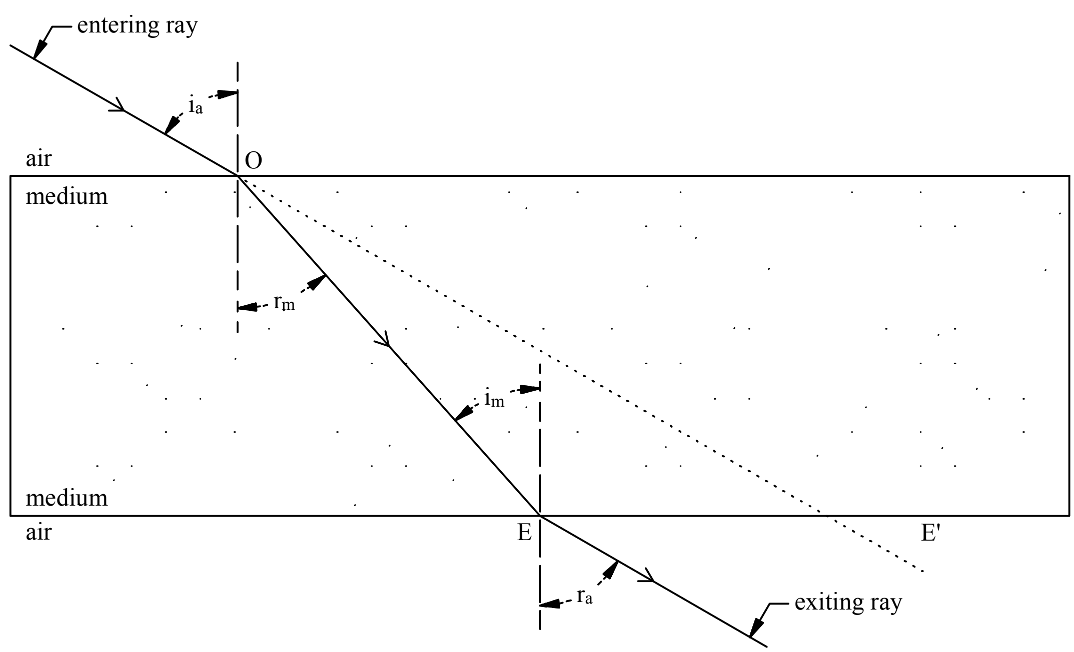

Figure 3.6.4.1

- The entering angle of incidence from the outside air is equal to the exiting angle of refraction into air.

$$i_a = r_a$$

Where
  - i_a is the ray's entering angle of incidence from air, in degrees;
  - r_a is the ray's exiting angle of refraction into air, in degrees.

- The angle of refraction after entering into the medium is equal to the angle of incidence just before exiting the medium.

$$r_m = i_m$$

Where
  - r_m is the angle of refraction inside the medium, in degrees;
  - i_m is the angle of incidence inside the medium as the ray is just about to exit, in degrees.

Experiment to Determine a Solid's Absolute Refractive Index Using Angle Comparison

I Procedure

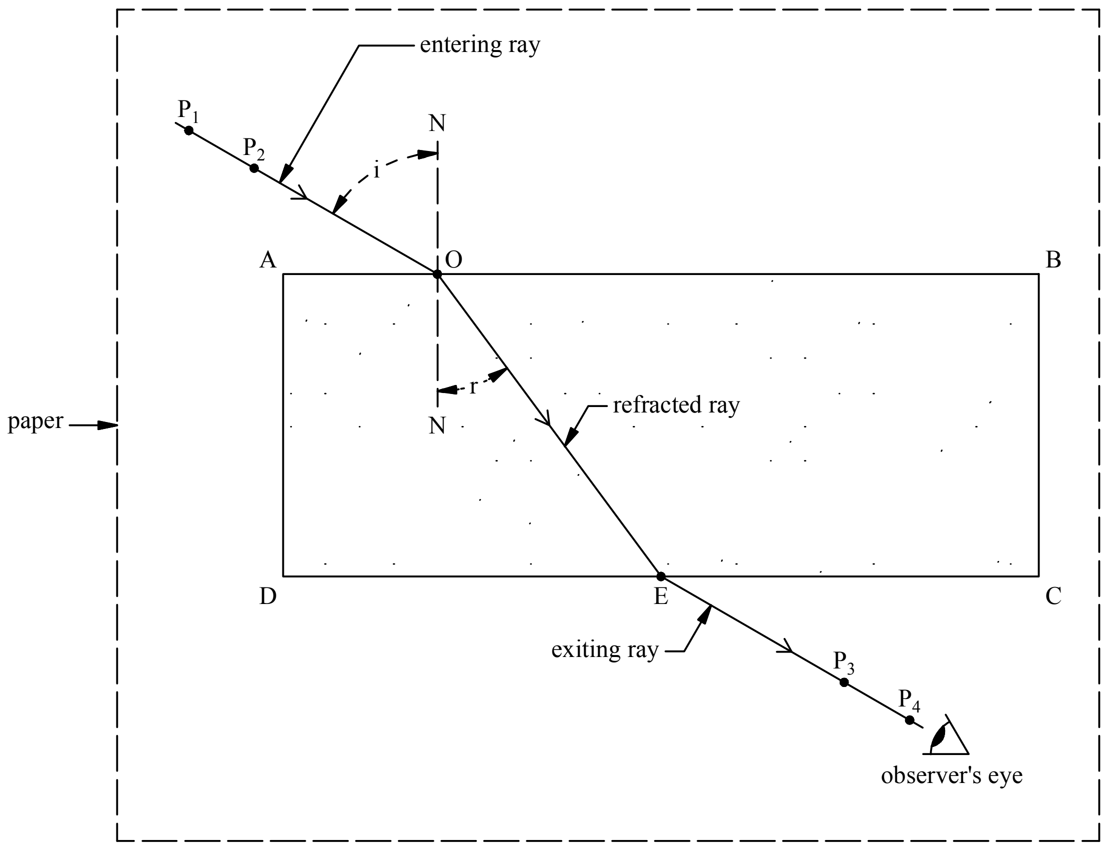

Figure 3.6.4.2

1 A piece of white paper is placed on a flat, horizontal surface like a desk or lab table.
2 A glass block is placed on the paper.
3 The block's outline is traced.
4 The outline's corners are labelled clockwise A, B, C and D.
5 Pins labelled P_1 and P_2 are placed on side AB.
6 The observer places their eye on the side CD so that P_2 appears to be directly in front of P_1.
7 Two more pins labelled P_3 and P_4 are placed on side DC blocking P_2 from the observer.
8 The block is lifted and the observer removes their eye.
9 An entering ray is traced that connects P_1 and P_2 and that terminates at the outline.
10 The point where this entering ray touches the outline is labelled O.
11 A ray emerging from the block is traced that begins at the outline and connects P_3 and P_4.
12 The point where this emerging ray departs the block's outline is labelled E.
13 A refracted ray is traced connecting O and E.
14 A normal line is drawn through O, perpendicular to the block's outline.
15 A protractor is used to measure i, the angle of incidence between the normal line and the entering ray.
16 A protractor is used to measure r, the angle of refraction between the refracted ray and the normal line.
17 Steps 5 through 16 are repeated with different P_1 and P_2 alignments to achieve varying values of i and r.
18 The values of i and r for each trial are recorded in a table.

II Calculations

- The sine of each angle of incidence is calculated.
- The sine of each angle of reflection is calculated.
- The data is then recorded in a tabular format.

trial	**i /°**	**r /°**	**sin(i)**	**sin(r)**
1	i_1	r_1	$\sin(i_1)$	$\sin(r_1)$
2	i_2	r_2	$\sin(i_2)$	$\sin(r_2)$
3	i_3	r_3	$\sin(i_3)$	$\sin(r_3)$
4	i_4	r_4	$\sin(i_4)$	$\sin(r_4)$

Table 3.6.4.1

- These values are used to create a graph where values of $\sin(i)$ are plotted along the y-axis and values of $\sin(r)$ are plotted along the x-axis.

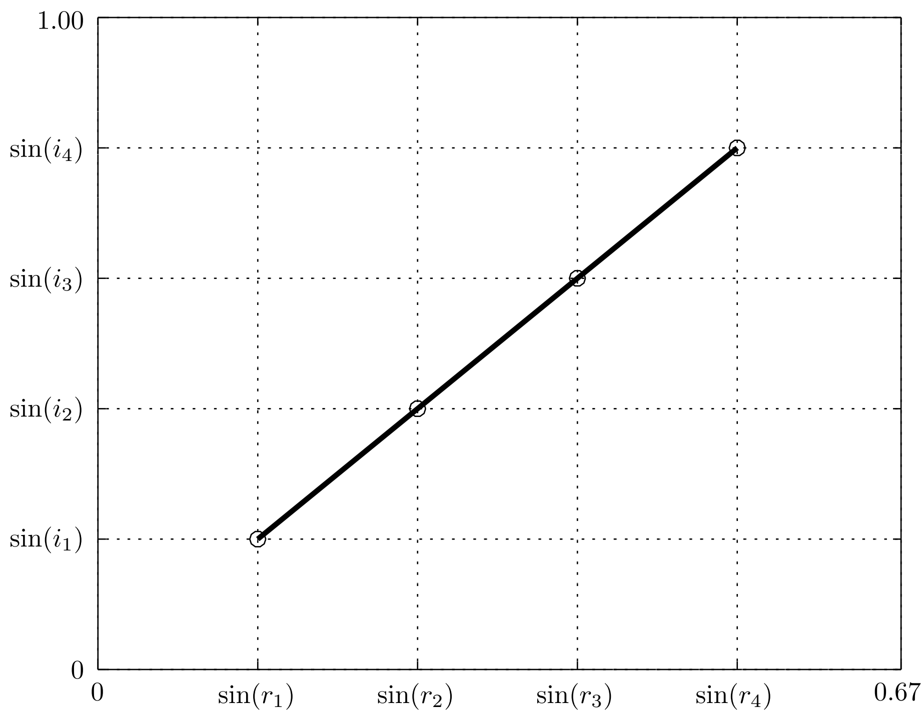

Figure 3.6.4.3

- Given the outside medium is air, the block's absolute refractive index is calculated as this plot's gradient.

$$n_{\text{glass block}} = \frac{\Delta \sin(i)}{\Delta \sin(r)}$$

III Precautions

- The experiment is repeated several times with each value of $n_{\text{glass block}}$ averaged to avoid random error.
- The pins are placed as vertically straight as possible to avoid lateral distortion.
- With each trial, the glass block is placed exactly in the same outline to avoid shifting values of i and r.
- With each trial, the observer removes their eye before tracing the exiting ray to avoid injury.
- Pins P_1 and P_2 are aligned differently amongst all trials to achieve a wide variety of i across 0° to 90°.

GCE Paper 1 Questions

1. Light entering glass from air is refracted at an angle of 24°. If the refractive index of glass is 1.5, then the incident angle is

   A 15.7° B 24° C 36° D 37.6°

2. Light is incident at an angle of 30° to a transparent object from air. If the resulting angle of refraction is 19°, the material's absolute refractive index is approximately

   A 1.22 B 1.54 C 1.33 D 1.80

3. A ray of light is leaving a material of refractive index 1.5 and strikes the interior surface at an incident angle of 30°. The angle of refraction in the outside air is approximately

   A 1.5° B 48.6° C 30° D 19.47°

Questions 4 through 6 refer to figure 3.6.4.4 which shows a beam of light that enters a prism.

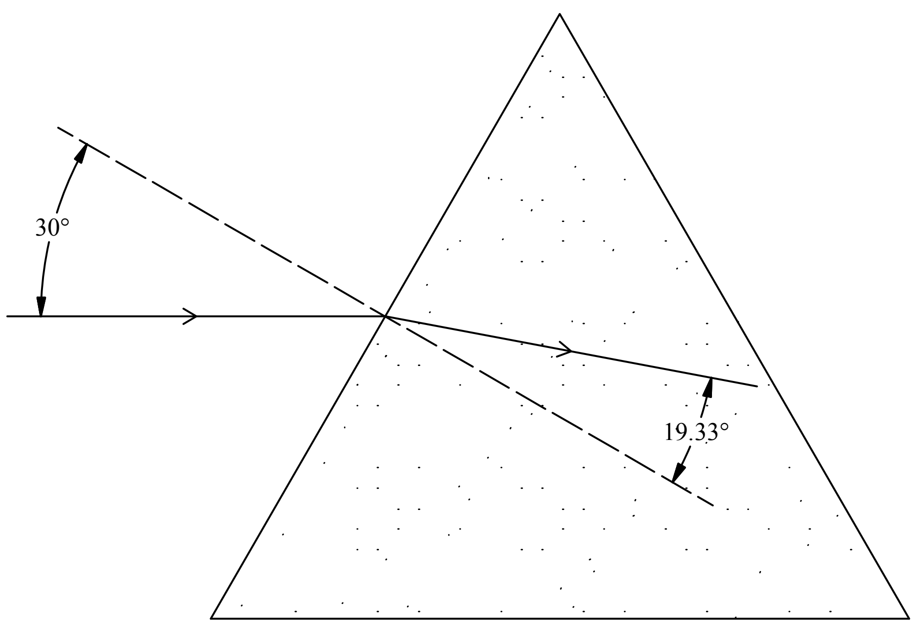

Figure 3.6.4.4

4. The phenomenon shown is

   A refraction.
   B density.
   C reflection.
   D total internal reflection.

5. The prism's absolute refractive index is

   A 0.61 B 1.49 C 1.51 D 1.33

6. Given the speed of light in air is $3 \times 10^8\ m\ s^{-1}$, the speed of light in the prism is

   A $1.98 \times 10^7\ m\ s^{-1}$ B $5.0 \times 10^{-9}\ m\ s^{-1}$ C $3.0 \times 10^6\ m\ s^{-1}$ D $1.98 \times 10^8\ m\ s^{-1}$

Questions 7 through 10 refer to figure 3.6.4.5 which shows a ray of light entering a refracting medium as well as four different possible paths for the exiting ray.

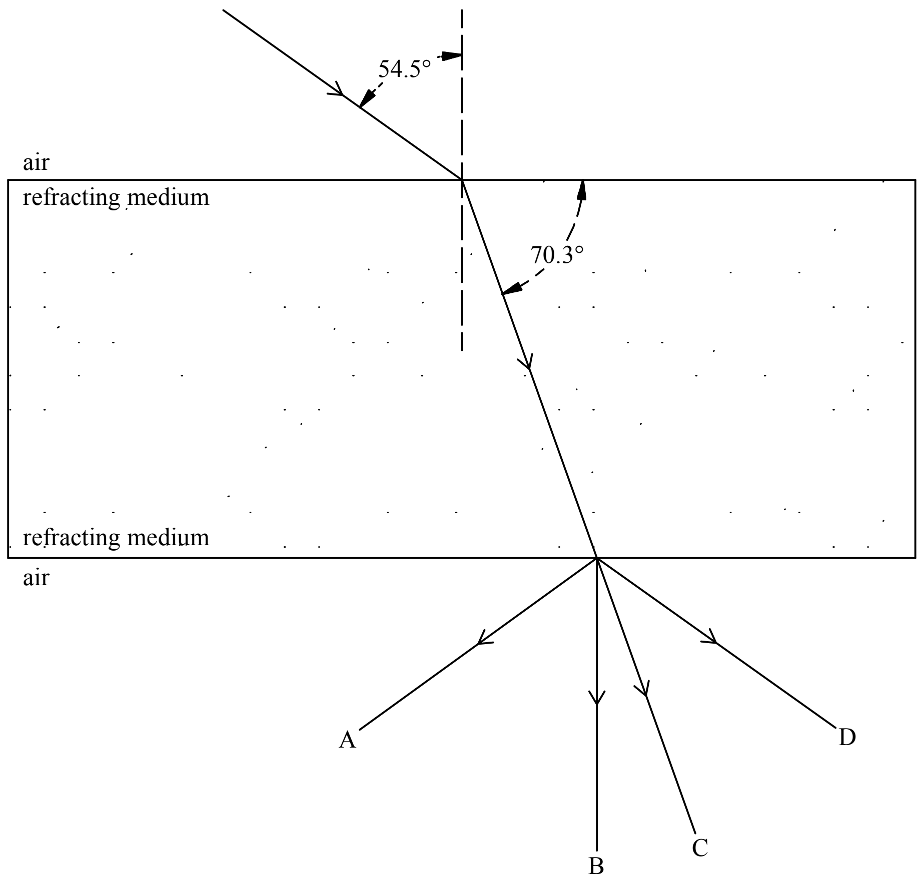

Figure 3.6.4.5

7. The ray emerging from the bottom of the refracting medium will follow which of the labelled paths?

8. If the incident ray is made to be parallel to the normal line, the ray emerging from the bottom of the refracting medium will follow which of the labelled paths?

9. The material's absolute refractive index is approximately

   A 2.42 B 1.50 C 1.33 D 0.86

10. The refracting medium is most likely

    A air B water C glass D diamond

GCE Paper 1 Solutions

1. D 2. B 3. B 4. A 5. C 6. D 7. D 8. B 9. A 10. D

GCE Paper 2 Questions

1. A light ray at a speed of $3 \times 10^8\ m\ s^{-1}$ in air enters a pool of paraffin oil with an angle of incidence of 25°. Its angle of refraction in the pool is measured to be 17.07° .

   (a) Calculate the liquid wax's absolute refractive index. **(3 mks)**
   (b) Calculate the speed of the light in the oil. **(2 mks)**
   (c) Draw a ray diagram illustrating this refraction scenario. **(2 mks)**

   ....

   Solution

   (a) *It is assumed that the absolute refractive index of the surrounding air is 1.*

   $$\text{given equation for refraction of light incident from air: } n_2 = \frac{\sin(i)}{\sin(r)}$$

   $$\text{subtituting known values: } n_2 = \frac{\sin(25°)}{\sin(17.07)}$$

   $$\text{final answer: } \boxed{n_2 \approx 1.44}$$

   (b) *The ray's speed is reduced when it enters the oil.*

   $$\text{given equation for light's speed when refracted: } n = \frac{c}{v}$$

   $$\text{turning light's refracted speed into subject: } v = \frac{c}{n}$$

   $$\text{substituting known values: } v = \frac{3 \times 10^8\ m\ s^{-1}}{1.44}$$

   $$\text{final answer: } \boxed{v \approx 2.08 \times 10^8\ m\ s^{-1}}$$

   (c) *See figure 3.6.4.6*

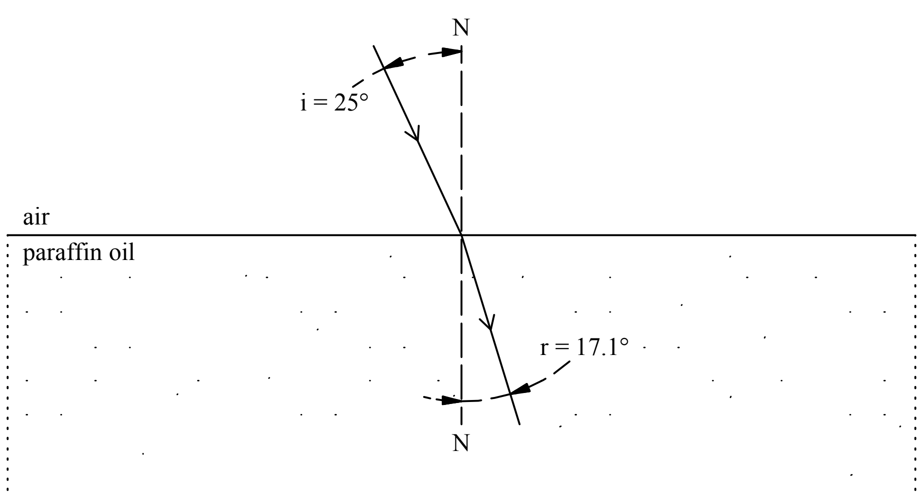

Figure 3.6.4.6

2. In an experiment to verify Snell's Law, the angles of incidence (i) and corresponding angles of refraction (r) were measured for light moving from air into a glass block and recorded in the table below.

i /°	20	30	40	50	60	80
r /°	13	19	25	30	35	41
$\sin(i)$	0.34			0.77		
$\sin(r)$	0.22			0.50		

(a) Copy and complete the table. **(3 mks)**
(b) Plot a graph of $\sin(i)$ along the y-axis against $\sin(r)$ along the x-axis. **(4 mks)**
(c) Determine the slope of the graph. **(2 mks)**
(d) State the significance of the slope. **(1 mk)**

. . . .

Solution

(a) *Solutions in bold*

i /°	20	30	40	50	60	80
r /°	13	19	25	30	35	41
$\sin(i)$	0.34	**0.50**	**0.64**	0.77	**0.87**	**0.98**
$\sin(r)$	0.22	**0.33**	**0.42**	0.50	**0.57**	**0.66**

(b) *See figure 3.6.4.7*

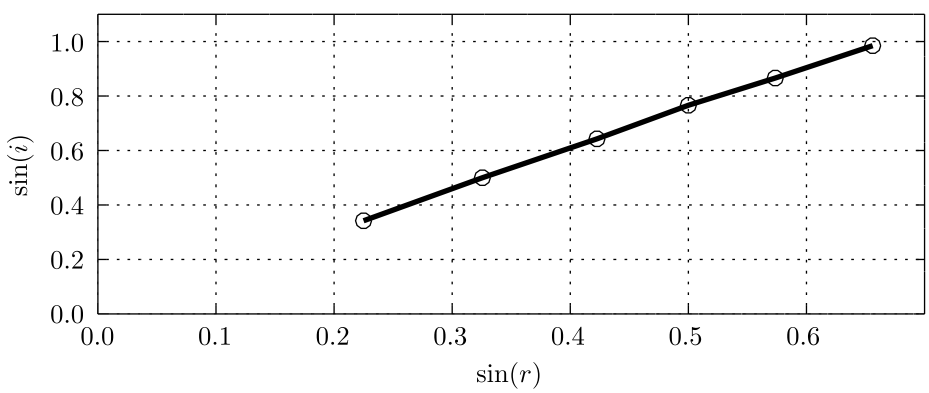

Figure 3.6.4.7

(c) *Given the approximately linear plot, the gradient, slope, or proportionality is most easily calculated as the difference in the first and last values of* $\sin(i)$ *per the difference in the first and last values of* $\sin(r)$.

$$\text{calculating gradient:} \quad k = \frac{\Delta \sin(i)}{\Delta \sin(r)}$$

$$\text{substituting first and last data points:} \quad k = \frac{0.98 - 0.34}{0.66 - 0.22}$$

$$\text{final answer:} \quad \boxed{\text{gradient} = k \approx 1.45}$$

3. Figure 3.6.4.8 shows an incident ray AO being refracted as it crosses the border between air and glass media. It emerges into the glass as the ray OB.

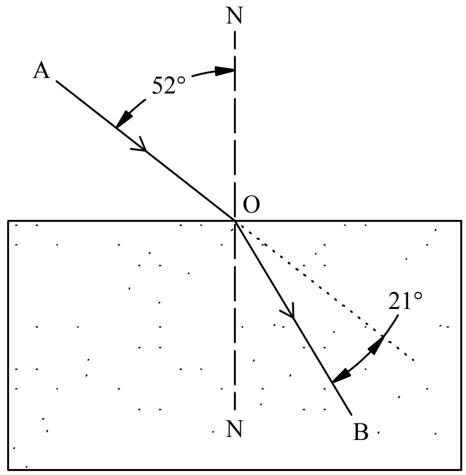

Figure 3.6.4.8

(a) Calculate the angle of refraction of the ray OB in the glass. **(3 mks)**
(b) Calculate glass' refractive index. **(3 mks)**
(c) The light's speed in air is $3 \times 10^8\ m\ s^{-1}$. Calculate its speed in the glass. **(2 mks)**

. . . .

Solution

(a) *The given angle of incidence is complimentary with the top angle that is opposite an angle on the bottom. This bottom angle is complimentary with the* 21° *angle as well as the angle of refraction.*

$$\text{summing bottom angles: } (90° - 52°) + 21° + r = 90°$$

$$\text{turning angle of refraction into subject: } r = 90° - (90° - 52°) - 21°$$

$$\text{final answer: } \boxed{r = 31°}$$

(b) *It is assumed that the absolute refractive index of the surrounding air is 1.*

$$\text{given equation for refraction of light incident from air: } n_2 = \frac{\sin(i)}{\sin(r)}$$

$$\text{subtituting known values: } n_2 = \frac{\sin(52°)}{\sin(31)}$$

$$\text{final answer: } \boxed{n_2 \approx 1.53}$$

(c) *The ray's speed is reduced when it enters the glass.*

$$\text{given equation for light's speed when refracted: } n = \frac{c}{v}$$

$$\text{turning light's refracted speed into subject: } v = \frac{c}{n}$$

$$\text{substituting known values: } v = \frac{3 \times 10^8\ m\ s^{-1}}{1.53}$$

$$\text{final answer: } \boxed{v \approx 1.96 \times 10^8\ m\ s^{-1}}$$

3.6.5 Total Internal Reflection

Objectives

By the end of the lesson, students should be able to

1. define total internal reflection.
2. explain, with the aid of a diagram, what is meant by total internal reflection.
3. state the conditions necessary for total internal reflection.
4. define a medium's critical angle.
5. explain the importance of total internal reflection in aquatic life, periscopes, binoculars and fibre optics.
6. discuss the advantages and disadvantages of fibre optics in telecommunication.

Total Internal Reflection

- Figure 3.6.5.1 shows three parallel rays, A, B and C inside an irregular glass prism of $n = 1.5$.

- Each strikes the prisms edge from the inside at a different angle of incidence.

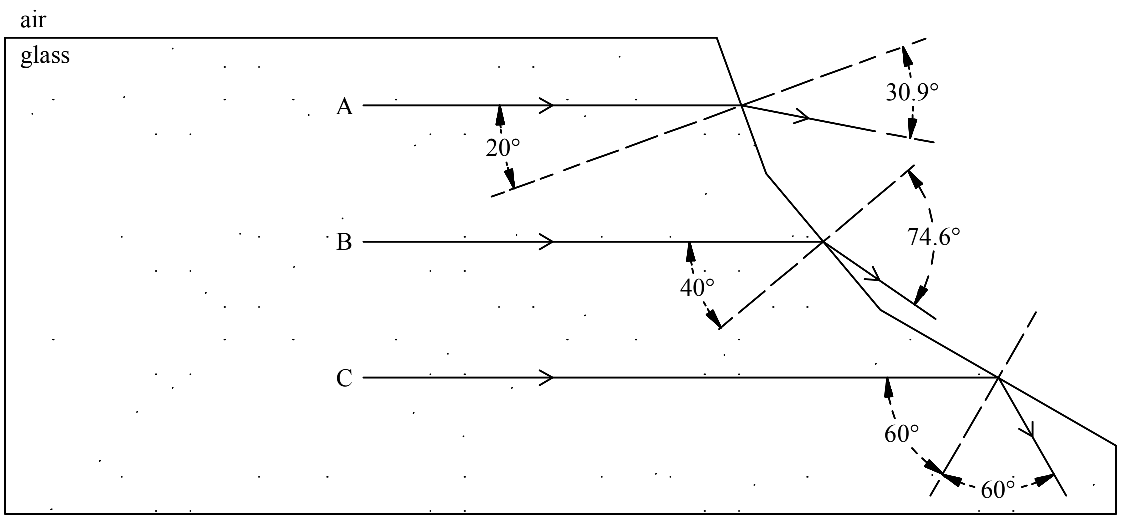

Figure 3.6.5.1

- Ray A strikes at an angle of incidence of $i = 20°$ causing an angle of refraction of $r = 30.9°$.
  - This is because the outside air has a lower n than the glass.
  - Therefore, the refracted ray bends away from the normal line.

- Ray B strikes at an angle of incidence of $i = 40°$, causing an angle of refraction of $r = 74.6°$.
  - This is because the incidence angle as increased from that of ray A.
  - Therefore, the refracted angle bends even further away from the normal line.

- Ray C strikes at an angle of incidence so large that the refracted ray doesnt exist.
  - This is because the refracted ray ends up having a value greater than 90°, which is physically impossible.
  - Therefore, the ray reflects back within the medium.
  - This is referred to as total internal reflection.

- **Total internal reflection** is the return of a light ray as it strikes the boundary between transparent media.

- The **critical angle**, or **c**, is the angle of incidence in a medium that causes the refracted ray in an adjacent, less dense medium to be 90°

- The conditions of total internal refection of are
  - light strikes a boundary between transparent media of differing absolute refractive indices ($n_1 \neq n_2$);
  - the absolute refractive index of the first media is greater than that of the second ($n_1 > n_2$).
  - the light has an angle of incidence at the boundary above the critical angle ($i > c$).

- As shown in figure 3.6.5.2, as a ray emerges from a curved glass prism into air,
  - rays emerge refracted with $r < 90°$ where $i < c$;
  - rays emerge refracted with r exactly equal to 90° where $i = c$;
  - rays are totally internally reflected where $i > c$;

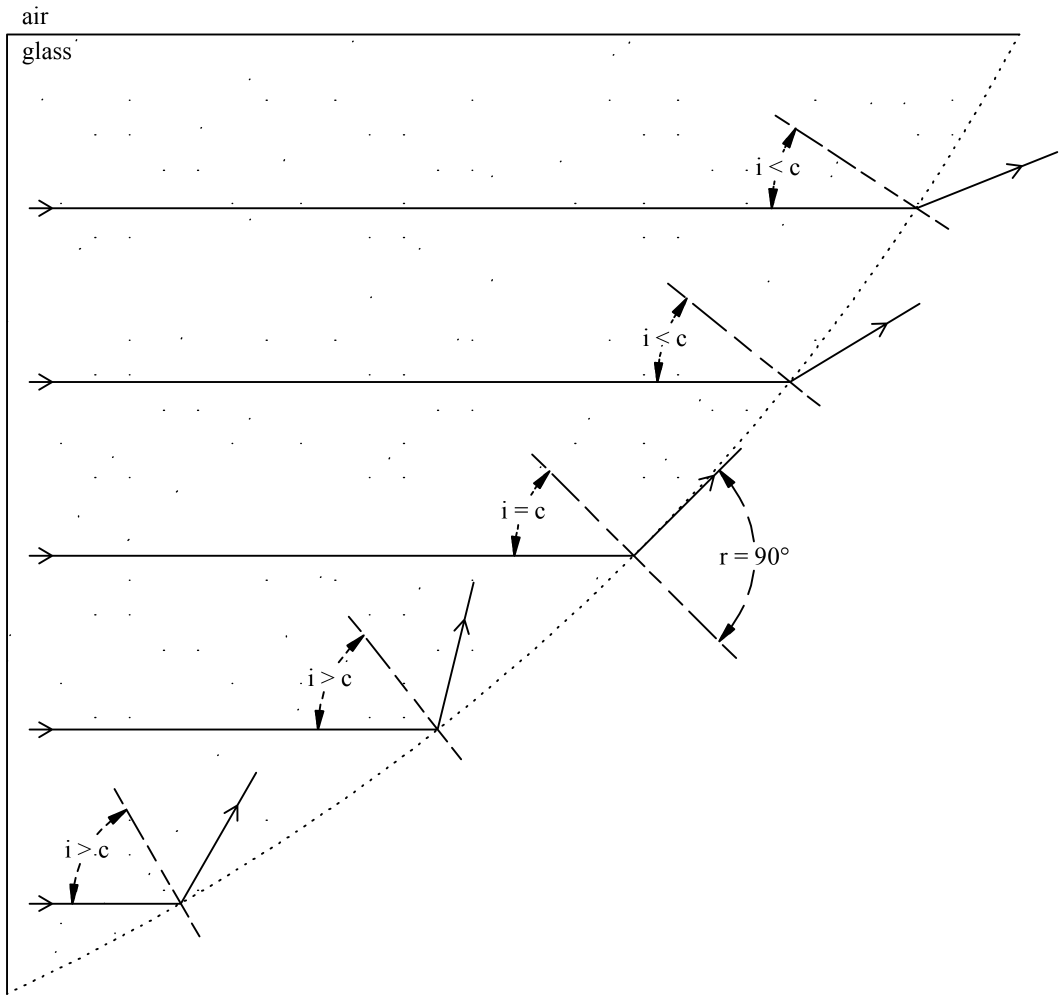

Figure 3.6.5.2

- The critical angle in any given scenario depends on the absolute refractive indices of both media involved.

given equation for refraction: $n_1 \sin(i) = n_2 \sin(r)$

substituting critical angle and corresponding angle of refraction: $n_1 \sin(c) = n_2 \sin(90°)$

simplifying: $n_1 \sin(c) = n_2(1)$

$$\text{turning critical angle into subject: } c = \sin^{-1}\left(\frac{n_2}{n_1}\right) \quad (3.6.5.1)$$

$$\text{considering second medium to be air: } c = \sin^{-1}\left(\frac{1}{n_1}\right) \quad (3.6.5.2)$$

Where
- c is the critical angle, in degrees;
- n_1 is the refractive index of the first medium (unit-less);
- n_2 is the refractive index of the second medium, equal to 1 if air (unit-less).

Critical Angles of Different Transparent Media

- The critical angle of any reflection/refraction scenario depends on both media involved.

$$\text{glass into air: } c = \sin^{-1}\left(\frac{1}{1.50}\right) \approx 41.81^\circ$$

$$\text{water into air: } c = \sin^{-1}\left(\frac{1}{1.33}\right) \approx 48.75^\circ$$

$$\text{diamond into air: } c = \sin^{-1}\left(\frac{1}{2.42}\right) \approx 24.41^\circ$$

$$\text{glass into water: } c = \sin^{-1}\left(\frac{1.33}{1.50}\right) \approx 62.46^\circ$$

$$\text{diamond into water: } c = \sin^{-1}\left(\frac{1.33}{2.42}\right) \approx 33.34^\circ$$

Total Internal Reflection and Aquatic Life

- The critical angle of light moving from water into air is about 48°.
- Therefore, a fish in water can see different parts of its environment depending on the viewing angle.
  - It can see the region above the surface across a range of $2c$, centred vertically above.
  - For viewing angles beyond that, it can only see objects below the surface that reflected back into its eye.
  - Certain aquatic life like cover pot fish have eyes facing directly upwards to exploit total internal reflection.

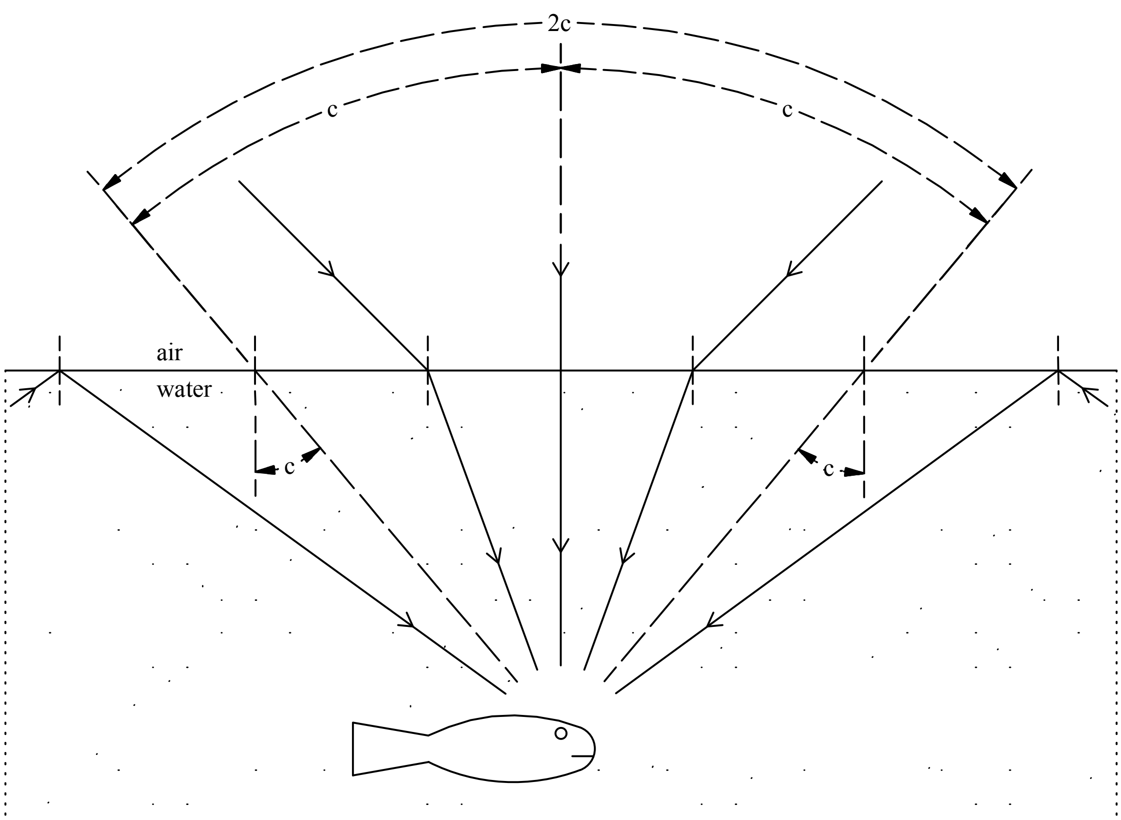

Figure 3.6.5.3

Use of Total Internally Reflecting Media as Mirrors

- A transparent prism that lets light in perpendicularly from one side can serve to reorient the image of an object.

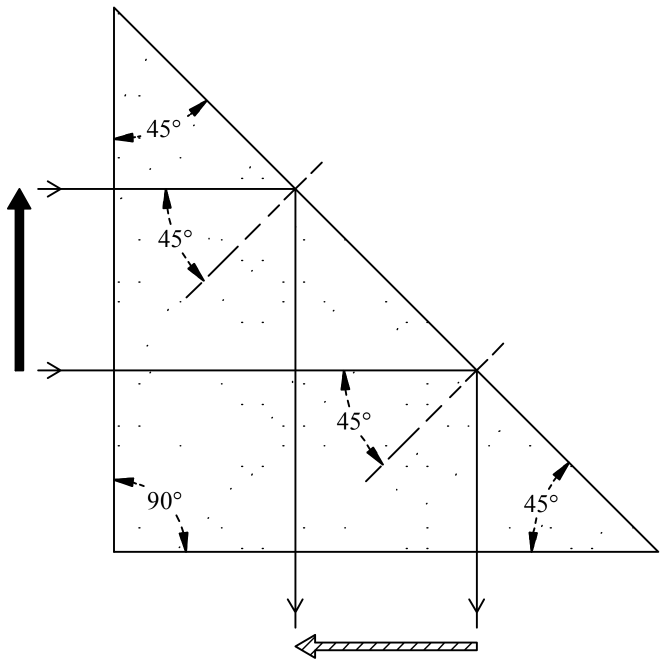

Figure 3.6.5.4

- Such a system works best when the object is held parallel to the either of the equal-sided legs of a right isosceles (90°, 45°, 45°) triangular-based prism.
- Any other orientation or prism shape could lead to image distortion or inversion (being upside-down).

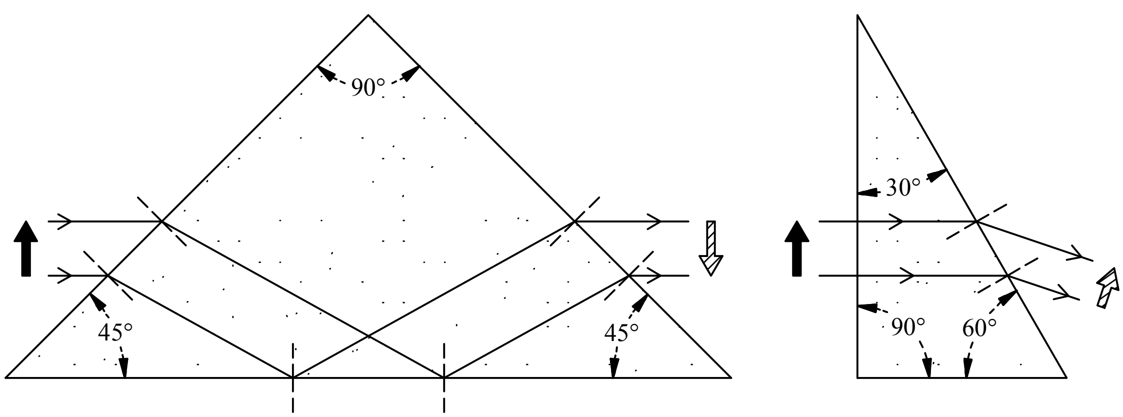

Figure 3.6.5.5

- Glass prisms are preferable to plane mirrors for reflection because
  - they reflect all light while mirrors only reflect most.;
  - their images are less blurry than mirrors because there is only one true reflecting surface;
  - they are stronger and more durable than thin mirrors;
  - they are not as easily scratched and damaged as chemically-treated plane mirrors;
  - they do not rust over time like plane metallic mirrors.
- However, the largest benefit of plane mirrors over glass prisms is a lower volume per area reflected.
- This reduces the cost of material as well as the cost of shipping plane mirrors.

Total Internal Reflection in Periscopes

- Two right-isosceles triangular-based prisms can be used in place of plane mirrors inside a periscope.

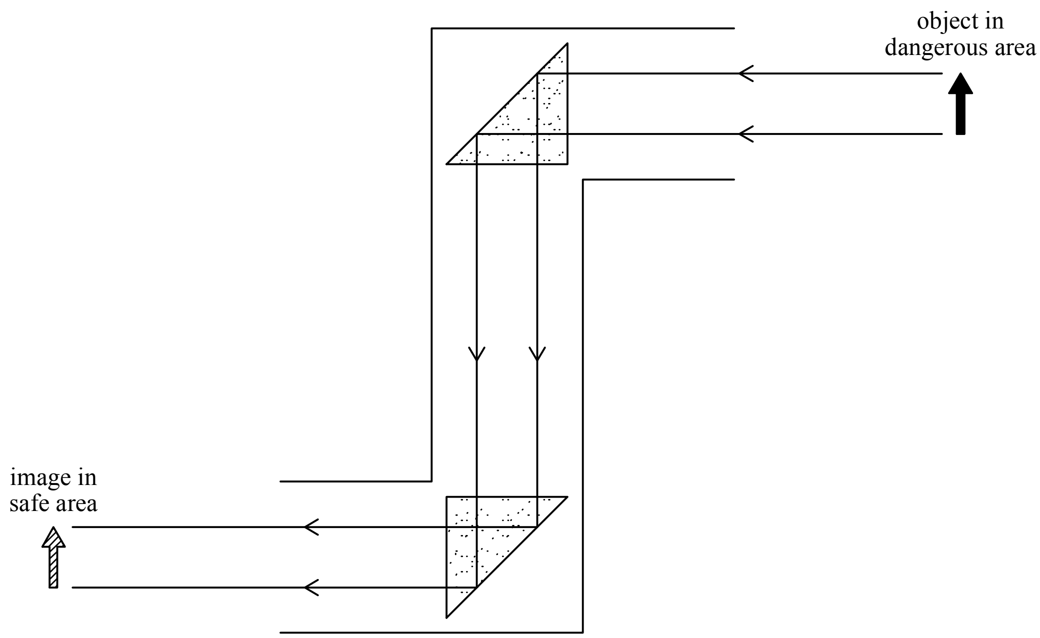

Figure 3.6.5.6

Total Internal Reflection in Binoculars

- After magnifying an image, binoculars produce an inverted image.
- A single, right-isosceles triangular-based prism can be used to render the image upright.

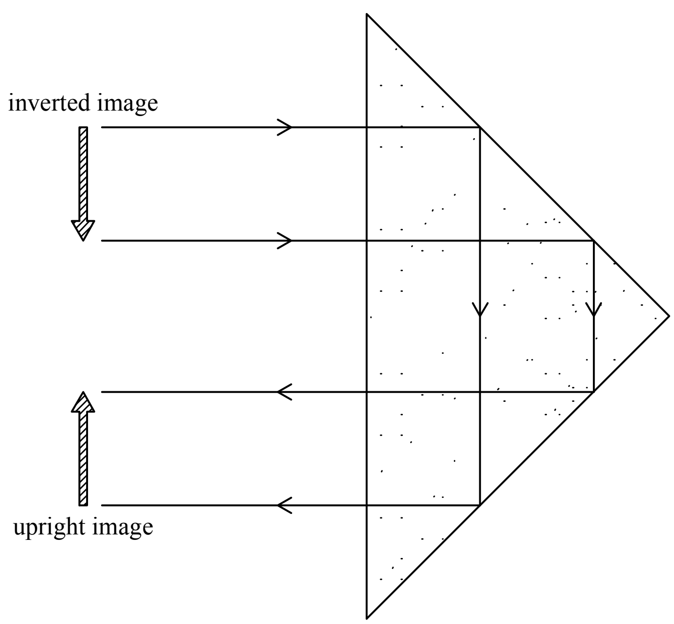

Figure 3.6.5.7

Total Internal Reflection in Fibre Optic Cables

- Fibre optics cables are composed of long thin fibres of glass used in telecommunication.
- Light signals are sent quickly and undiminished from one side to the other using total internal reflection.

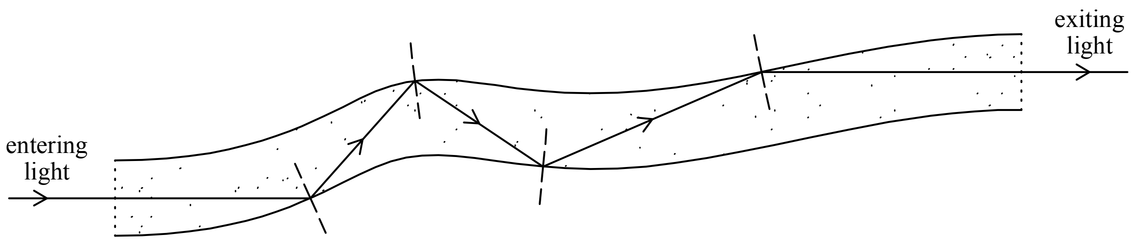

Figure 3.6.5.8

- The three main methods of telecommunication are
  1. light signals sent over fibre optic cables;
  2. airborne signals from satellites, radio and cellphone towers;
  3. electric signals sent over electric conductors like copper wires.
- Fibre optics are advantageous in telecommunications because their signals
  - travel much quicker than electric signals;
  - do not diminish over long distances like electric signals;
  - are not effected by weather like airborne signals;
  - require much less power to operate than electric signals.
- The largest disadvantages of fibre optic cables are
  - cost of installation;
  - speciality of equipment needed to decipher information sent as light pulses.
- Airborne signals from satellites, radio and cellphone towers are often preferred because their signals
  - require no wires or cables to operate;
  - can be sent and received anywhere in the world;
- Electric signals sent over electric conductors like copper wires are still commonly used in many regions of the world because their signals
  - are not effected by weather like airborne signals;
  - are transferred more reliably than airborne signals;
  - require equipment and infrastructure that was already common before the rise of more recent methods.

GCE Paper 1 Questions

1. Total internal reflection occurs when light travels from

   A water to air. B air to water. C a vacuum to water. D air to glass.

2. Glass has a critical angle of about 42°. If light incident through glass happens to be totally internally reflected, the angle of incidence is likely

   A 40° B 41.5° C 42° D 43.5°

3. The absolute refractive index of a particular medium is given to be 2.0. When placed in air, its critical angle is

   A 42° B 30° C 45° D 50°

4. If water has $n = 1.33$, the critical angle of light moving from its interior to air is approximately

   A 42.7° B 30° C 45° D 48.8°

5. Figure 3.6.5.9 shows an incident ray of light travelling from water to air.

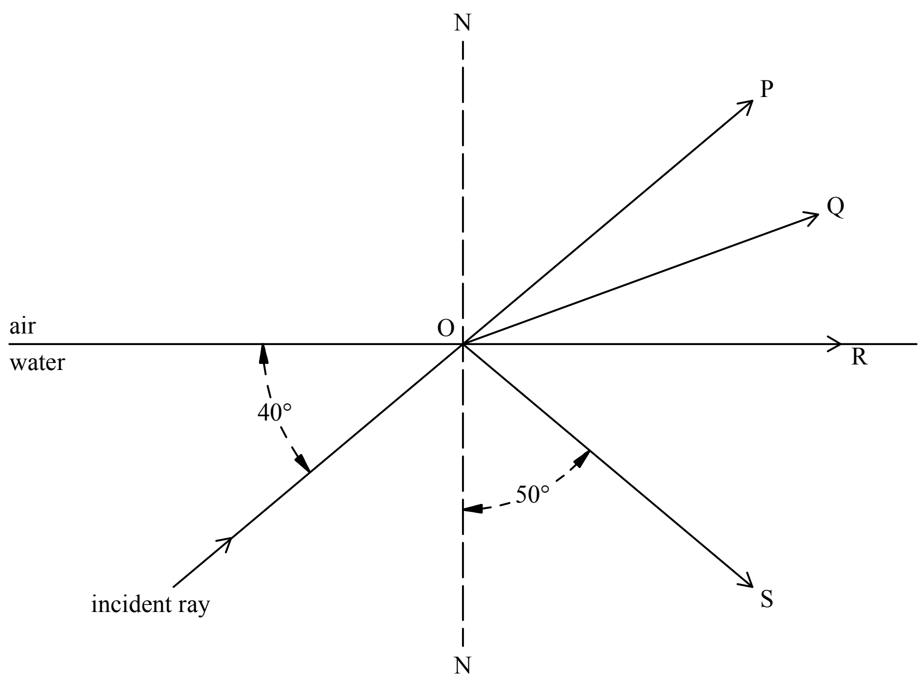

Figure 3.6.5.9

   If it is given that the refractive index of water is 1.33, the emergent ray is

   A *OP* B *OQ* C *OR* D *OS*

6. Which of the following is not a condition for total internal reflection to occur?

   A Light must be about to cross the boundary between two transparent media.
   B The first medium must have a greater absolute refractive index than the second.
   C The first medium must have a smaller absolute refractive index than the second.
   D The angle of incidence at the boundary must be greater than the critical angle.

7. Figure 3.6.5.10 shows a light ray incident normally on a glass prism.

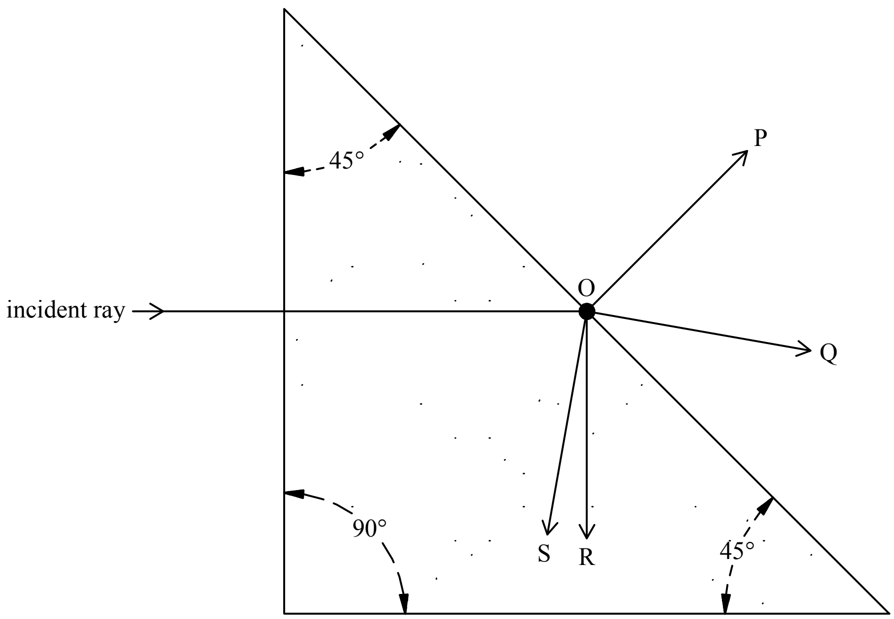

Figure 3.6.5.10

The ray continuing from point O is

A OP B OQ C OR D OS

8. If a fibre optic cable is made of glass with an absolute refractive index of 1.5, light signals travel through it at an average speed of

A $3.0 \times 10^8\ m\ s^{-1}$ B $4.5 \times 10^8\ m\ s^{-1}$ C $2.0 \times 10^8\ m\ s^{-1}$ D $1.5 \times 10^8\ m\ s^{-1}$

9. Given the answer to question 8, which of the following statements most accurately describes v_{es}, the effective speed of an electric signal sent through a copper wire?

A $v_{es} < 2.0 \times 10^8\ m\ s^{-1}$ B $v_{es} = 2.0 \times 10^8\ m\ s^{-1}$ C $v_{es} > 2.0 \times 10^8\ m\ s^{-1}$ D $v_{es} = 0\ m\ s^{-1}$

10. Which of the following is not an advantage of fibre optic cables over copper wires in telecommunication?

A speed of data sent
B reduced cost of installation
C long-distance durability of data sent
D power-efficiency of data sent and received

GCE Paper 1 Solutions

1. A 2. D 3. B 4. D 5. D 6. C 7. C 8. C 9. A 10. B

GCE Paper 2 Questions

1. Figure 3.6.5.11 shows a ray of light AB incident on the side of a semi-circular perspex block. The critical angle of perspex is given to be 40°.

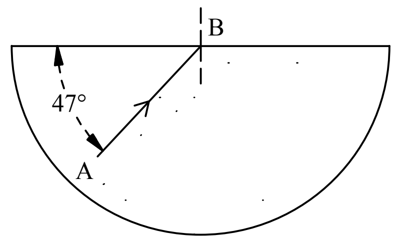

Figure 3.6.5.11

(a) Copy and complete the diagram to show the ray after incidence. Give a reason to explain why the ray is drawn the way its shown. **(3 mks)**

(b) What is the common name used to describe the effect drawn? **(2 mks)**

. . . .

Solution

(a) The given angle of 47° is complimentary with the angle of incidence. Therefore, $i = 90° - 47° = 43°$, which is greater than the given critical angle. This means that total internal reflection occurs.

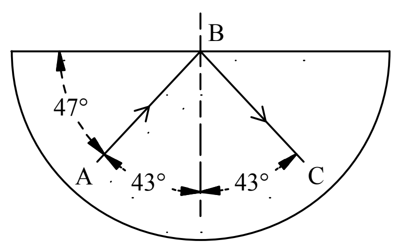

Figure 3.6.5.12

(b) Total internal reflection is the common name used to describe this effect.

---

2. (a) Describe two everyday occurrences of refraction. **(2 mks)**

(b) Name one practical use of refractive indices of materials. **(1 mks)**

(c) Explain the meaning of water having a critical angle of 48°. **(2 mks)**

. . . .

Solution

(a) *Below is a non-exhaustive list of occurrences. Only two are required.*
- Pools of water appear less deep than they truly are.
- Straight objects appear to be bent in water.
- Objects viewed through a glass prism appear to be inside the prism.
- Air above hot surfaces appears blurry.

(b) *Below is a non-exhaustive list of devices. Only one is required.*
- periscopes
- binoculars
- fibre optic cables
- underwater viewing by fish looking upwards

(c) This means that if light is leaving from water to air at any angle of incidence above 48°, it will be reflected back down from the surface.

---

3. A light ray is travelling in air towards a glass block.
   (a) Can it be totally internally reflected at the glass surface? Explain. **(2 mks)**
   (b) State the conditions necessary for total internal reflection to occur. **(3 mks)**
   (c) If the critical angle of glass is 42°, what is its absolute refractive index? **(2 mks)**
   (d) What will happen to a ray of light travelling as an incident ray from glass to air at $i = 43°$? **(1 mk)**
   (e) If the absolute refractive index of a diamond is given to be 2.5, what is the smallest angle of incidence by which it can totally internally reflect light? **(3 mks)**

. . . .

Solution

(a) No it cannot. This is because light can only be totally internally reflected as it moves between transparent media where the second medium has a lower absolute refractive index than the first.

(b) If a ray of light is about to pass from a medium of absolute refractive index n_1 to another medium whose absolute refractive index is n_2 and the critical angle between the media is c, it will be totally internally reflected if

$$n_1 \neq n_2 \qquad n_1 > n_2 \qquad i > c$$

(c) *It is assumed that the given critical angle is for light moving from glass to air.*

$$\text{given equation for critical angle: } n_1 \sin(c) = n_2 \sin(90°)$$

$$\text{turning absolute refractive index of first medium into subject: } n_1 = \frac{n_2 \sin(90°)}{\sin(c)}$$

$$\text{substituting known values: } n_1 = \frac{(1)(1)}{\sin(42°)}$$

$$\text{final answer: } \boxed{n_1 \approx 1.49}$$

(d) It will be totally internally reflected because the angle of incidence is greater than the critical angle.

(e) *It is assumed that the given reflection/refraction scenario is from diamond into air.*

$$\text{given equation for critical angle: } c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

$$\text{substituting known values: } c = \sin^{-1}\left(\frac{1}{2.5}\right)$$

$$\text{final answer: } \boxed{c \approx 23.58°}$$

That is, the diamond will totally internally reflect any light approaching its boundary from the inside at an angle of incidence greater than 23.58°.

3.6.6 Introduction to Lenses

Objectives

By the end of the lesson, students should be able to

1. describe six different types of lenses.
2. explain the action of a converging lens in terms of refraction by a composition of prisms.
3. define various lens-related terms.
4. draw ray diagrams showing the action of a convex lens on a parallel beam of light.
5. draw ray diagrams to determine the properties of images formed by biconvex lenses.
6. state the optical properties of a formed image.
7. discuss several applications of converging lenses, including microscopes, cameras, photocopiers and projectors.

Lens Classification

- A **lens** is a transparent media with one or two regular curved surfaces.
- While lenses come in many shapes and sizes, this chapter will focus on six common types.
- **Converging** or **convex** lenses are lenses that are thickest at their centres or middles.
  - **Biconvex** lenses are converging lenses with both sides of equal curvature;
  - **Planoconvex** lenses are converging lenses with only one curved side;
  - **Converging meniscus** lenses are converging lenses with two sides of different curvature.

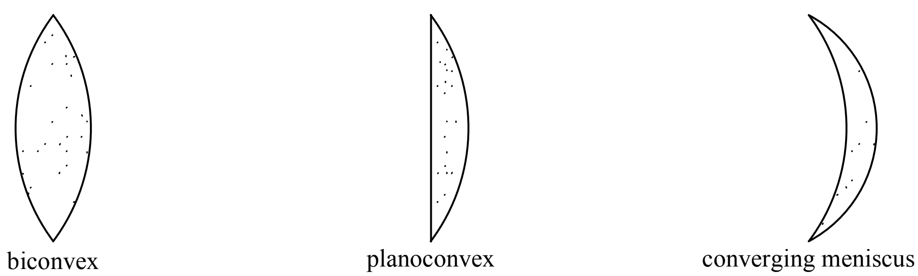

Figure 3.6.6.1

- **Diverging** or **concave** lenses are lenses that are thinnest at their centres or middles.
  - **Biconcave** lenses are diverging lenses with both sides of equal curvature;
  - **Planoconcave** lenses are diverging lenses with only one curved side;
  - **Diverging meniscus** lenses are diverging lenses with two sides of different curvature.

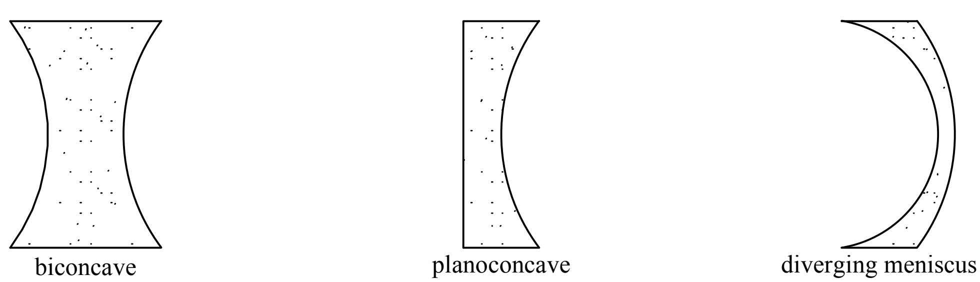

Figure 3.6.6.2

- NB: Both converging meniscus and diverging meniscus lenses have two sides of different curvature, but they differ in that the first is thickest at its centre while the second is thinnest.

Biconvex Lenses as Multiple Prisms

- A biconvex lens can be thought of as several refracting trapezoidal glass prisms.
- Each prism is angled increasingly inward with increasing distance from the lens' centre.
- This causes rays entering the prisms as a parallel beam to refract towards each other as they exit.
- If the prisms' angles have a certain proportion, all exiting rays converge at a single point of focus.

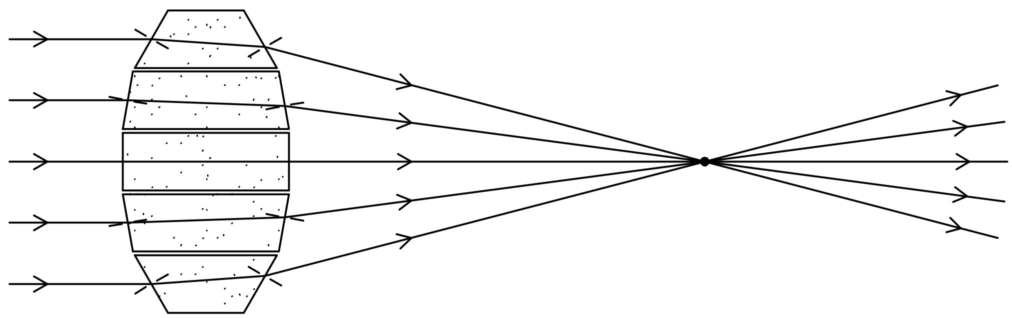

Figure 3.6.6.3

Lens Terminology

- The **optical centre**, or **P**, of a lens is the point at its centre where light rays pass without being deviated.
- The **aperture** of a lens is the diameter of the circular area through which it allows light to pass.
- The **principal axis** of a lens is the line passing through its optical centre as well as its sides' centres of curvature.

  It may be worth explaining that the flat side of a planoconvex lens can be thought of as a curve with an infinite radius. Therefore, its centre is still on its principal axis.

- The **centre plane** of a lens is the plane perpendicular to its principal axis passing through its optical centre.
- The **principal focus**, or **F**, of a converging lens is the point on its principal axis at which exiting rays converge.
- The **focal length**, or **f**, of a converging lens is the distance between its optical centre and its principal focus.
- The **focal plane** of a lens is the plane perpendicular to its principal axis passing through its principal focus.

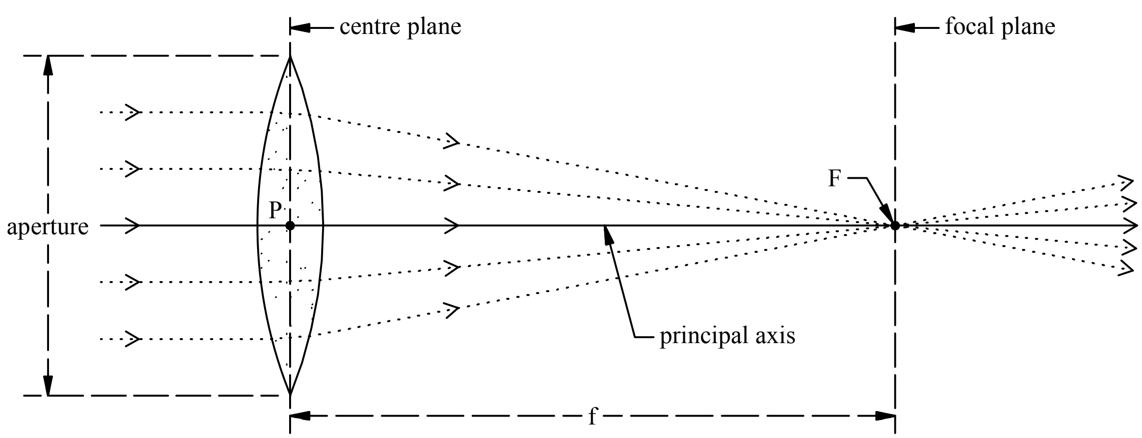

Figure 3.6.6.4

Image Classification

- Formed images are classified according to three separate criteria.

1 **Lateral location**
- **Real** images are formed by the actual intersection of rays and can be formed on a screen. (For a lens, real images are formed on the side of the centre plane opposite the object.)
- **Virtual** images are formed by the apparent intersection of rays and cannot be formed on a screen. (For a lens, virtual images are formed on the same side of the centre plane as the object.)

2 **Vertical location**
- **Upright** or **erect** images are formed on the same side of the principal axis as the object.
- **Inverted** images are formed on the side of the principal axis opposite the object.

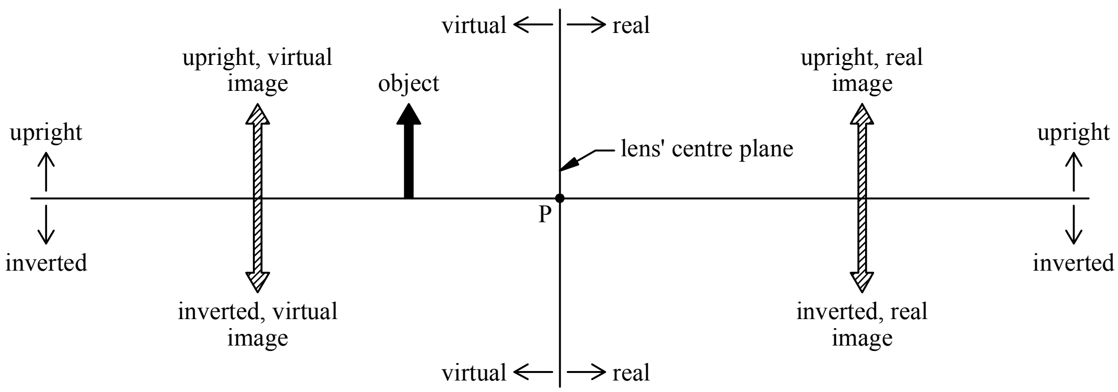

Figure 3.6.6.5

3 **Size change** - Considering h, the original object's size and h', the formed image's size,
- **magnified** images are larger than the original object ($h' > h$);
- **diminished** images are smaller than the original object ($h' > h$).

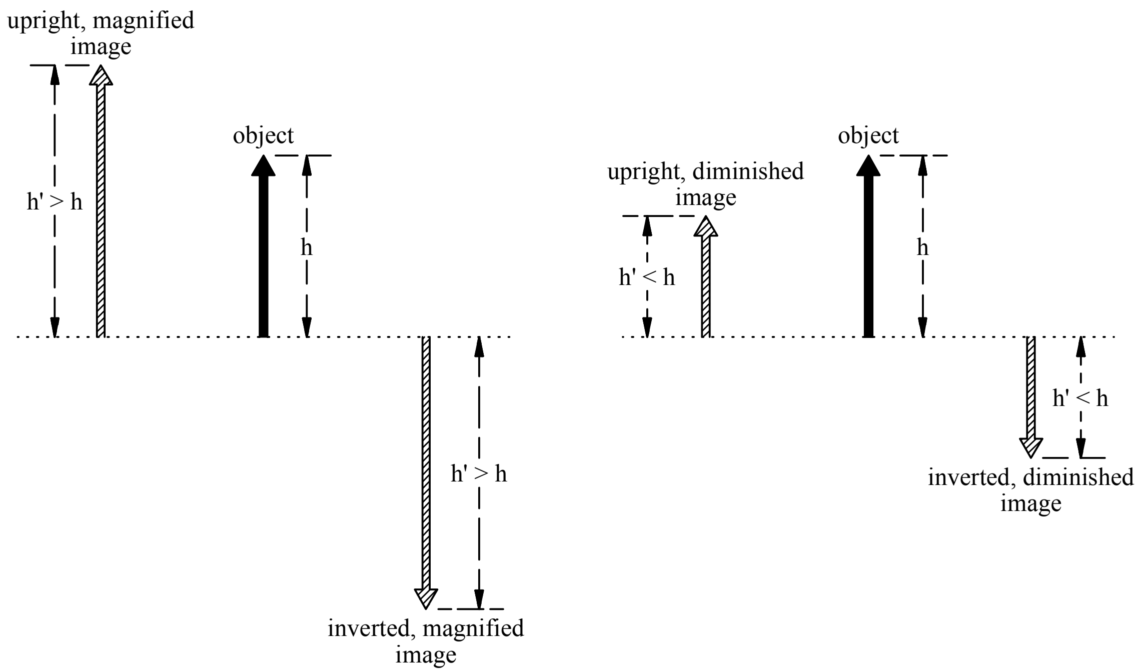

Figure 3.6.6.6

Ray Diagrams with Converging Lenses

- **Ray diagrams** are figures constructed to show how lenses form images.
- A biconvex lens has two focal points for each curved side.
- Assuming a left-to-right ray direction,
  - the focal point to the left of P is often labelled F';
  - the focal point to the right of P is often labelled F;
  - objects left of F are said to be "before the focal point".
- Ray diagrams often involve three separate rays.

1 The first ray is drawn connecting
   - the object's top and the centre plane as a line parallel to the principal axis.
   - this point on the centre plane and F.

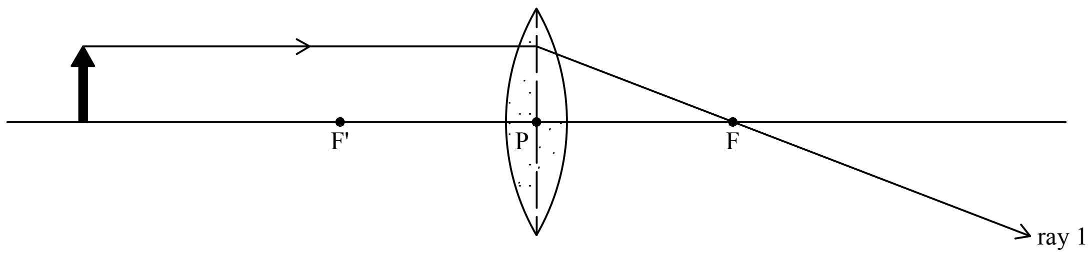

Figure 3.6.6.7

2 The second ray is drawn connecting
   - the object's top and P.

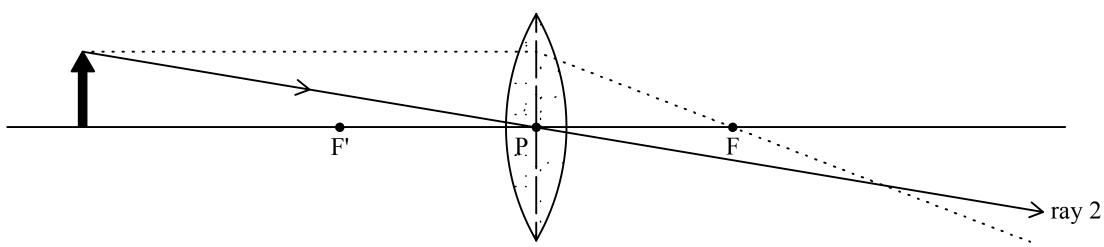

Figure 3.6.6.8

3 The third ray is drawn connecting
   - the object's top, F' and the centre plane;
   - this point on the centre plane and onward as a line parallel to the principal axis.

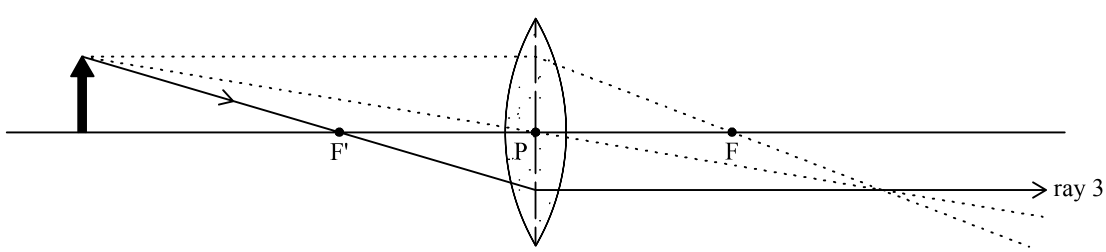

Figure 3.6.6.9

- The image is perpendicular to the principal axis, terminating at the intersection point of the three rays.

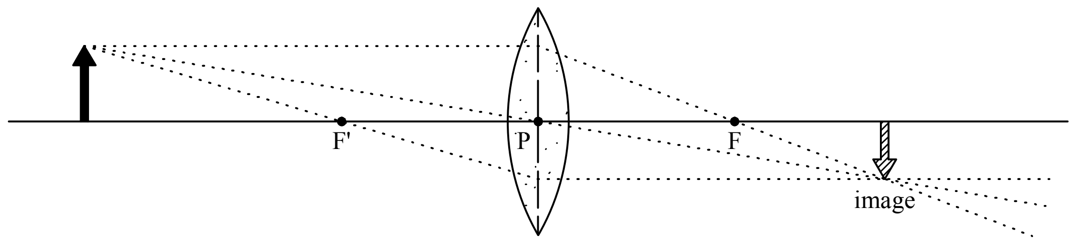

Figure 3.6.6.10

- NB: While only two rays are needed to determine the image, the third helps to correct errors.

Image Formation with Converging Lenses

- The properties of the image formed by a biconvex lens depend on the object's location in relation to
  - the lens' optical centre, P;
  - the lens' principal focus on the object's side, F;
  - a distance of two times the lens' principal focus on the object's side, $2F'$.
  - all distances further than two times the lens' principal focus on the object's side, ∞' (infinity).

- An object placed **directly at F′** forms no image.
  - The rays emerge from the lens' other side as parallel beam.
  - This is used in search lights and projectors which produce a parallel beam by placing a light source at F'.

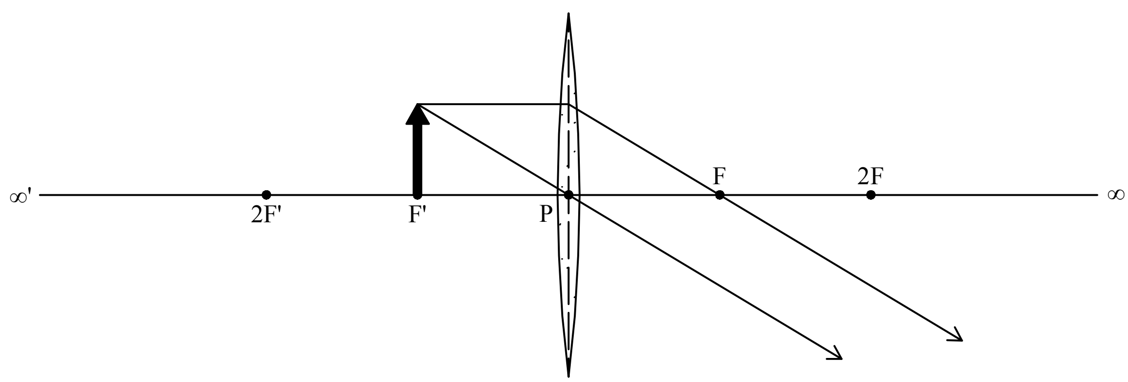

Figure 3.6.6.11

- An object placed **between F′ and 2F′** forms a real, inverted, magnified image.
  - This is used in microscopes as well as the enlarging features of some photocopiers and cameras.

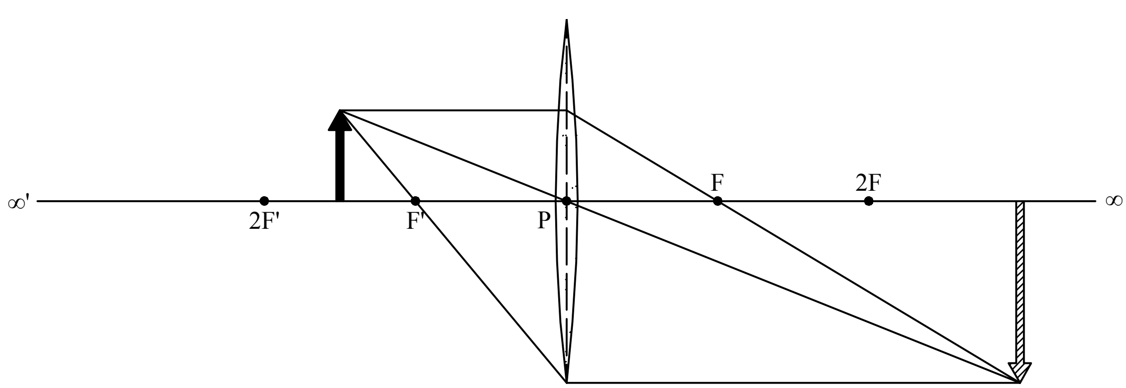

Figure 3.6.6.12

- An object placed **at 2F′** forms a real, inverted image of the same size as the object.
  - This is used in some photocopiers and cameras while taking equal-sized pictures and copies.

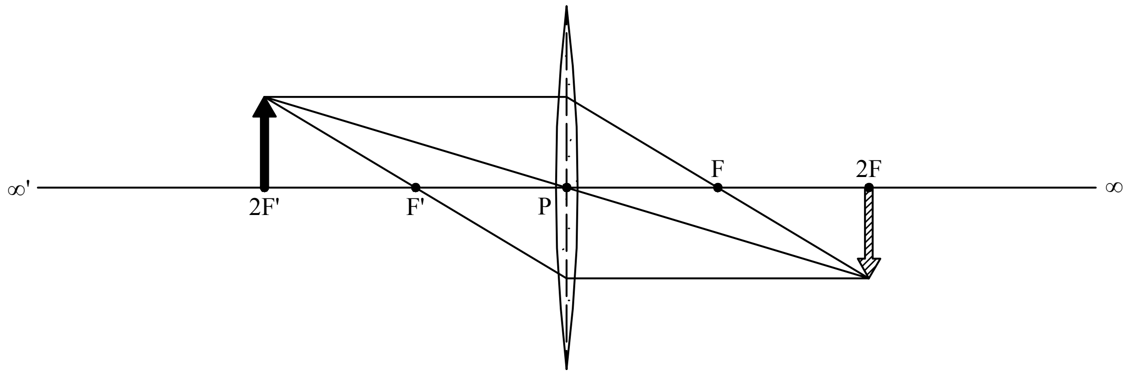

Figure 3.6.6.13

- An object placed **before 2F′** forms a real, inverted, diminished image.
  - This is used in the human eye as well as the shrinking features of some photocopiers and cameras.

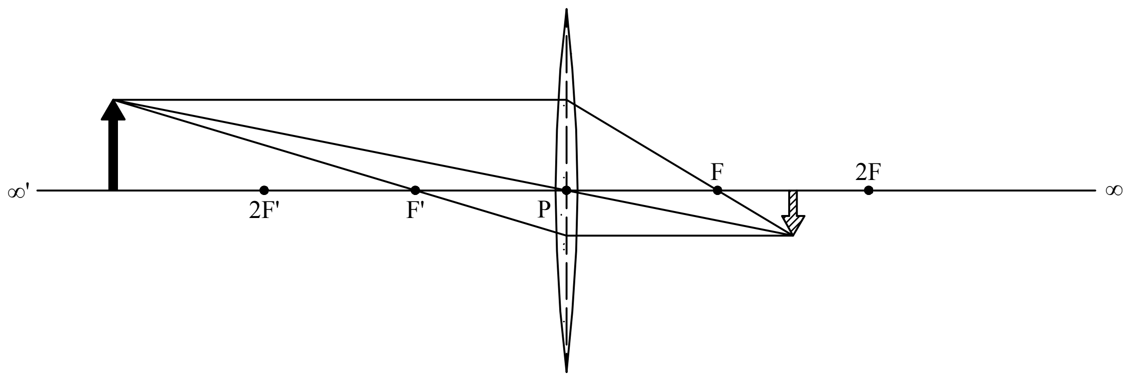

Figure 3.6.6.14

- An object placed **between F′ and P** forms a virtual, upright, magnified image.
  - The first ray is drawn with an extension of the exiting ray drawn on the other side of the centre plane.
  - The second ray is drawn with an extension drawn away from the centre plane.
  - The third ray is drawn with an extension towards the centre plane resulting in a line parallel to the principal axis returning back from the centre plane.
  - This is used in magnifying glasses, eyepieces and spectacles that correct for long-sightedness.

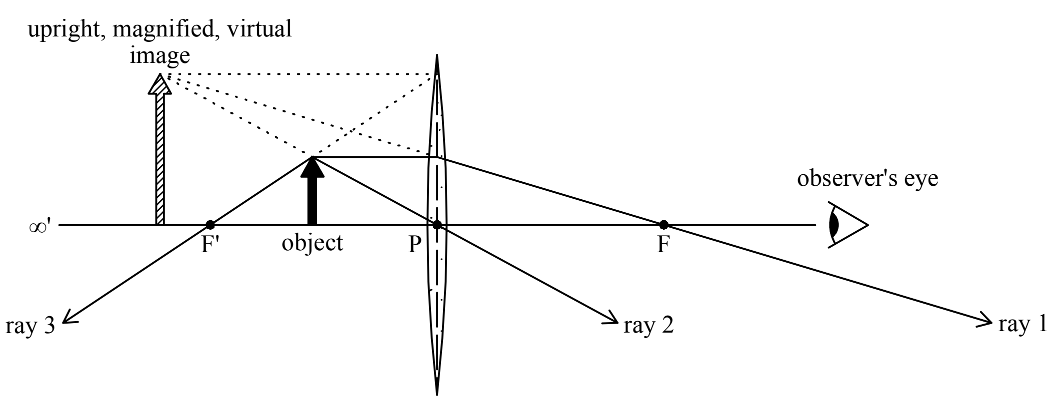

Figure 3.6.6.15

GCE Paper 1 Questions

1. An object is placed 5 *cm* from a converging lens of focal length 10 *cm*. The image formed is

   A real and on the same side as the object.
   B real and inverted.
   C virtual and upright.
   D virtual and on the opposite side of the object.

2. Which of the following instruments uses an object at a position between the lens and the principal focus?

   A projector B photocopier C simple camera D magnifying glass

3. A real image has all except for which of the following properties?

   A It can be formed on a screen.
   B It is formed by the actual intersection of rays.
   C In a ray diagram, it is formed on the opposite side of the centre plane as the object.
   D In a ray diagram, it is formed on the same side of the centre plane as the object.

4. Which of the following lenses has one flat side?

   A biconvex B planoconvex C diverging meniscus D converging meniscus

5. A virtual image has all except for which of the following properties?

   A In a ray diagram, it is formed on the same side of the centre plane as the object.
   B In a ray diagram, it is formed on the opposite side of the centre plane as the object.
   C It cannot be formed on a screen.
   D It is formed by the intersection of apparent rays only.

6. Placing an object exactly at the principal focus of a convex lens produces

   A an inverted image B an upright image C no distinct image D 4 images

7. Which of the following can be the characteristics of an image formed by the lens of a camera?

   A diminished, inverted and real
   B diminished, erect and real
   C diminished, inverted and virtual
   D diminished, erect and virtual

8. A magnifying glass works best when an object is placed between

   A P and F'
   B F' and $2F'$
   C $2F'$ and ∞'
   D a rock and a hard place

9. When an object is above a lens' principal axis, its inverted image is ______ this axis.

   A to the right of B behind C above D below

10. Which of the following lenses has two round sides each of different curvature?

    A converging meniscus B biconvex C planoconcave D biconcave

GCE Paper 1 Solutions

1. C 2. D 3. D 4. B 5. B 6. C 7. A 8. A 9. D 10. A

GCE Paper 2 Questions

1. A student uses a biconvex lens to magnify a very tiny object. The lens has a focal length of 20 *cm* and is held 10 *cm* away from the tiny object.

   (a) Draw a ray diagram to show how the lens forms the magnified image. **(3 mks)**
   (b) Give two properties of the image formed. **(2 mks)**

....

Solution

(a) *See figure 3.6.6.16.*

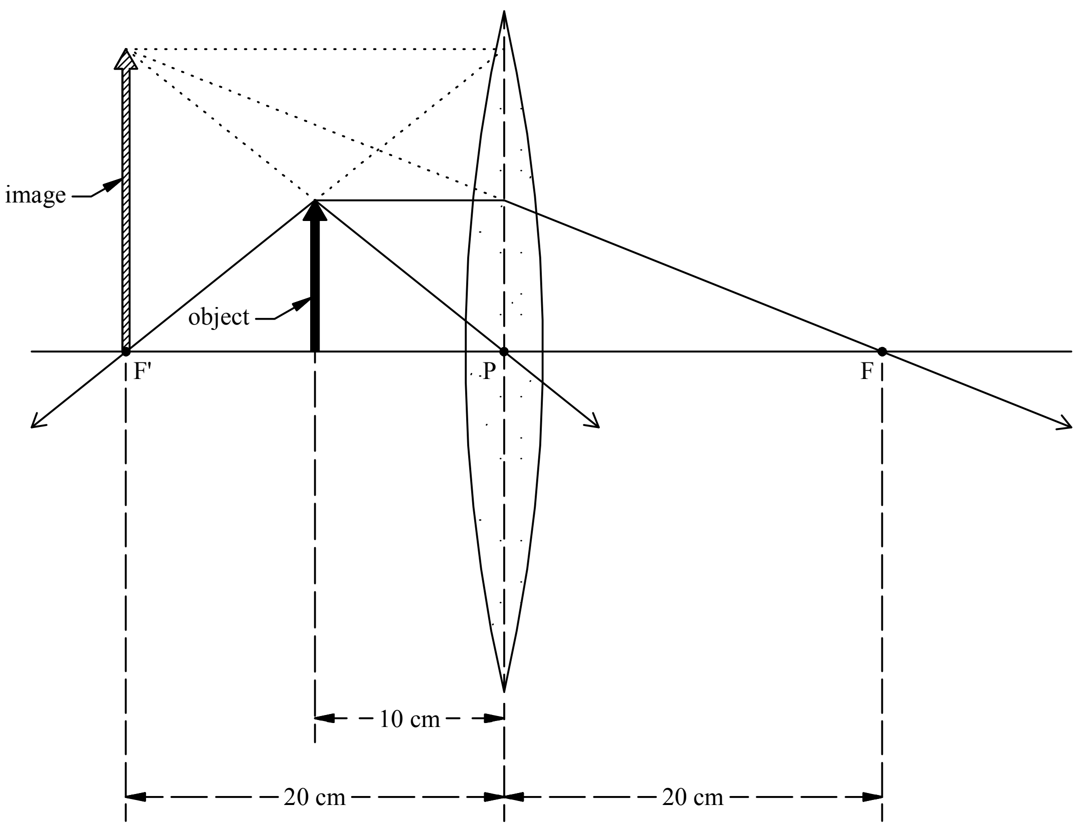

Figure 3.6.6.16

(b) *Below is a list of three image properties. Only two are required.*

The image formed is

– virtual – upright – magnified

2. A photocopier with a focal length of 2 *cm* is used to make three separate copies. While the first copy is smaller than the original, the second is equal and the third is larger. The machine is built such that the object of the document can be placed only at a distance of 3 *cm*, 4 *cm* and 5 *cm* from the lens' optical centre.

   Draw the corresponding ray diagram for
   (a) the smaller copy. **(3 mks)**
   (b) the equal-sized copy. **(3 mks)**
   (c) the larger copy. **(3 mks)**

. . . .

Solution

(a) *See figure 3.6.6.17. The object should be past* $2F'$*, which means the* 5 *cm position is best.*

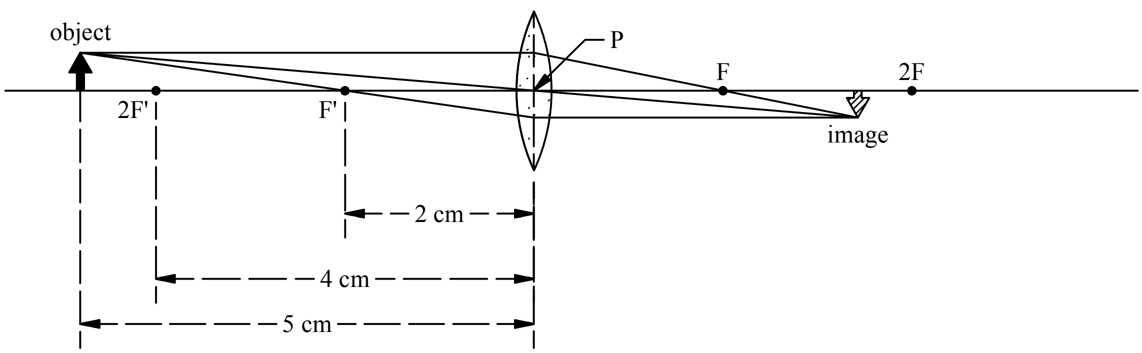

Figure 3.6.6.17

(b) *See figure 3.6.6.18. The object should be exactly at* $2F'$*, which means the* 4 *cm position is best.*

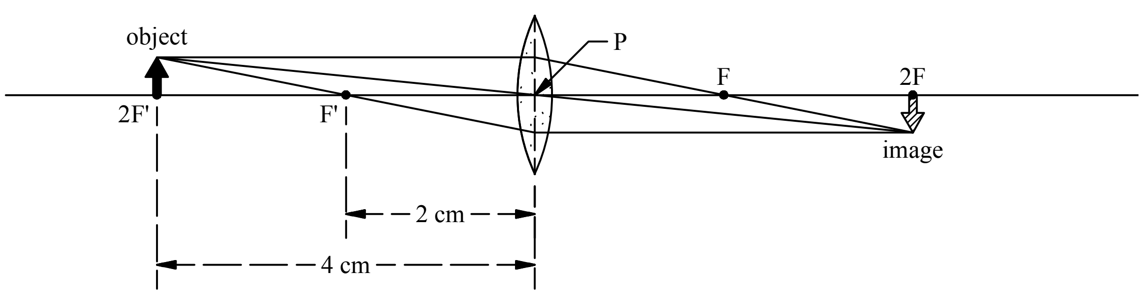

Figure 3.6.6.18

(c) *See figure 3.6.6.19. The object should be between* $2F'$ *and* F'*, which means the* 3 *cm position is best.*

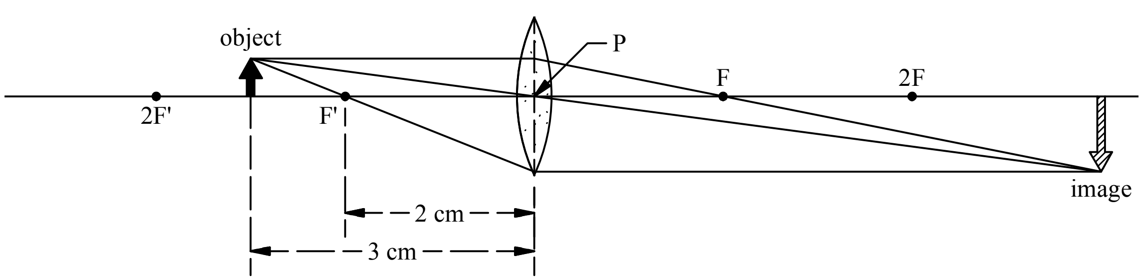

Figure 3.6.6.19

3.6.7 Convex Lenses in Detail

Objectives

By the end of the lesson, students should be able to

1. describe the auxiliary plane mirror method of determining the focal length of a converging lens.
2. describe the distant object method of determining the focal length of a converging lens.
3. discuss the human eye as a converging lens.
4. solve problems involving the distance and size of a lens' object and its image.

Experiment to Determine the Focal Length of a Converging Lens: Auxiliary Plane Mirror Method

I Procedure

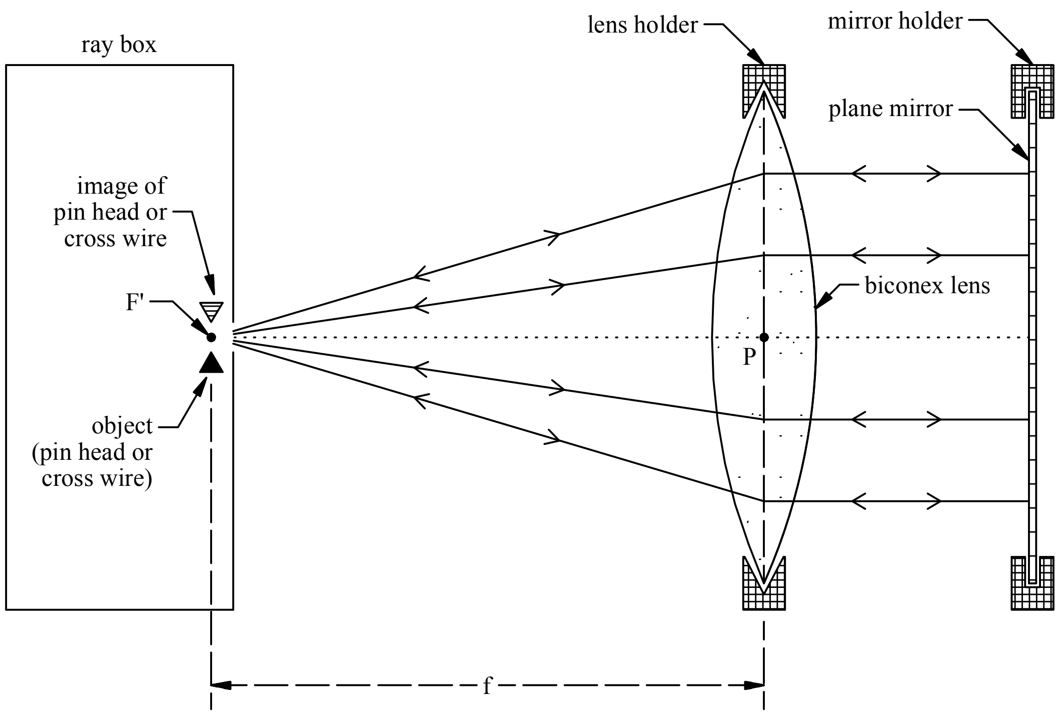

Figure 3.6.7.1

1 A biconvex lens is placed in a holder.
2 An optical pin or cross wire object is placed at the opening of a ray box.
3 This ray box is placed near one side of the lens.
4 A plane mirror is placed parallel to the lens on the side opposite the object.
5 The distance between the lens and the object is adjusted by shifting the holder slowly.
6 The adjustment is stopped when a sharp image of the object appears directly near the object.
7 A metre rule is used to measure the distance between the lens' centre plane and the object/image.

II Conclusion

- The lens' focal length, f, is the distance at which the object's image is sharpest and beside itself.
- This is because, if the object is directly at the lens' principal focus, its light rays
  - enter the lens and exit the other side as a parallel beam;
  - are reflected by the mirror back into the lens;
  - are converged once more as a sharp image, directly at the lens' principal focus.

III Precautions

- The experiment is repeated several times to get multiple values of f. The final value is then taken as the mean of these values in order to minimize error.

Experiment to Determine the Focal Length of a Converging Lens: Distant Object Method

I Procedure

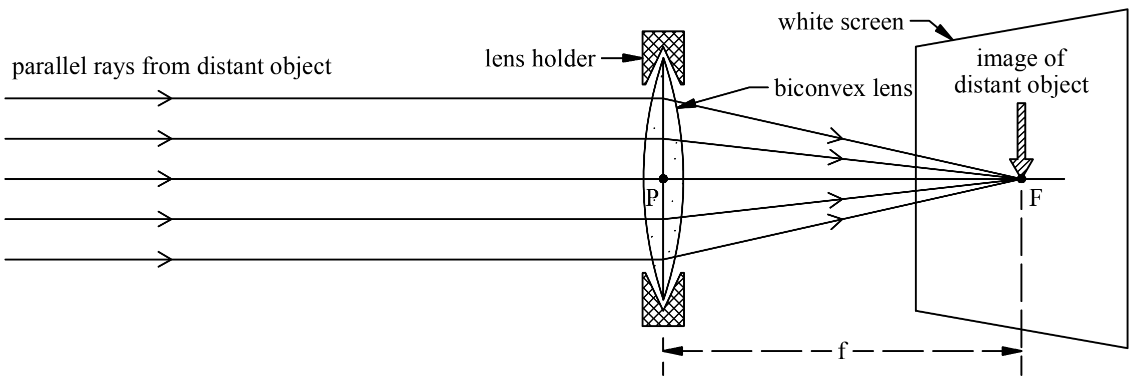

Figure 3.6.7.2

1 A biconvex lens is placed in a holder near a white screen.
2 A light source as an object is placed at least 10 m from a convex lens on the other side of the screen.
3 The distance between lens and the screen is adjusted by shifting the holder slowly.
4 The adjustment is stopped when the image appears sharpest on the white screen.
5 A metre rule is used to measure the distance between the lens' centre plane and the screen.

II Conclusion

- The converging lens' focal length f is the distance at which the image is sharpest on the screen.
- This is because light rays from the distant object
  - enter one side of the lens as an almost perfectly parallel beam;
  - exit the other side and converge at the lens's principal focus.

III Precautions

- The experiment is repeated several times to get multiple values of f. The final value is then taken as the mean of these values in order to minimize error.
- The object is placed far enough to warrant a parallel-ray approximation.

The Eye as a Converging Lens

- The human eye acts as an optical device using a converging lens.
  - The **cornea** and **aqueous humour** are watery structures that "pre-converge" light before the lens.
  - The **lens** is a flexible transparent structure which changes shape to fine-adjust the eye's focal length.
  - The **ciliary muscles** strain and relax to adjust the lens' shape.
  - The **retina** acts as the screen upon which images are formed.
  - The **optic nerve** sends information from the retina to the brain.

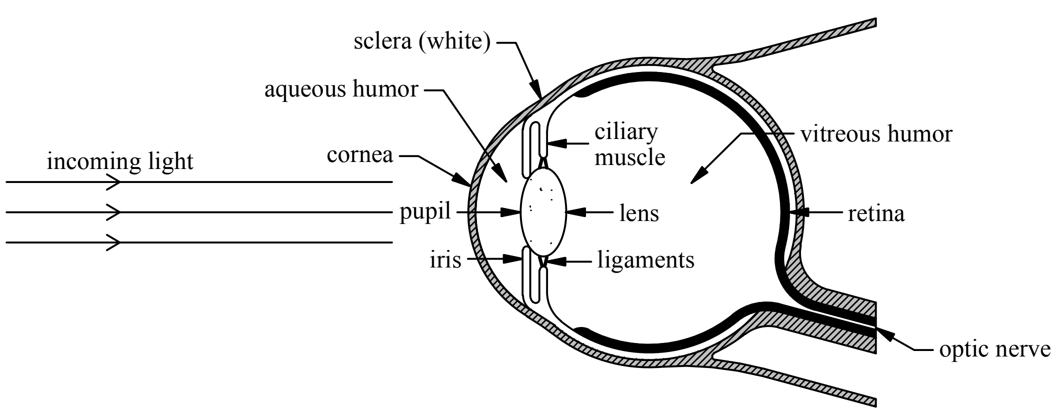

Figure 3.6.7.3

- **Accommodation** is the ability of the lens of the eye to change shape in order to change its principal focus.
- The **near point** of the eye is the closest point at which an object can be seen clearly.
- At this point, which is about 25 *cm* for a normal, healthy eye,
  - the most accommodation is needed to view an object clearly;
  - the ciliary muscles strain as hard as they can;
  - the lens takes on its maximum, thickest curvature;
  - the lens' focal length is minimized to the nearby object.

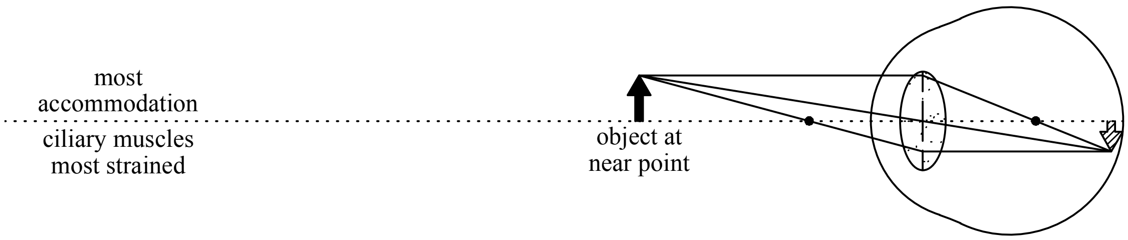

Figure 3.6.7.4

- When the object is moved a somewhat further away,
  - the ciliary muscles reduce their strain but don't relax completely;
  - the lens reduces its curvature;
  - the lens' focal length is increased to the object's new distance;
  - some accommodation is still provided by the eye.

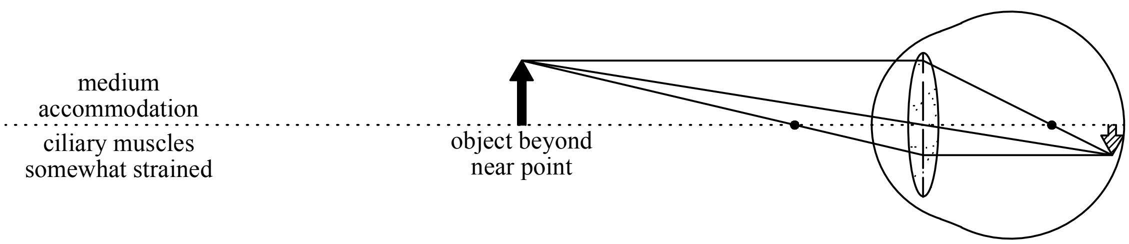

Figure 3.6.7.5

- The **far point** of the eye is the farthest point at which an object can be seen clearly.
- At this point, which is at infinity for a normal healthy eye,
  - the least accommodation is needed;
  - the ciliary muscles relax completely
  - the lens takes on its minimum, thinnest curvature;
  - the lens' focal length is set directly on the retina;
  - the eye focuses only on parallel incoming light;
  - all objects at a decent distance are in focus;

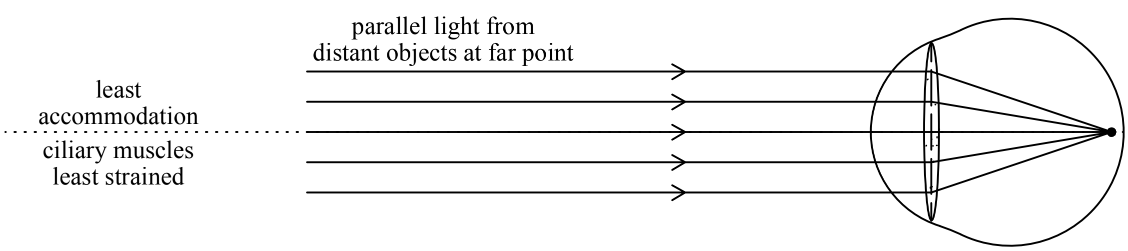

Figure 3.6.7.6

- While eyes focus by changing the shape of their lens, cameras focus by shifting the position of their lens.

Calculating Lens-Object and Lens-Image Distances

- The distance between a formed image and a converging lens (v) can be calculated from its focal length (f) as well as the distance between its optical centre and the object (u).

$$\frac{1}{f} = \frac{1}{u} + \frac{1}{v} \tag{3.6.7.1}$$

Where
- f is the lens' focal length, in m;
- u is the distance between the object and the lens' optical centre, in m;
- v is the distance between the image and the lens' optical centre, in m.

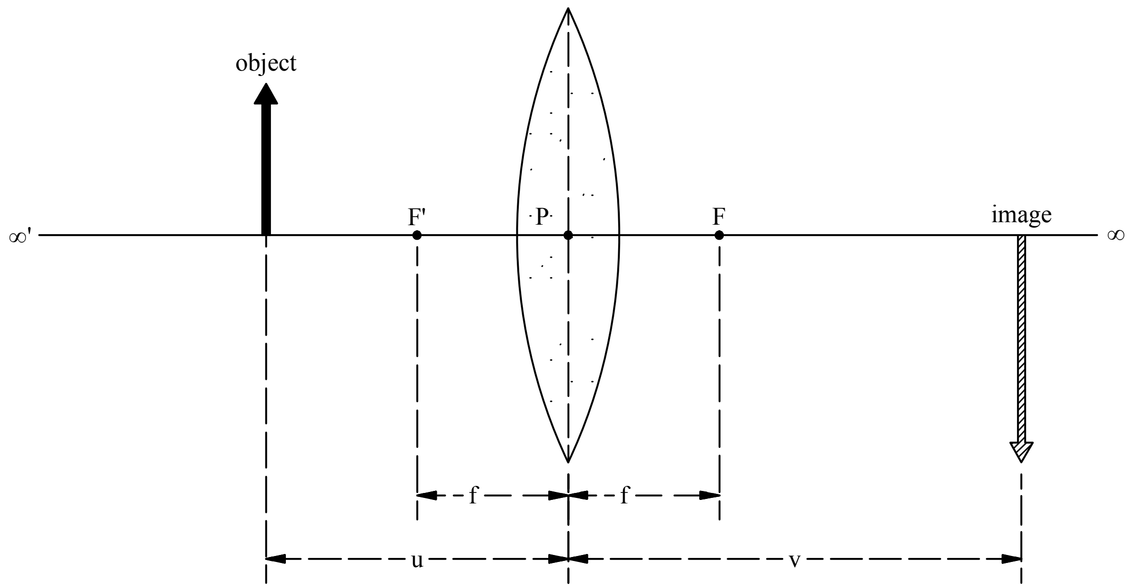

Figure 3.6.7.7

- The sign of each variable used in this formula must be regarded with caution.

for biconvex (converging) lenses: $f > 0$
for biconcave (diverging) lenses: $f < 0$
for real images on the opposite side of the lens as the object: $v > 0$
for virtual images, on the same side of the lens as the object: $v < 0$

- This equation can be modified such the focal length (and not its reciprocal) is the subject.

given equation for image/object-lens distance: $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$

creating common denominator: $\frac{1}{f} = \frac{1}{u}\left(\frac{v}{v}\right) + \frac{1}{v}\left(\frac{u}{u}\right)$

combining like terms: $\frac{1}{f} = \frac{v+u}{vu}$

inverting: $f = \frac{vu}{v+u}$

Linear Magnification

- The **linear magnification**, or **M**, of an image is the ratio of its height to that of the original object.

$$M = \frac{h'}{h} \tag{3.6.7.2}$$

Where
  - M is an image's linear magnification (unit-less);
  - h' is the image's height, in m;
  - h is the object's height, in m.

- It is a scalar.

- It has no units given that it is the value of one distance (height) divided by another.

- NB: The "magnification" of an object can result in a smaller image, in which case $0 < M < 1$. However, the image's size-property would be "diminished" as opposed to "magnified".

magnification	**image property**	**relation between heights**
$M < 1$	diminished	$h' < h$
$M = 1$	same size as object	$h' = h$
$M > 1$	magnified	$h' > h$

Table 3.6.7.1

- This formula allows for image properties to be tabulated in terms of how u and f relate for a biconvex lens.

object position	**image properties**			**applications**
	location	**orientation**	**size**	
$u < f$	virtual	upright	magnified	magnifying glasses eyepieces spectacles for long-sightedness
$u = f$	-	-	-	parallel beams from search lights parallel beams from projectors
$f < u < 2f$	real	inverted	magnified	microscopes photocopiers (enlarging) cameras (zooming in)
$u = 2f$	real	inverted	same	photocopiers (equal size) cameras (equal size)
$u > 2f$	real	inverted	diminished	photocopiers (shrinking) cameras (zooming out)

Table 3.6.7.2

- Linear magnification can be calculated from the image-lens distance and the object-lens distance.
- If the image is real, its distance from the lens takes on a positive value ($v > 0$).
- In such a case, the object and image heights are legs of two similar, right triangles sharing a vertex at P.

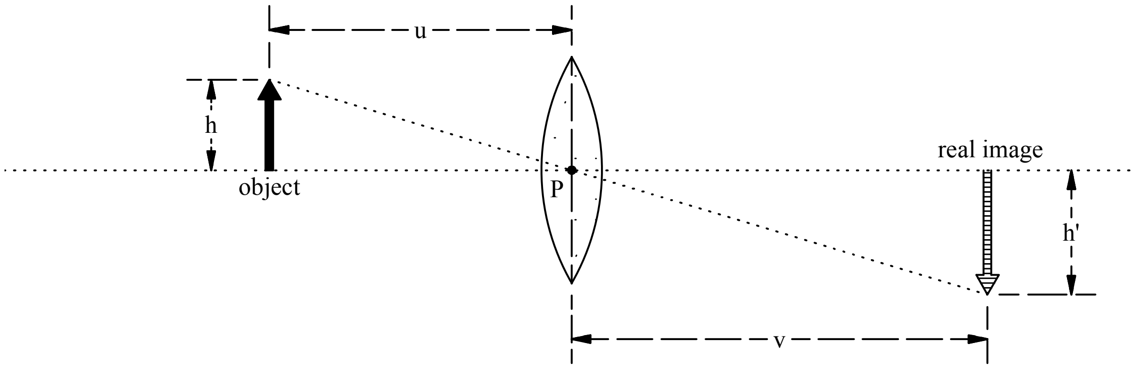

Figure 3.6.7.8

$$\text{assuming similar triangles: } \frac{h}{u} = \frac{h'}{v}$$

$$\text{separating height and distance ratios: } \frac{h'}{h} = \frac{v}{u}$$

- With a virtual image where $v < 0$, this equation still applies when considering the absolute value of v.

$$\frac{h'}{h} = \frac{|v|}{u} \tag{3.6.7.3}$$

- In such a case, the object and the image heights are legs of two similar, right triangles with collinear hypotenuses.

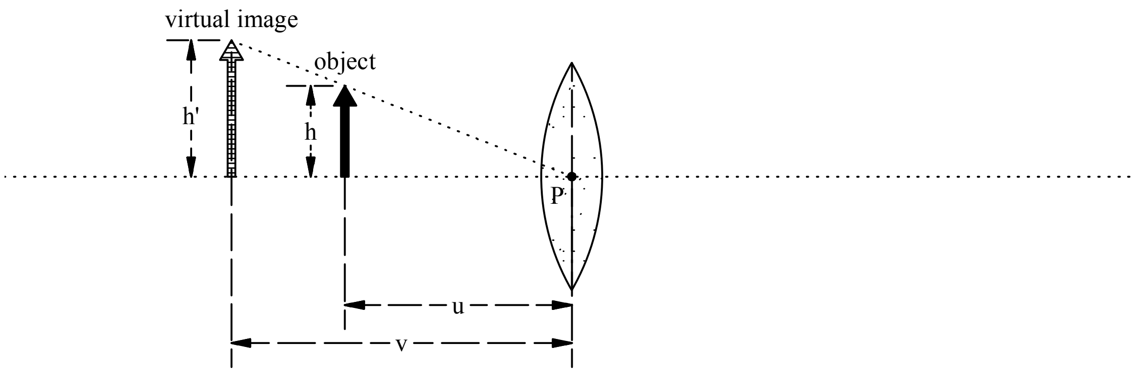

Figure 3.6.7.9

- This requires the equation of an image's magnification to depend on whether it is real or virtual.

$$\text{for a real image: } M = \frac{v}{u} \tag{3.6.7.4}$$

$$\text{for a virtual image: } M = \frac{-v}{u} \tag{3.6.7.5}$$

Where
- M is the image's linear magnification (unit-less);
- v is the distance between the image and the lens' optical centre, in m;
- u is the distance between the object and the lens' optical centre, in m.

- If a real image's magnification is known along with its lens' focal length, the object-lens distance required for this size-change can be calculated.

$$\text{given equation for magnification of real images: } M = \frac{v}{u}$$

$$\text{turning image-lens distance into subject: } v = Mu$$

$$\text{substituting image-lens distance into equation for object/image-lens distance: } \frac{1}{f} = \frac{1}{u} + \frac{1}{Mu}$$

$$\text{creating common denominator: } \frac{1}{f} = \frac{1}{u}\left(\frac{M}{M}\right) + \frac{1}{Mu}$$

$$\text{combining like terms: } \frac{1}{f} = \frac{M+1}{Mu}$$

$$\text{turning object-lens distance into subject: } u = \frac{f(M+1)}{M}$$

- Applying the same approach to a virtual image yields a modified equation.

$$\text{for a virtual image: } u = \frac{f(M-1)}{M}$$

- If a real image's magnification is known along with its lens' focal length, the image-lens distance at which this size-change occurs can be calculated.

$$\text{given equation for magnification of real images: } M = \frac{v}{u}$$

$$\text{turning object-lens distance into subject: } u = \frac{v}{M}$$

$$\text{substituting object-lens distance into equation for object/image-lens distance: } \frac{1}{f} = \frac{1}{\frac{v}{M}} + \frac{1}{v}$$

$$\text{simplifying: } \frac{1}{f} = \frac{M}{v} + \frac{1}{v}$$

$$\text{combining like terms: } \frac{1}{f} = \frac{M+1}{v}$$

$$\text{turning image-lens distance into subject: } v = f(M+1)$$

- Applying the same approach to a virtual image yields a modified equation.

$$\text{for a virtual image: } v = f(1-M)$$

- Thus, the distances needed for a particular image magnification with a given lens focal length depend on whether the image is real or virtual.

$$\text{for a real image:} \quad u = \frac{f(M+1)}{M}, \quad v = f(M+1)$$

$$\text{for a virtual image:} \quad u = \frac{f(M-1)}{M}, \quad v = f(1-M)$$

  Where
  - u is the distance between the object and the lens' optical centre, in m.
  - v is the distance between the image and the lens' optical centre, in m.
  - f is the lens' focal length, in m;
  - M is the image's magnification (unit-less);

GCE Paper 1 Questions

1. Which of the following similarities between the eye and a camera is true?

   A Focusing is done by moving the lens.
   B The image produced is real and inverted.
   C Objects in any position can form sharp images.
   D The thickness of the lens does not change.

2. An object is placed 40 *cm* in front of a convex lens of focal length 15 *cm*. If the image of that object is formed 24 *cm* behind the lens, then the linear magnification is

   A $\frac{3}{5}$ B $\frac{15}{16}$ C $\frac{16}{15}$ D $\frac{5}{3}$

3. The human eye lens produces an image on the retina that is

   A concave B virtual C real and upright D real and inverted

4. When an object is brought closer to the human eye for clearer vision, the eye lens

   A moves forward. B moves backward. C becomes thicker. D becomes thinner.

5. Accommodation is the ability of the eye to

   A bring objects near the lens.
   B bring light rays from an object into focus.
   C focus on objects at different distances.
   D provide dinner and a room to sleep.

6. An object having a height of 2 *cm* is placed 40 *cm* from a biconvex converging lens of focal length 20 *cm*. How far from the lens is the image formed on the opposite side?

   A 40 *cm* B 30 *cm* C 20 *cm* D 10 *cm*

7. What is the size of the image formed in question 6?

   A 1 *cm* B 1.5 *cm* C 2 *cm* D 4 *cm*

8. An object is placed 20 *cm* in front of a converging lens of $f = 15$ *cm*. The distance between P and the image is

   A 20 *cm* B 5 *cm* C 15 *cm* D 60 *cm*

9. A biconvex lens has a focal length of 25 *cm*. An upright arrow is located 10 *cm* from the optical centre of the lens. The image is

   A upright and bigger than the object.
   B virtual and smaller than the object.
   C real, inverted and smaller than the object.
   D virtual, inverted and smaller than the object.

10. The closest point at which an eye can clearly see an object is called its

    A far point B near point C retina D iris

GCE Paper 1 Solutions

1. B 2. A 3. D 4. C 5. C 6. A 7. C 8. D 9. A 10. B

GCE Paper 2 Questions

1. A photocopier reduces the size of a picture by positioning it at a distance of 5.2 cm from its biconvex lens. Its focal length is given to be 2 cm.

   (a) What is the distance between the lens and the formed image? **(3 mks)**
   (b) Give three properties to describe the image formed **(3 mks)**
   (c) Draw a labelled ray diagram to show how this size reduction is accomplished. **(4 mks)**

. . . .

Solution

(a) *It is assumed that the requested distance starts at the lens' optical centre.*

$$\text{given equation for image/object-lens distance: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{turning image distance into subject: } v = \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$$\text{substituting known values: } v = \frac{1}{\frac{1}{2\ cm} - \frac{1}{5.2\ cm}}$$

$$\text{final answer: } \boxed{v = 3.25\ cm}$$

(b) The image is real, inverted and diminished.

(c) *See figure 3.6.7.10*

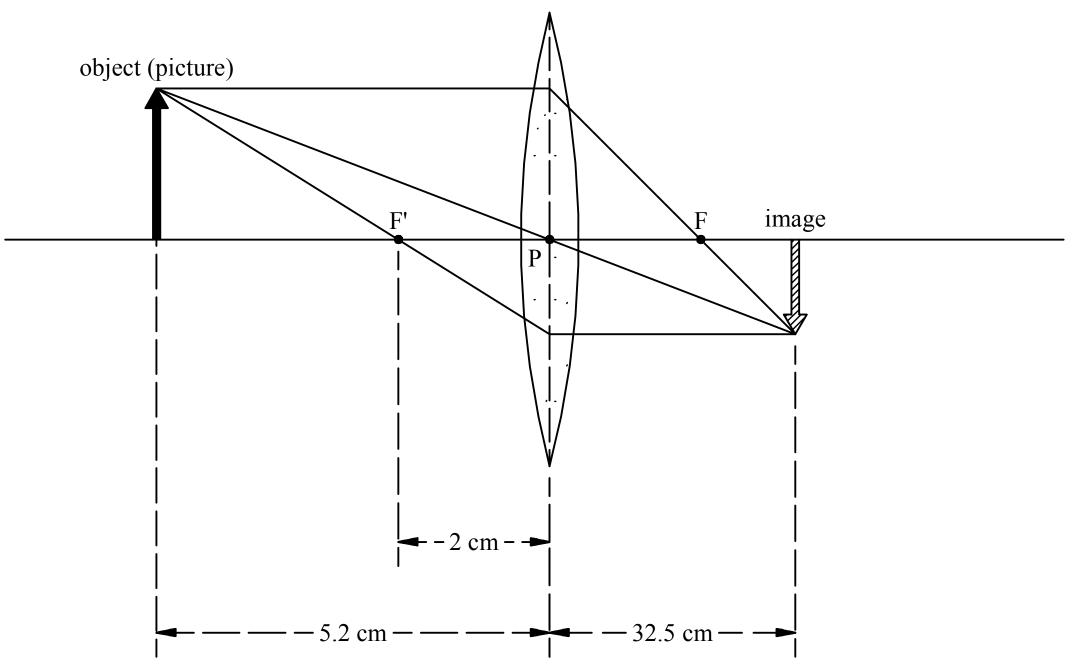

Figure 3.6.7.10

2. A biconvex lens with a focal length of 20 *cm* can be used to form both real and virtual magnified images. It is given that such a lens is arranged to magnify a 10 *cm* into an image 2.5 times as large.

   (a) Determine the object-lens spacing of the real image. **(3 mks)**
   (b) Determine the image-lens spacing of the real image. **(2 mks)**
   (c) Determine the object-lens spacing of the real image. **(3 mks)**
   (d) Determine the image-lens spacing of the virtual image. **(2 mks)**
   (e) Determine the height of the image. **(2 mks)**
   (f) On two separate, vertically-aligned ray diagrams, draw the real and virtual images including the object's corresponding position for each. **(5 mks)**

. . . .

Solution

(a) *The linear magnification is given to be 2.5.*

$$\text{given equation for object-lens spacing of real images: } u = \frac{f(M+1)}{M}$$

$$\text{substituting known values: } u = \frac{(20\ cm)(2.5+1)}{2.5}$$

$$\text{final answer: } \boxed{u_{\text{real}} = 28\ cm}$$

(b) *Given the larger size of the real image, its distance from the lens should be greater than that of the object.*

$$\text{given equation for image-lens spacing of real images: } v = f(M+1)$$

$$\text{substituting known values: } v = (20\ cm)(2.5+1)$$

$$\text{final answer: } \boxed{v_{\text{real}} = 70\ cm}$$

(c) *Even for the virtual image, the linear magnification is still the given value of 2.5.*

$$\text{given equation for object-lens spacing of virtual images: } u = \frac{f(M-1)}{M}$$

$$\text{substituting known values: } u = \frac{(20\ cm)(2.5-1)}{2.5}$$

$$\text{final answer: } \boxed{u_{\text{virtual}} = 12\ cm}$$

(d) *Even for the virtual image, the image-lens spacing should still be greater than the object-lens spacing.*

$$\text{given equation for image-lens spacing of virtual images: } v = f(1-M)$$

$$\text{substituting known values: } v = (20\ cm)(1-2.5)$$

$$\text{final answer: } \boxed{v_{\text{virtual}} = -30\ cm}$$

(e) *The object height and image magnification are given.*

$$\text{given equation for linear magnification: } M = \frac{h'}{h}$$

$$\text{turning image height into subject: } h' = Mh$$

$$\text{substituting known values: } h' = (2.5)(10\ cm)$$

$$\text{final answer: } \boxed{h' = 25\ cm}$$

(f) *See figure 3.6.7.11*

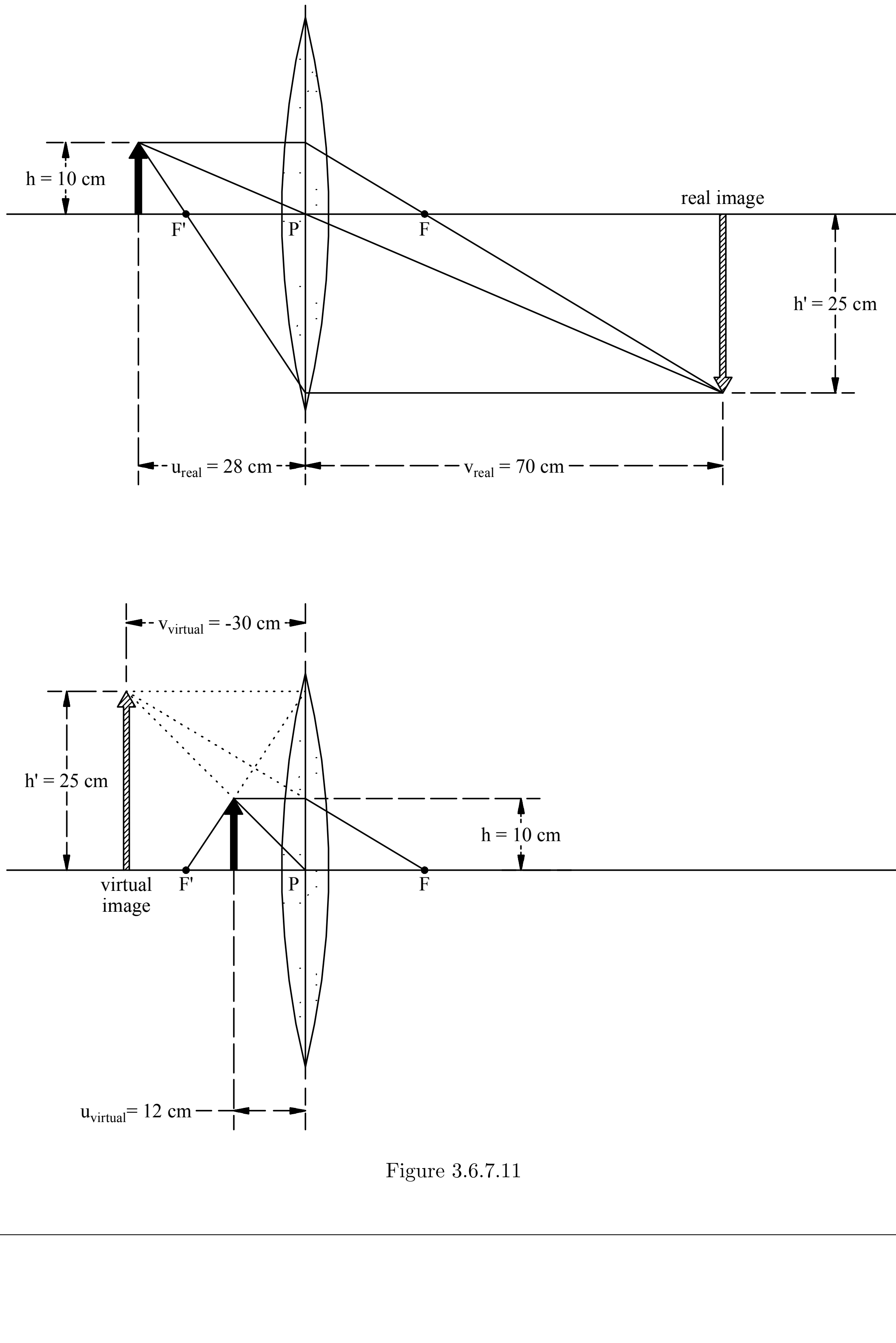

Figure 3.6.7.11

3. An object having a height of 5 *cm* is placed 10 *cm* in front of a biconvex lens. This forms an image of the same size on the opposite side.

(a) Determine the image distance from the lens. **(2 mks)**
(b) Calculate the focal length of the lens. **(2 mks)**
(c) State two properties of the image formed. **(2 mks)**
(d) Name one device which is an application of such an arrangement of lens and object. **(1 mk)**

. . . .

Solution

(a) *The image is real and its magnification is 1.*

$$\text{given equation for magnification of real images: } M = \frac{v}{u}$$

$$\text{turning image-lens distance into subject: } v = Mu$$

$$\text{substituting known values: } v = (1)(10\ cm)$$

$$\text{final answer: } \boxed{v = 10\ cm}$$

(b) *Since $M = 1$, it should be calculated that both the object and the image are located two times the focal length from the lens' centre plane.*

$$\text{given equation for image/object-lens distance: } f = \frac{vu}{v+u}$$

$$\text{substituting known values: } f = \frac{(10\ cm)(10\ cm)}{10\ cm + 10\ cm}$$

$$\text{final answer: } \boxed{f = 5\ cm}$$

(c) *Below is a list of three image properties. Only two are required.*

The image formed is
- real
- inverted
- of the same size the object (neither magnified nor diminished)

(d) *Below is a non-exhaustive list of devices. Only one is required.*

- photocopiers for same-size images
- cameras for same-size images

3.6.8 Interaction of Lenses

Objectives

By the end of the lesson, students should be able to:

1. draw ray diagrams showing the action of a concave lens on a parallel beam of light.
2. draw ray diagrams to determine the properties of images formed by biconcave lenses.
3. discuss several defects of the eye.
4. show how spectacle lenses can be used to correct defects of the eye.

Biconcave Lenses as Multiple Prisms

- Just like a biconvex lens, a biconcave lens can be thought of as several refracting trapezoidal glass prisms.
- Each prism is angled increasingly outward with increasing distance from the lens' centre.
- This causes rays entering the prisms as a parallel beam to refract away from each other as they exit.
- If the prisms' angles have a certain proportion, all exiting rays diverge as if from a single point of focus.

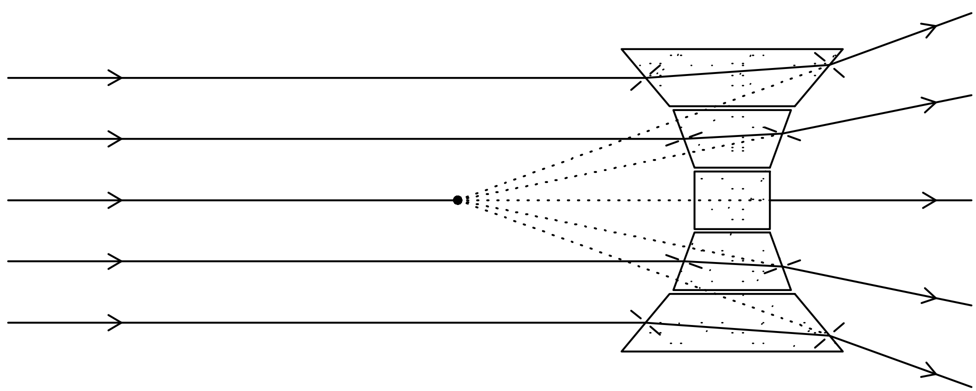

Figure 3.6.8.1

- Therefore, a smooth biconcave lens has the same effect.

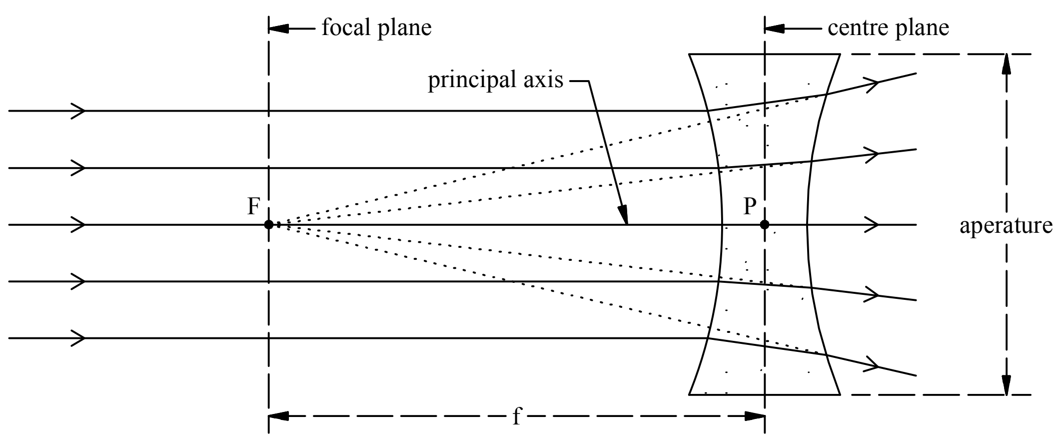

Figure 3.6.8.2

- The principal focus is on the side of the incoming rays.
- Therefore, the focal length, f, takes on a negative value ($f < 0$).

Ray Diagrams with Diverging Lenses

- While very similar to converging lens ray diagrams, those constructed for diverging lenses have the principal focus (F) on the same side as the object.

1 The first ray is drawn
   - connecting the object's top and the centre plane as a line parallel to the principal axis;
   - from this point on the centre plane diverging outward from F.

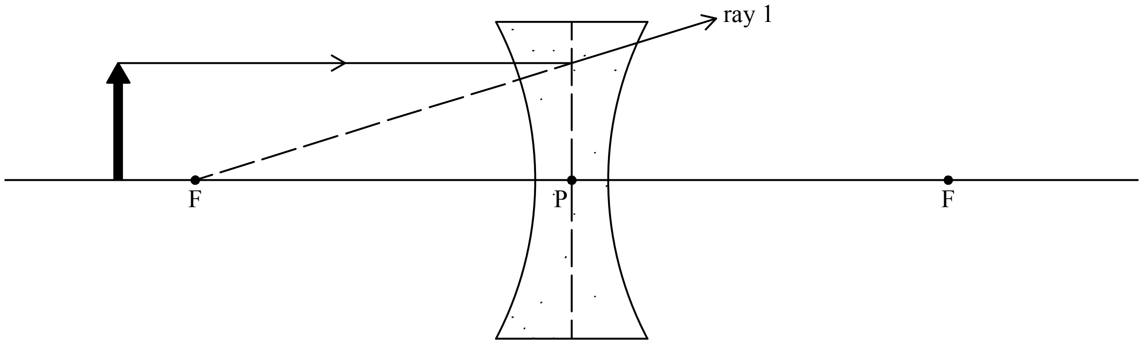

Figure 3.6.8.3

2 The second ray is drawn connecting
   - the object's top and P.

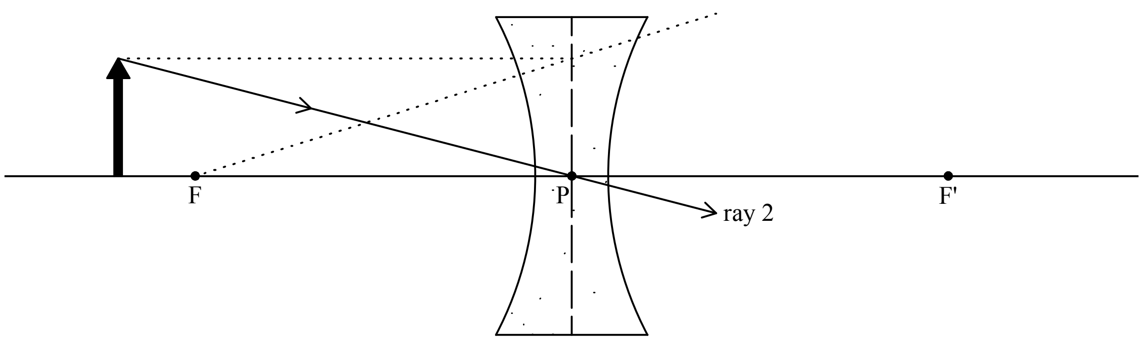

Figure 3.6.8.4

3 The third ray is drawn
   - connecting the object's top to F;
   - as a line parallel to the principal axis from the point where this line intersects the centre plane.

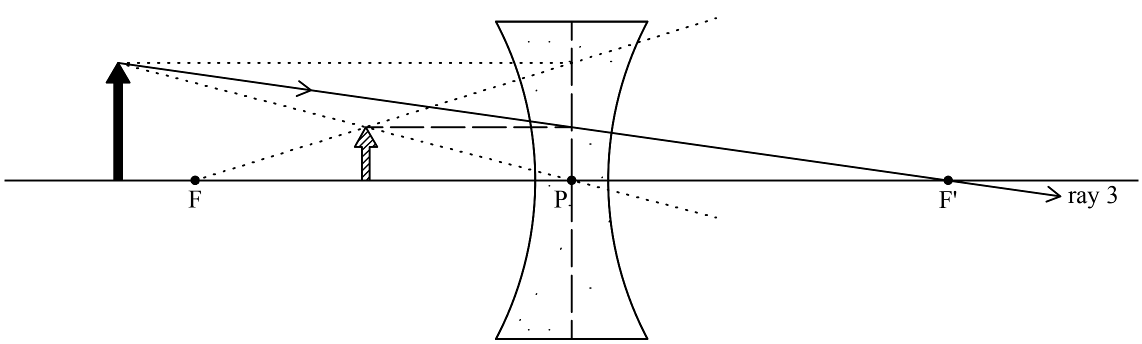

Figure 3.6.8.5

- The image is perpendicular to the principal axis, terminating at the intersection point of the three rays.

Image Formation with Diverging Lenses

- A diverging lens forms a virtual, upright, diminished object, regardless of the object-lens distance.
- This is the case for $u < f$, $u = f$, $f < u < 2f$, $u = 2f$ and $u > 2f$.

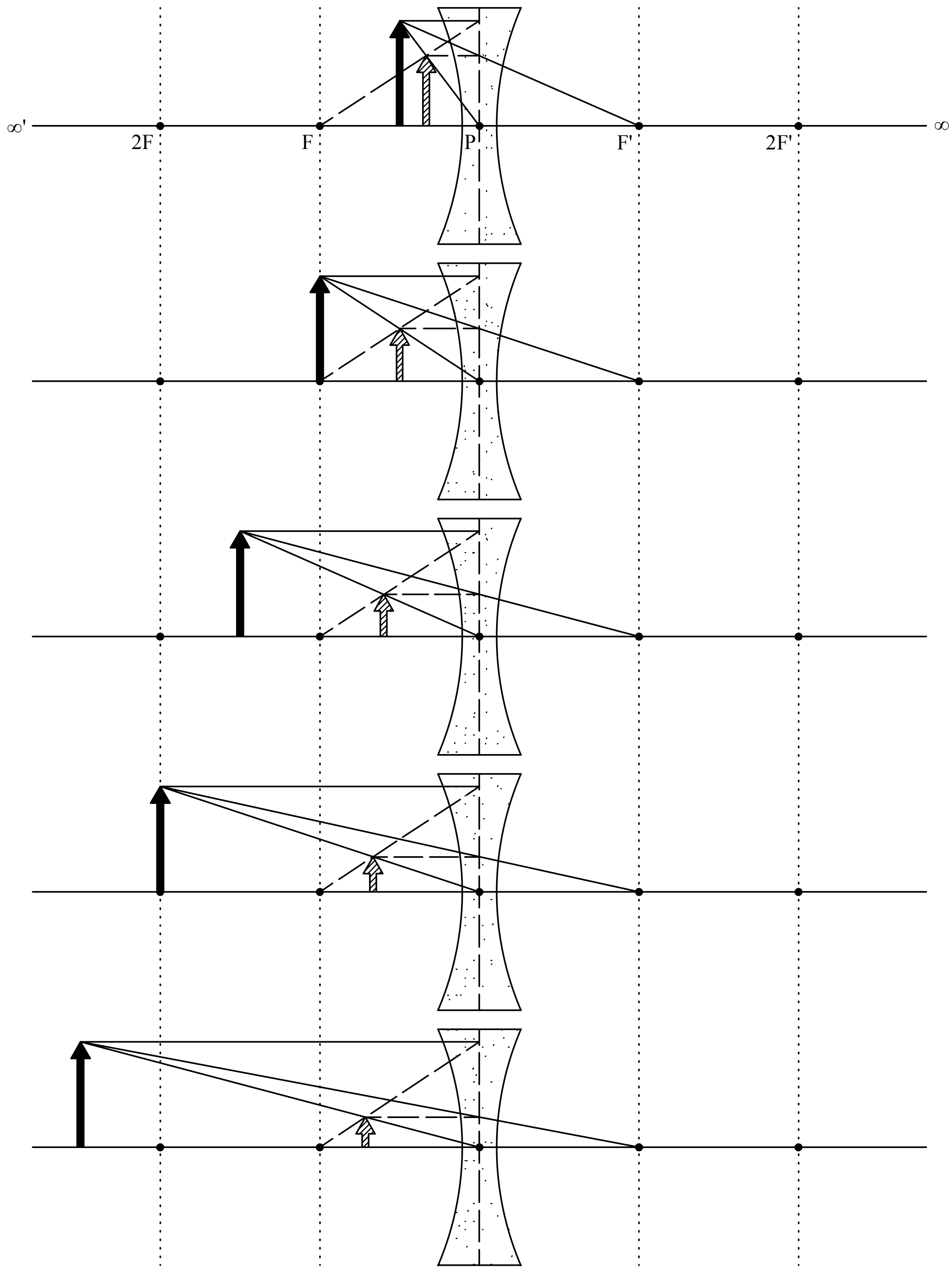

Figure 3.6.8.6

Calculations with Biconcave Lenses

- The position/size formulas that are used for images formed in a converging lenses also apply to diverging lenses.
- The only difference is that the focal length of a diverging lens takes on a negative value.

$$\text{for diverging lenses: } f < 0$$

- This causes the image to always be virtual and diminished.

$$\text{for diverging lenses: } v < 0, \ M < 1$$

Example: An object 4 cm tall is viewed 7 cm from the centre plane of a diverging lens of focal length 5 cm.

(a) How far is the image from the centre plane?

$$\text{given equation for image/object-lens distance: } \frac{1}{f} = \frac{1}{u} + \frac{1}{v}$$

$$\text{turning image-lens distance into subject: } v = \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$$\text{substituting known values: } v = \frac{1}{\frac{1}{-5\ cm} - \frac{1}{7\ cm}}$$

$$\text{final answer: } \boxed{v \approx -2.92\ cm}$$

(b) What is the size of this image formed?

$$\text{given equation for image/object size ratio: } \frac{h'}{h} = \frac{|v|}{u}$$

$$\text{turning image size into subject: } h' = h\left(\frac{|v|}{u}\right)$$

$$\text{substituting known values: } h' = (4\ cm)\left(\frac{|-2.92\ cm|}{7\ cm}\right)$$

$$\text{final answer: } \boxed{h' = 1.67\ cm}$$

(c) Draw a ray diagram showing how the image is formed.

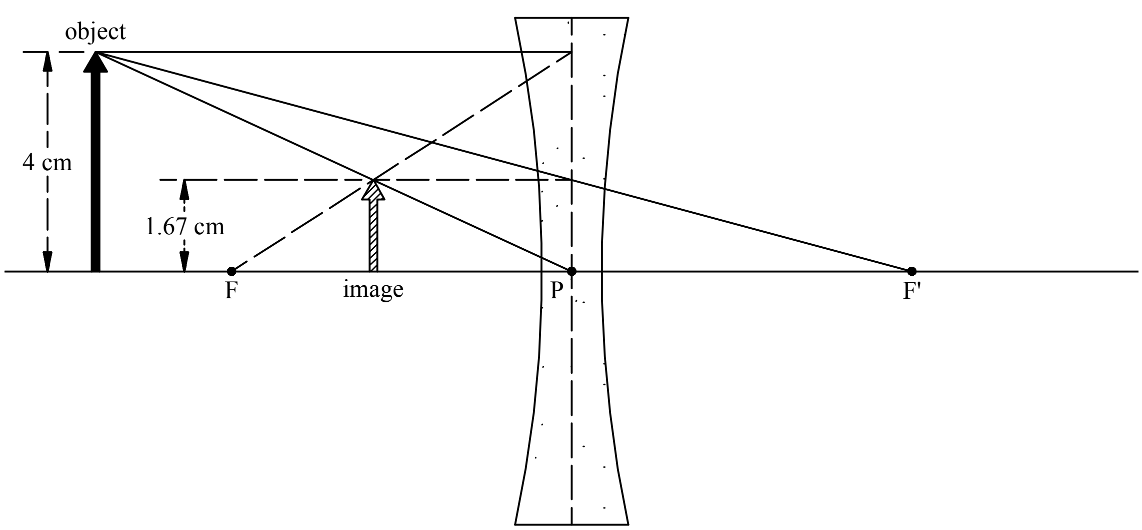

Figure 3.6.8.7

Defects of the Eye

- If an eye forms images before the retina, only nearby objects can be seen clearly.
- **Short-sightedness** or **myopia** is an eye defect allowing one to only view nearby objects clearly.

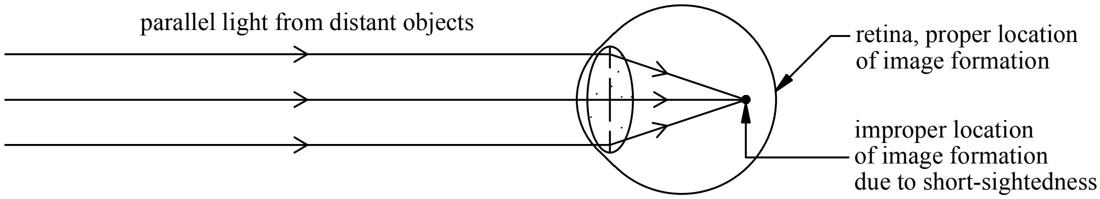

Figure 3.6.8.8

- Short-sightedness can be corrected with spectacles that make uses of a concave, diverging meniscus lens.
  - Light exits the spectacles and enters the eye's lens slightly diverged.
  - It then exits the eye's lens and converges over a greater distance than it would uncorrected.
  - Images are then formed at the retina, allowing for clear vision of distant objects.

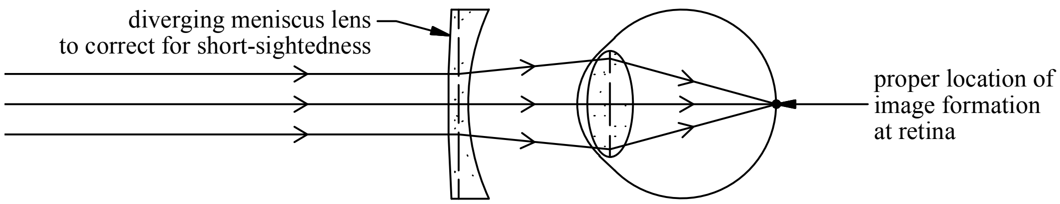

Figure 3.6.8.9

- If an eye forms images after the retina, only distant objects can be seen clearly.
- **Far-sightedness** or **hypermetropia** is an eye defect allowing one to only view distant objects clearly.

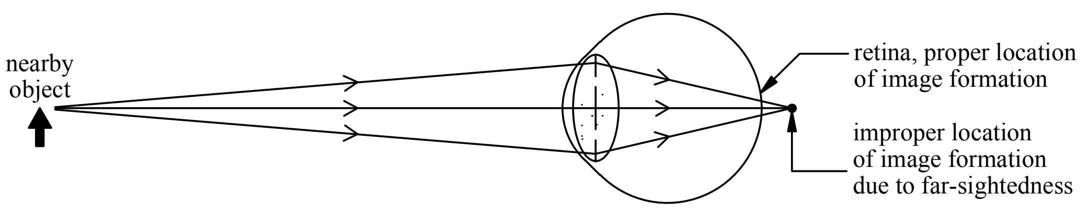

Figure 3.6.8.10

- Far-sightedness can be corrected with spectacles that make uses of a convex, converging meniscus lens.
  - Light exits the spectacles and enters the eye's lens already slightly converged.
  - It then exits the eye's lens and converges over a shorter distance than it would uncorrected.
  - Images are then formed at the retina, allowing for clear vision of distant objects.

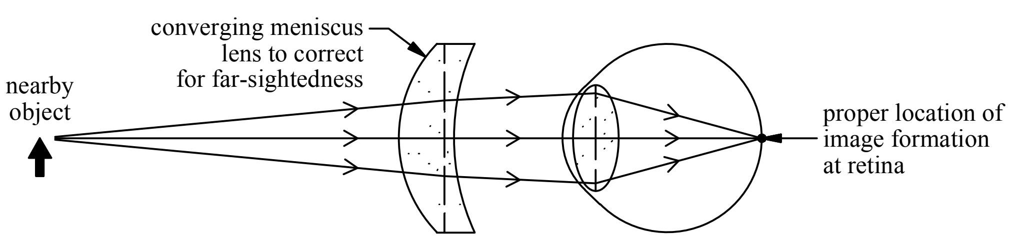

Figure 3.6.8.11

- **Loss of accommodation** or **presbyopia** is an eye defect limiting the focus of nearby or distant objects.
- This condition often occurs at older ages.
- It is most often caused by
  - a weakening of the ciliary muscles;
  - a loss of eye-lens elasticity.
- **Bifocal lens spectacles** have two separate effective lenses and can be used to correct for this defect.
- The upper section is a diverging meniscus which corrects for long-distance viewing.

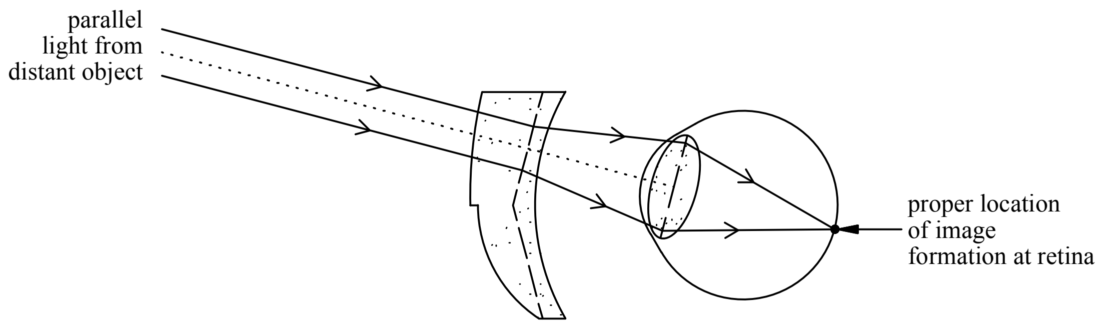

Figure 3.6.8.12

- The lower section is a converging meniscus which corrects for short-distance viewing.

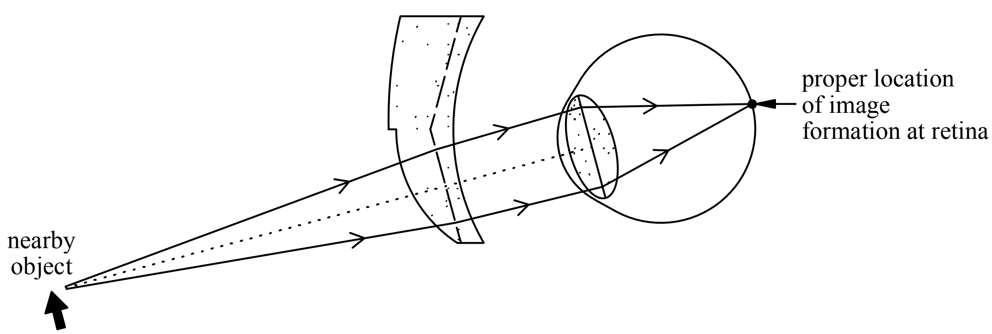

Figure 3.6.8.13

- The property table of images formed with diverging lenses is much more simple than that of converging lenses.

object position	**image properties**			**applications**
	location	**orientation**	**size**	
$0 < u < \infty'$	virtual	upright	diminished	spectacles for short-sightedness bifocal spectacles

Table 3.6.8.1

GCE Paper 1 Questions

1. Figure 3.6.8.14 below shows a ray of light approaching a diverging lens at its optical center.

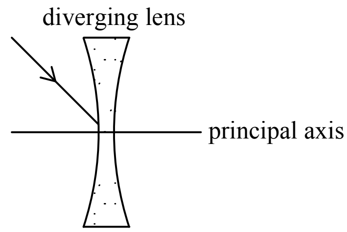

Figure 3.6.8.14

Which of the following diagrams in figure 3.6.8.15 shows the path of light after passing through the lens?

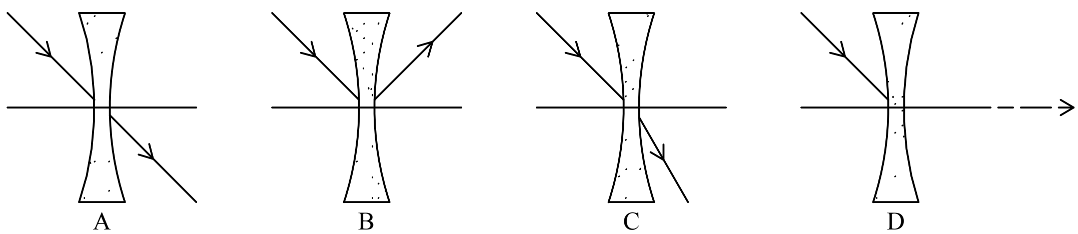

Figure 3.6.8.15

2. The images formed by a concave lens are always

A virtual, upright and diminished.
B real, upright and diminished.
C real, inverted and magnified.
D virtual, inverted and magnified.

3. The images formed by a concave lens are never

A virtual B upright C diminished D magnified

Questions 4 through 6 refer to figure 3.6.8.16 which shows an object placed in from of a diverging lens as well as 4 images *A*, *B*, *C* and *D*.

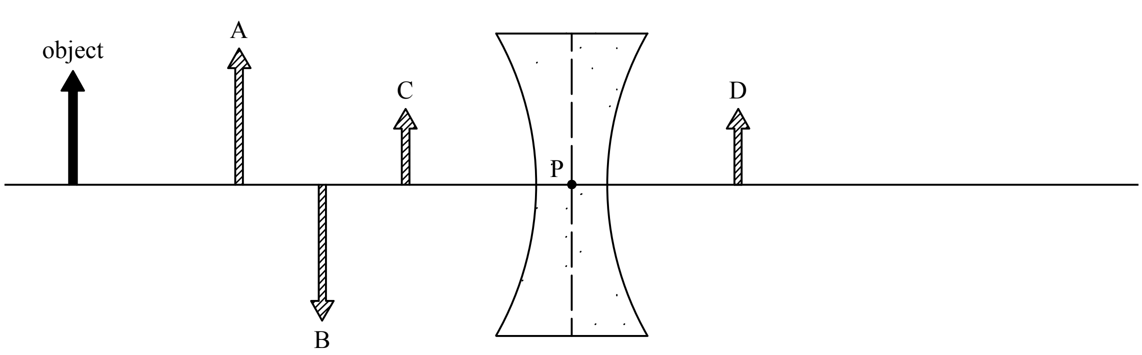

Figure 3.6.8.16

4. Which of these images is possibly formed from the object?

5. Which of these images is inverted?

6. Which of theses images is virtual, upright and magnified?

7. Loss of accommodation in older people means their eye

   A lenses become less elastic.
   B ciliary muscles become too strong.
   C near points are too close to their corneas.
   D abilities go from being near to short sighted.

8. Someone living with hypermetropia has trouble seeing objects that are ___________ without corrective lenses.

   A made in Nigeria
   B far away
   C nearby
   D inverted

Questions 9 through 10 refer to figure 3.6.8.17 which shows for different optical lenses.

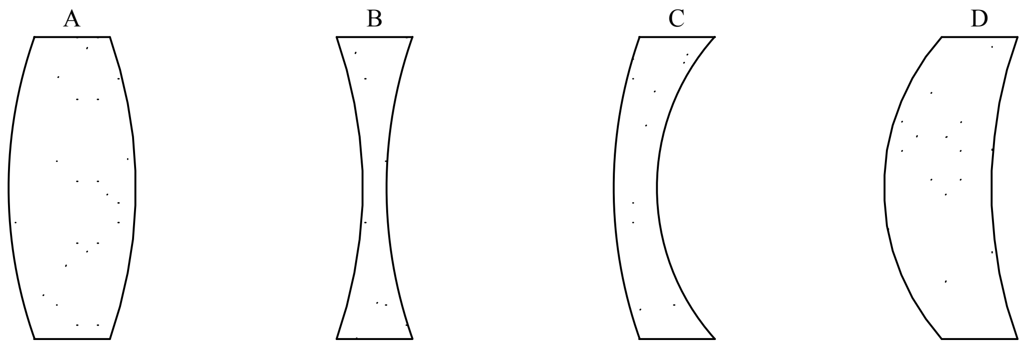

Figure 3.6.8.17

9. Which lens can be used to correct for short-sightedness?

10. Which lens can be used to correct for far-sightedness?

GCE Paper 1 Solutions

1. C 2. A 3. D 4. C 5. B 6. A 7. A 8. C 9. C 10. D

GCE Paper 2 Questions

1. Boris creates a 7 cm tall image by placing a 10 cm tall object 6 cm from the centre plane of a diverging lens.

   (a) Determine the linear magnification of the image. **(2 mks)**
   (b) Determine the lens' focal length. **(3 mks)**
   (c) Determine the distance of the image from lens' centre plane. **(3 mks)**
   (d) Boris moves the object until the image is only half the object's size with the same lens. Determine the object's new distance from the lens' centre. **(3 mks)**
   (e) Determine the image's new distance from the lens' given this new object distance. **(3 mks)**
   (f) On two separate, vertically-aligned ray diagrams, draw the formation of the images corresponding with both the original object position as well as that of (d) and (e) **(4 mks)**

. . . .

Solution

(a) *Given the object is diminished, M should be less than 1.*

$$\text{given equation for image magnification: } M = \frac{h'}{h}$$

$$\text{substituting known values: } M = \frac{7\ cm}{10\ cm}$$

$$\text{final answer: } \boxed{M = 0.7}$$

(b) *The diverging lens' focal length should be negative.*

$$\text{given equation for object-lens distance of virtual images: } u = \frac{f(M-1)}{M}$$

$$\text{turning focla length into subject: } f = \frac{Mu}{M-1}$$

$$\text{substituting known values: } f = \frac{(0.7)(6\ cm)}{0.7-1}$$

$$\text{final answer: } \boxed{f = -14\ cm}$$

(c) *Given the image is virtual, its distance from the lens' centre should be negative.*

$$\text{given equation for image-lens spacing of virtual images: } v = f(1-M)$$

$$\text{substituting known values: } v = (-14\ cm)(1-0.7)$$

$$\text{final answer: } v = \boxed{-4.2\ cm}$$

(d) *The focal length is unchanged while the new linear magnification is* $\frac{1}{2}$.

$$\text{given equation for object-lens spacing of virtual images: } u = \frac{f(M-1)}{M}$$

$$\text{substituting known values: } u = \frac{(-14\ cm)\left(\frac{1}{2}-1\right)}{\frac{1}{2}}$$

$$\text{final answer: } u = \boxed{14\ cm}$$

(e) *The image's distance from the lens should be less than that of the object*

$$\text{given equation for image-lens spacing of virtual images: } v = f(1 - M)$$

$$\text{substituting known values: } v = (-14\ cm)\left(1 - \frac{1}{2}\right)$$

$$\text{final answer: } v = \boxed{-7\ cm}$$

(f) *See figure 3.6.8.18*

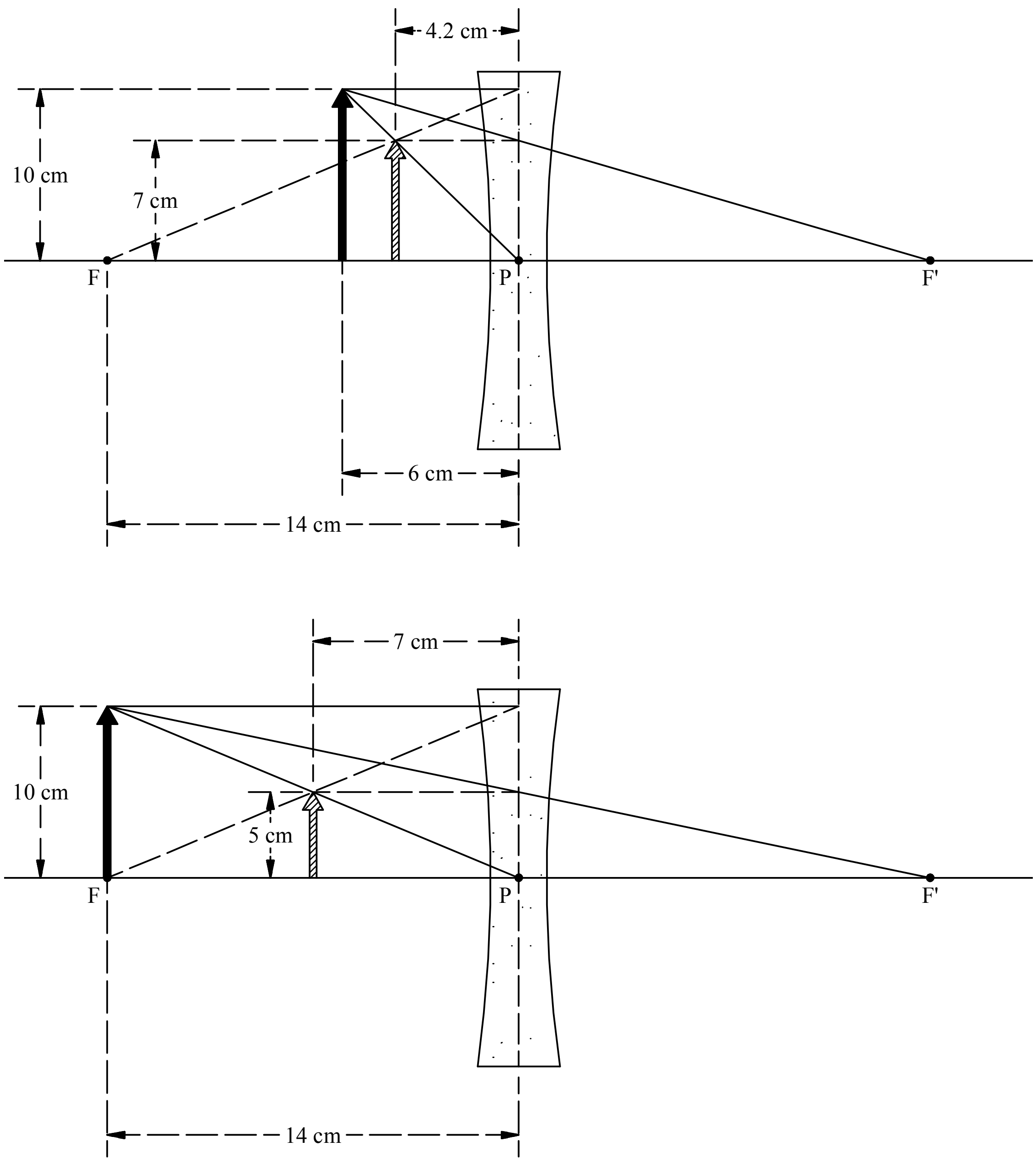

Figure 3.6.8.18

2. While carrying out an experiment using a lens, Nsame collected the following information and calculations for u, the object's distance from the lens' focal plane and v, the resulting image's distance from that same plane.

v/cm	-0.800	-1.333	-1.714	-2.000	-2.222	-2.400
u/cm	1.000	2.000	3.000	4.000	5.000	6.000
$v \times u/cm^2$				-8.000	-11.111	
$v + u/cm$			1.286			

Using this data,

(a) Copy and complete the table. **(4 mks)**
(b) Plot a graph of $v \times u$ along the y-axis against $v + u$ along the x-axis. **(4 mks)**
(c) Determine the gradient of the graph. **(2 mks)**
(d) Explain the gradient's physical significance and account for its negative value. **(2 mks)**

. . . .

Solution

(a) *Solutions in bold*

v/cm	-0.800	-1.333	-1.714	-2.000	-2.222	-2.400
u/cm	1.000	2.000	3.000	4.000	5.000	6.000
$v \times u/cm^2$	**-0.800**	**-2.667**	**-5.143**	-8.000	-11.111	**-14.400**
$v + u/cm$	**0.200**	**0.667**	1.286	**2.000**	2.778	**3.600**

(b) *See figure 3.6.8.19. Note the negative y-axis values.*

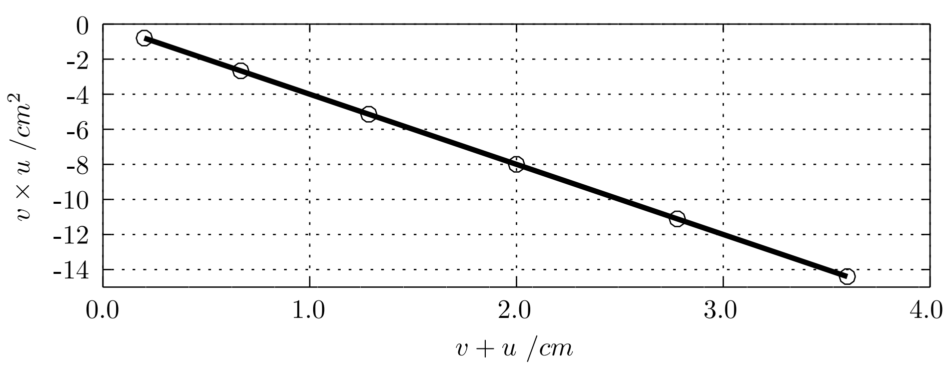

Figure 3.6.8.19

(c) *The gradient, slope, or proportionality is most easily calculated as the difference in the first and last values of $v + u$ per the first and last values of $v \times u$.*

$$\text{calculating gradient:} \quad k = \frac{\Delta(v \times u)}{\Delta(v + u)}$$

$$\text{substituting first and last data points:} \quad k = \frac{\left(-14.400\ cm^2\right) - \left(-0.800\ cm^2\right)}{3.600\ cm - 0.200\ cm}$$

$$\text{final answer:} \quad \boxed{\text{gradient} = k = -4}$$

(d) The slope is the lens' focal length. It is negative because the lens is diverging.

3.6.9 Spherical Mirrors

Objectives

By the end of the lesson, students should be able to

1. describe how images are formed with convex mirrors.
2. describe how images are formed with concave mirrors.
3. state applications of both concave and convex mirrors.
4. investigate the uses of curved mirrors.
5. investigate the uses of plane mirrors.

Curved Mirror Classification

- A **spherical mirror** is a curved mirror which forms part of a sphere.
- A **concave spherical mirror** is a mirror that forms part of a hollow sphere whose interior surface is reflective.

Figure 3.6.9.1

- A **convex spherical mirror** is a mirror that forms part of a hollow sphere whose exterior surface is reflective.

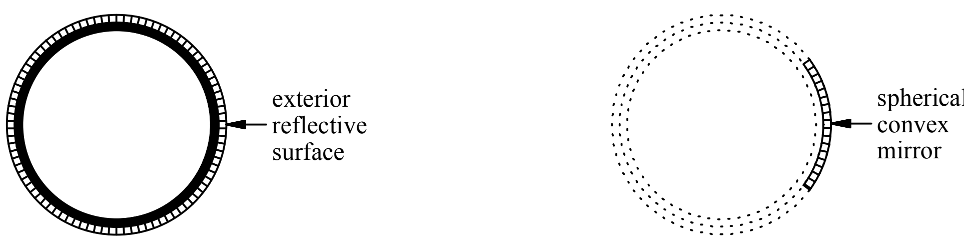

Figure 3.6.9.2

Spherical Concave Mirrors as Multiple Plane Mirrors

- A concave spherical mirror can be thought of as several plane, reflecting surfaces.
- Each mirror is angled increasingly inward with increasing distance from the middle mirror.
- This causes a parallel beam to reflect back inwards.
- If the mirrors' angles have a certain proportion, all reflected rays converge at a single point of focus.

Figure 3.6.9.3

Spherical Mirror Terminology

- The **pole**, or **P**, of a spherical mirror is the centre of its reflecting surface.
- The **centre of curvature**, or **C**, of a spherical mirror is the centre of the sphere formed partially by the mirror.
- The **principal axis** of a spherical mirror is the line passing through its pole as well as its centre of curvature.
- The **radius of curvature**, or **r**, of a spherical mirror is the radius of the sphere formed partially by the mirror.
- The **principal focus**, or **F** of a spherical mirror is the point at which all reflected rays converge.
- The **focal length**, or **f** of a spherical mirror is the distance between its pole and its principal focus.
- The **aperture** of a spherical mirror is the diameter of the circular area off which light is reflected.

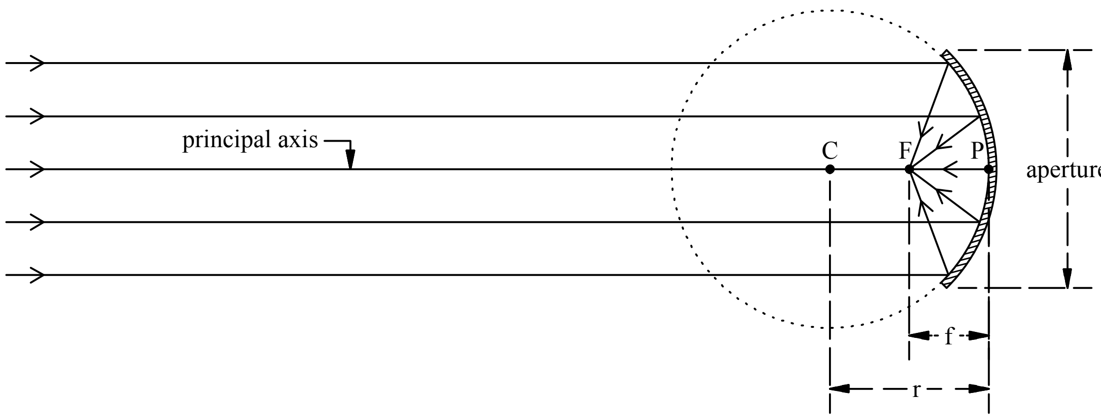

Figure 3.6.9.4

Real and Virtual Images with Spherical Mirrors

- As opposed to images formed with lenses, images formed with spherical mirrors are
  - real on the same side as the object;
  - Virtual on the opposite side as the object.

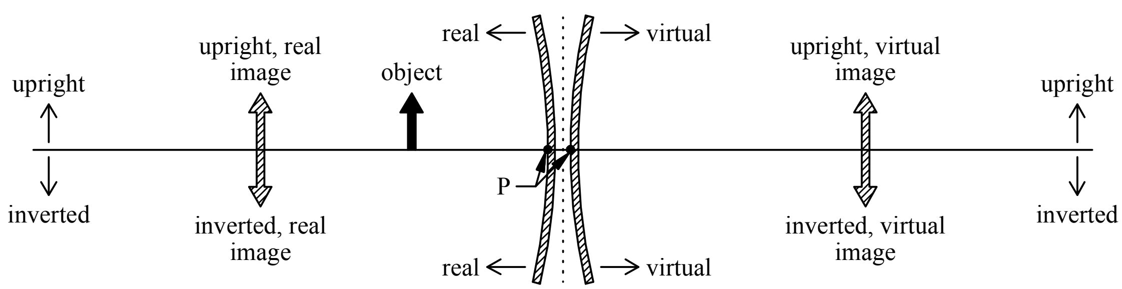

Figure 3.6.9.5

Radius of Curvature and Focal Length

- The focal length of a spherical mirror is half its radius of curvature.

$$f = \frac{r}{2} \tag{3.6.9.1}$$

Where
- f is the spherical mirror's focal length, in m;
- r is the spherical mirror's radius of curvature, in m.

Ray Diagrams with Spherical Concave Mirrors

- A spherical concave mirror's ray diagram involves its principal focus as well as its centre of curvature.

  1 The first ray is drawn
    - connecting the object's top and the mirror through the principal focus;
    - returning from this point on the mirror as line parallel to the principal axis.

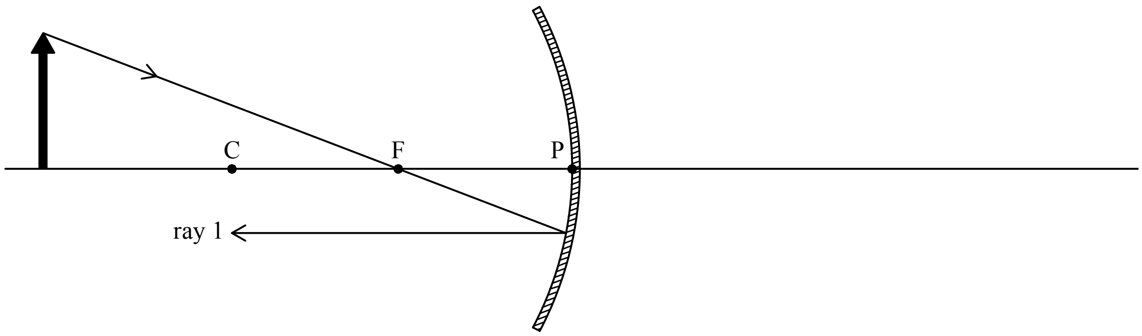

Figure 3.6.9.6

  2 The second ray is drawn
    - connecting the object's top and the mirror as a line parallel to the principal axis;
    - returning from this point on the mirror through the principal focus.

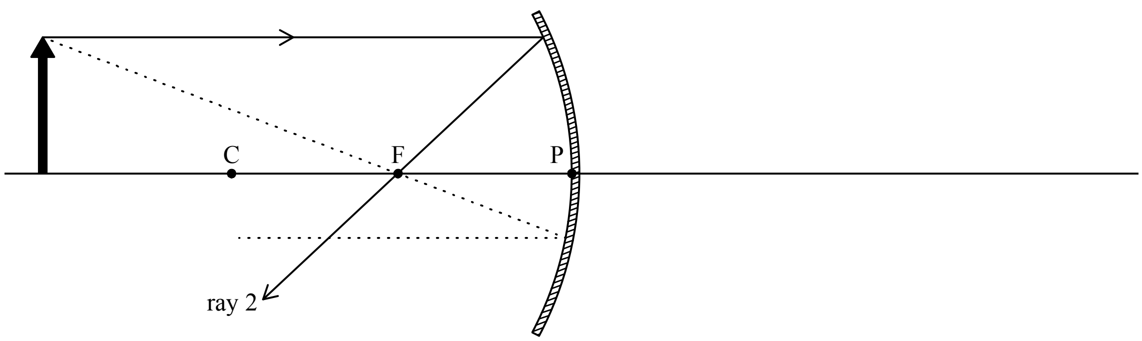

Figure 3.6.9.7

  3 The third ray is drawn
    - passing onward from the object's top through the centre of curvature.

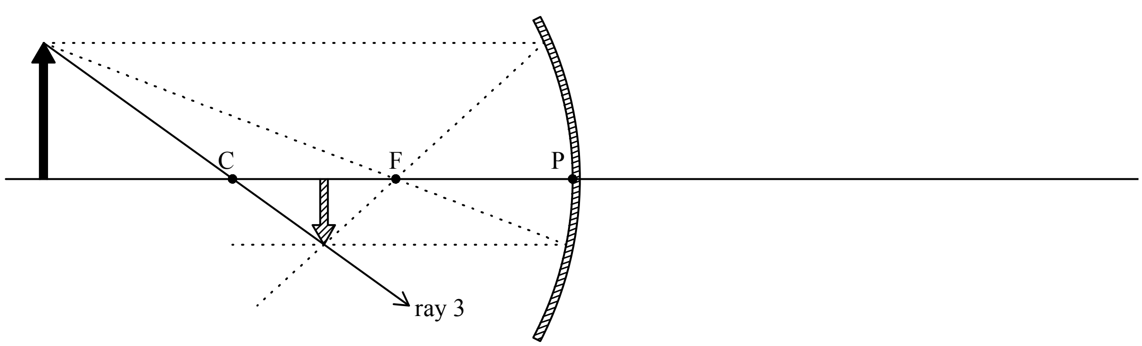

Figure 3.6.9.8

- The image is perpendicular to the principal axis, terminating at the intersection point of the three rays.

Image Formation with Spherical Concave Mirrors

- An object placed **between F and P** forms a virtual, upright, magnified image.
  - This is used for personal-viewing in shaving and make-up mirrors.

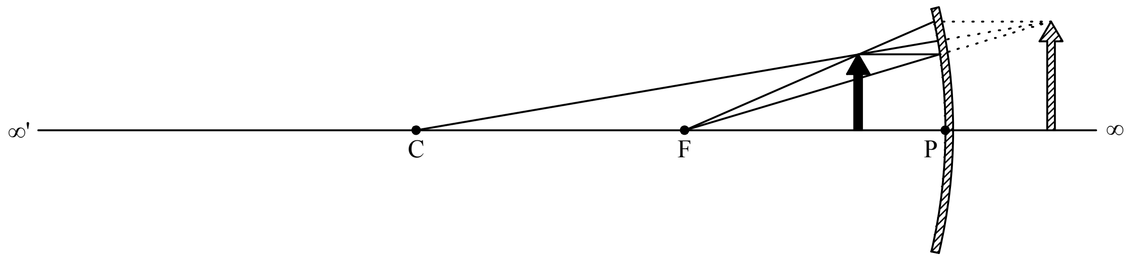

Figure 3.6.9.9

- An object placed **at F** forms no image.
  - The rays are reflected from the mirror as a parallel beam.
  - Many devices use this arrangement to produce a parallel light beam by placing a light source at F.
  - Solar cookers use this arrangement to converge incoming light at a single point of focus.
  - Satellite dishes use this arrangement to converge incoming electromagnetic signals onto a single receiver.

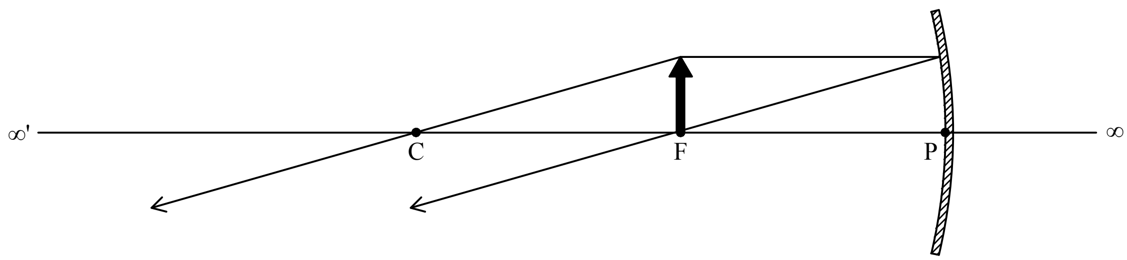

Figure 3.6.9.10

- An object placed **between C and F** forms a real, inverted, magnified image.

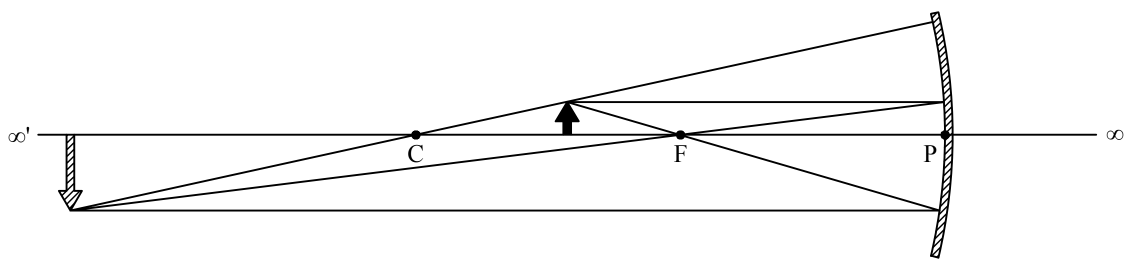

Figure 3.6.9.11

- An object placed **at C** forms a real, inverted image of the same size as the object.

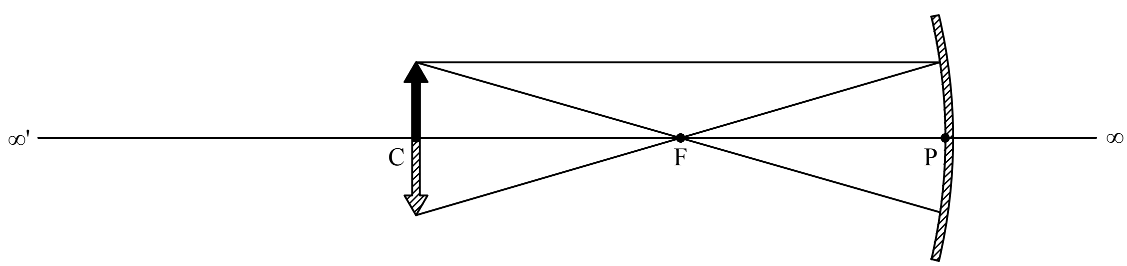

Figure 3.6.9.12

- An object placed **before C** forms a real, inverted, diminished image.

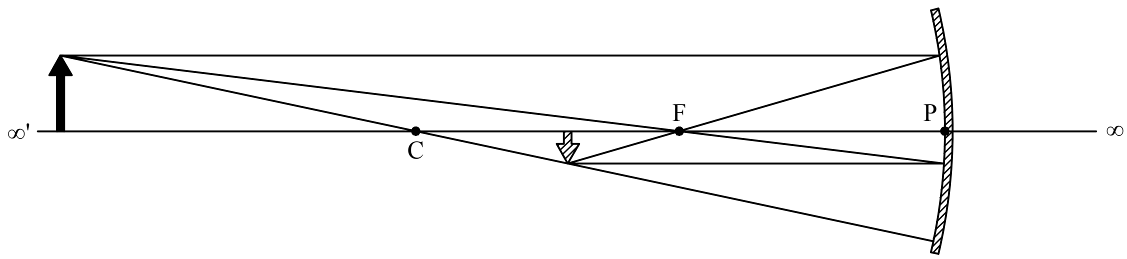

Figure 3.6.9.13

- The property table of images formed with concave mirrors is as follows.

object position	**image properties**			**applications**
	location	**orientation**	**size**	
$u < f$	virtual	upright	magnified	magnifying cosmetic mirrors
$u = f$	-	-	-	parallel beams from search lights parallel beams from vehicle headlights parallel beams from torches light concentration by solar cookers signal concentration by satellite dishes
$f < u < C$	real	inverted	magnified	-
$u = C$	real	inverted	same	-
$u > C$	real	inverted	diminished	-

Table 3.6.9.1

Spherical Convex Mirrors as Multiple Plane Mirrors

- A convex spherical mirror can be thought of as several plane, reflecting surfaces.
- Each mirror is angled increasingly outward with increasing distance from the middle mirror.
- This causes an incident parallel beam to diverge after reflection.
- If the mirrors' angles have a certain proportion, these rays diverge as if from a single point of focus.

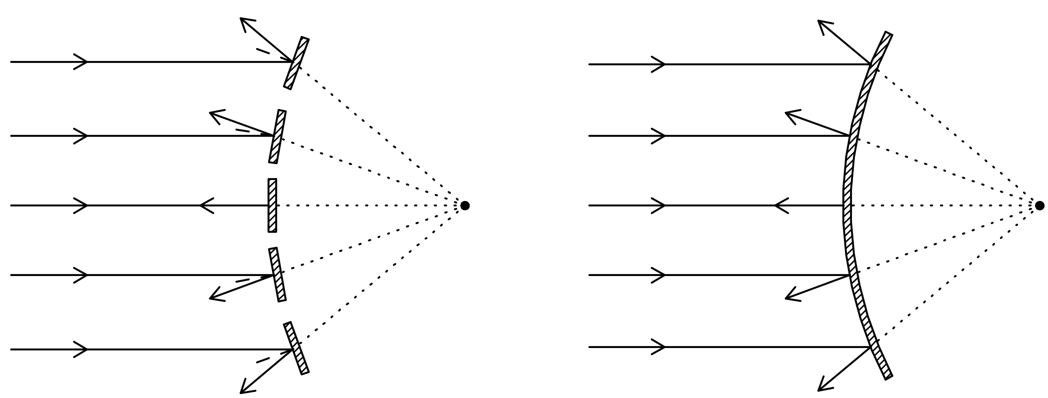

Figure 3.6.9.14

Ray Diagrams with Spherical Convex Mirrors

- A spherical convex mirror's ray diagram involves its principal focus as well as its centre of curvature.

  1 The first ray is drawn
    - connecting the object's top to the principal focus through the mirror;
    - from the point where this ray passes through the mirror onward as a line parallel to the principal axis.

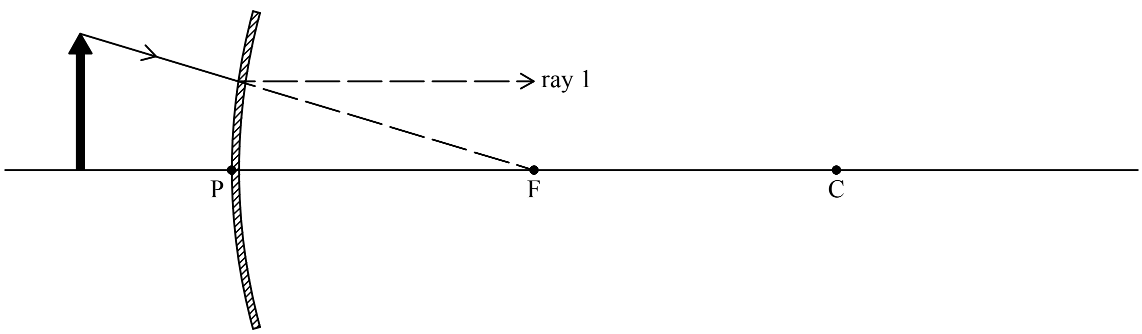

Figure 3.6.9.15

  2 The second ray is drawn
    - connecting the object's top to the mirror as a line parallel to the principal axis;
    - from this point on the mirror onward through the principal focus.

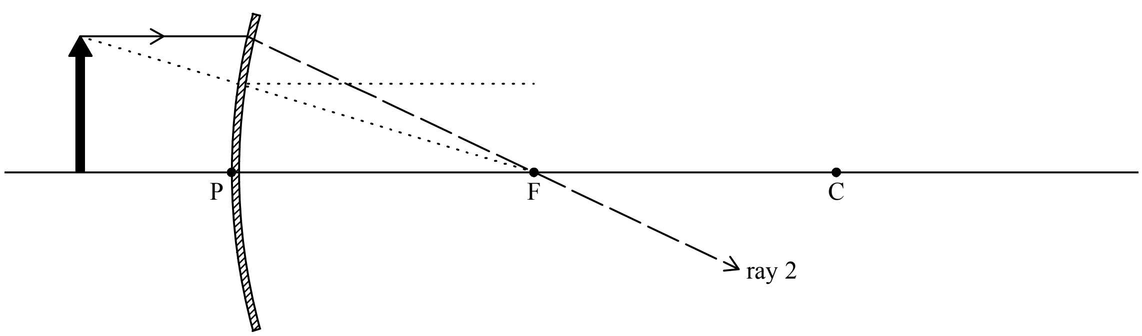

Figure 3.6.9.16

  3 The third ray is drawn
    - connecting the object's top to the mirror's centre of curvature and onward through the mirror.

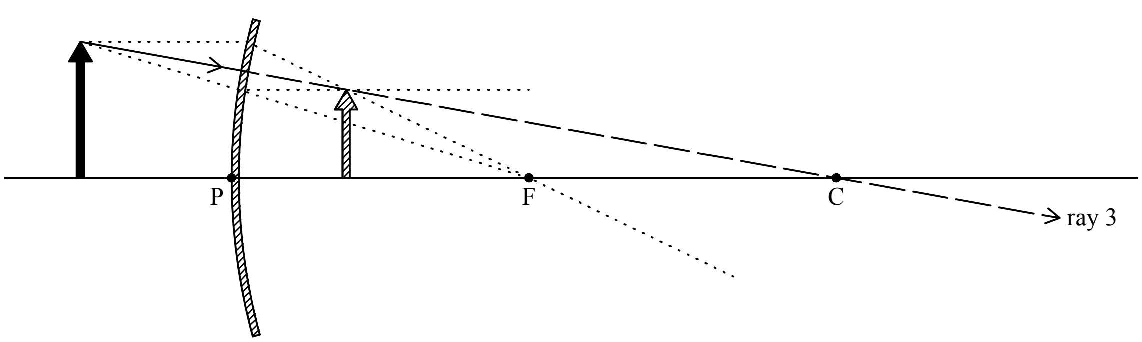

Figure 3.6.9.17

- The image is perpendicular to the principal axis, terminating at the intersection point of the three rays.

Image Formation with Convex Mirrors

- A convex mirror forms a virtual, upright, diminished object, regardless of the object-mirror distance.
- This is the case for $u < f$, $u = f$, $f < u < C$, $u = C$ and $u > C$.

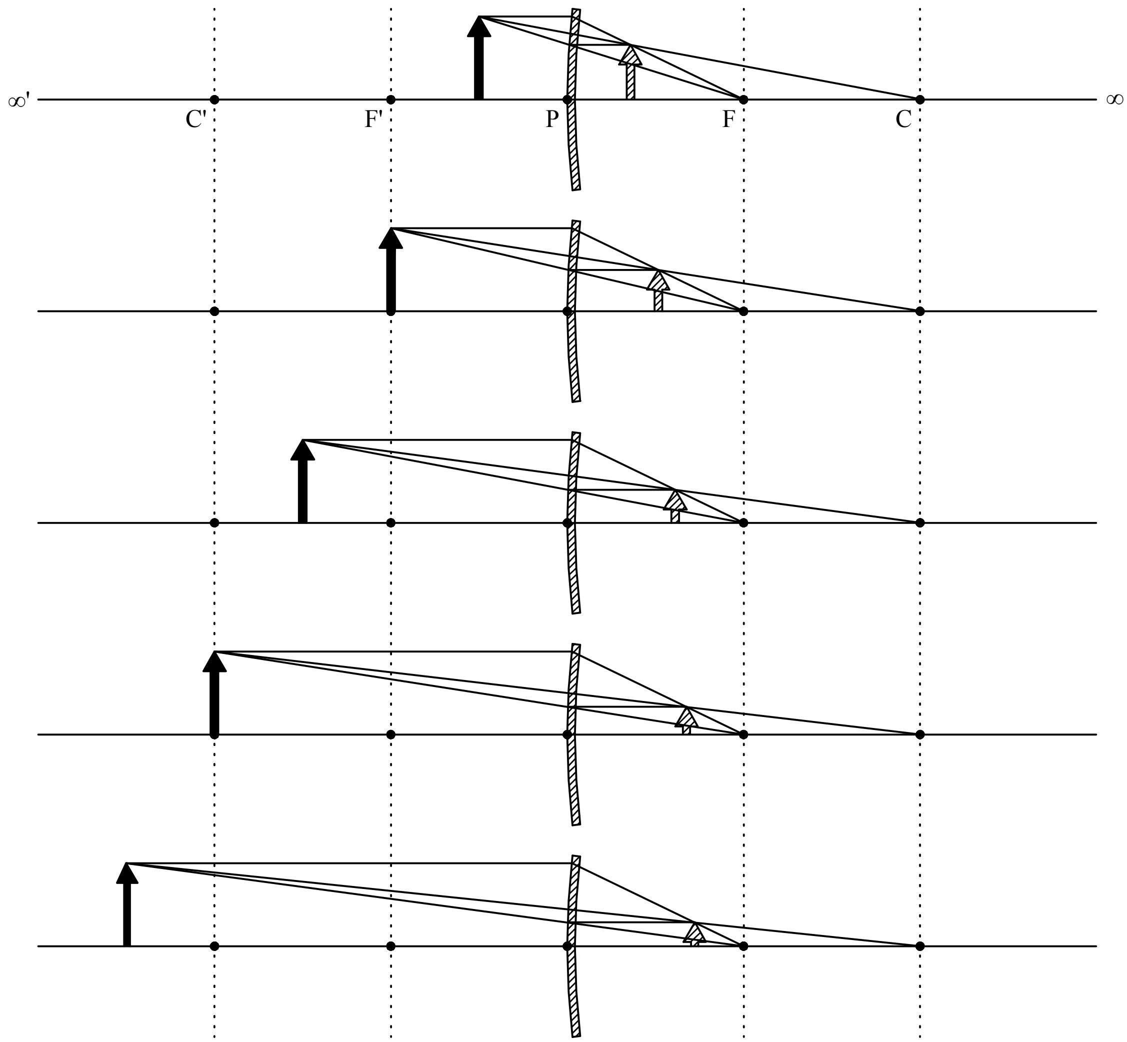

Figure 3.6.9.18

- Convex mirrors are used as vehicle rear-view mirrors because they allow the driver to see a wider field of view.

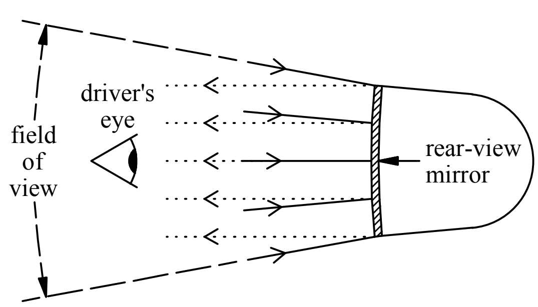

Figure 3.6.9.19

- This application has the disadvantage of objects appearing further away than they actually are given that their image is diminished by the mirror.

- The property table of images formed with convex mirrors is much more simple than that of concave mirrors.

object position	**image properties**			**applications**
	location	**orientation**	**size**	
$0 < u < \infty'$	virtual	upright	diminished	spectacles for short-sightedness bifocal spectacles

Table 3.6.9.2

Image Quantity with Multiple Inclined Mirrors

- When two mirrors are angled towards each other, an object placed between them is reflected multiple times.
  - The object itself is reflected across each mirror.
  - The the object's image in one mirror is reflected in the other mirror and vice versa.
  - Those reflected images are also reflected back across the mirrors.

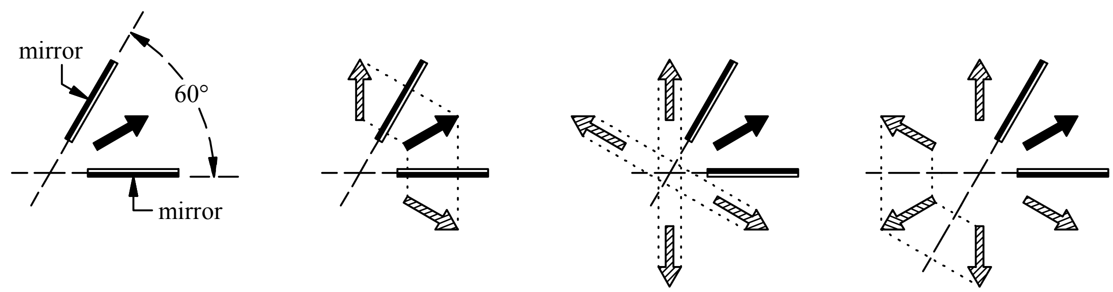

Figure 3.6.9.20

- The quantity of images formed in such an arrangement depends only on the angle between the mirrors.

$$N = \frac{360°}{\theta} - 1 \tag{3.6.9.2}$$

Where
  - N is the quantity of formed images;
  - θ is the angle between the mirrors, in degrees.

- This allows for image quantity to be tabulated for specific inter-mirror angles.

θ /°	90	72	60	45	40
N	3	4	5	7	8

Table 3.6.9.3

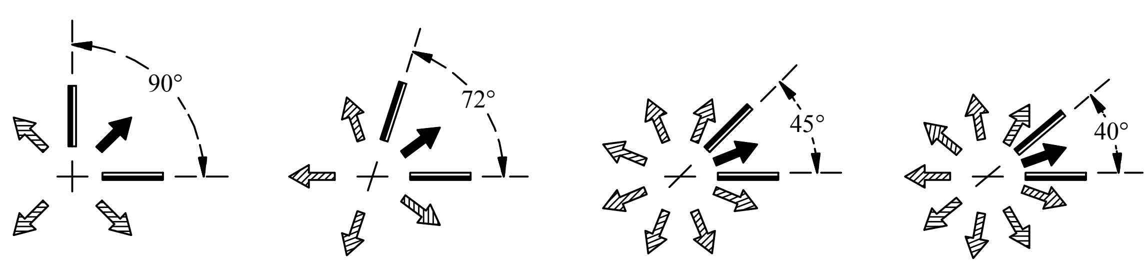

Figure 3.6.9.21

GCE Paper 1 Questions

1. When the image formed by a concave mirror of focal length f is real, inverted and magnified, the location of the object is

   A between ∞' and C. B exactly at C. C between C and F. D between ∞' and F.

2. The radius of curvature of a concave mirror is 15 cm. The focal length of the mirror is

   A 7.5 cm B 9.5 cm C 15 cm D 30 cm

3. Which of the following devices allows someone to see a magnified image of their own eye?

   A a converging, concave mirror
   B a torch
   C a diverging, convex mirror
   D a plane mirror

4. Convex mirrors are often used as driving mirrors because they

   A form erect images.
   B are magnified.
   C are cheaper.
   D provide a wider angle of view.

5. The image formed by a convex mirror is always

   A real, inverted and diminished.
   B virtual, erect and diminished.
   C virtual, inverted and diminished.
   D real, erect and diminished.

6. An object is placed 5 cm from a concave mirror of focal length 10 cm. The image formed is

   A real, inverted and magnified
   B real, upright and diminished
   C virtual, inverted and diminished
   D virtual, upright and magnified

7. When an object is placed 22 cm from a convex mirror of focal length 11 cm, the image formed is

   A virtual, upright and the same size as the object
   B virtual, inverted and the same size as the object
   C virtual, upright and diminished
   D virtual, upright and magnified

8. A virtual image formed by a plane mirror is an image which

   A has no actual light rays passing through it.
   B can only be viewed on a computer.
   C is inverted.
   D is formed at a smaller size than the real object.

9. Two plane mirrors are angled towards each other such that they form 3 images of an object placed between them. The mirrors are at what angle?

   A 60° B 90° C 120° D 180°

10. When an object is placed beyond the centre of curvature of a concave mirror, the nature of the image is

    A real, inverted, and larger than the object.
    B virtual, upright and larger than the object.
    C real, inverted and smaller than the object.
    D virtual, upright, and smaller than the object.

GCE Paper 1 Solutions

1. C 2. A 3. A 4. D 5. B 6. D 7. C 8. A 9. B 10. C

GCE Paper 2 Questions

1. A girl stands 2 m in front of a large mirror.

   Draw a ray diagram to show the formation of the girl's image if the mirror is

   (a) plane. **(2 mks)**

   (b) concave with a focal length of 4 m. **(3 mks)**

   (c) convex with a focal length of 2 m. **(3 mks)**

   ....

Solution

(a) *See figure 3.6.9.22*

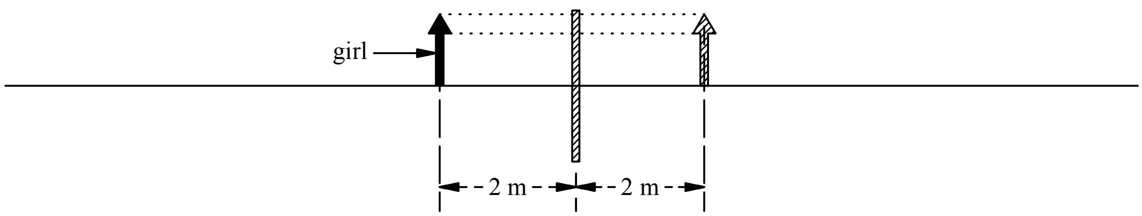

Figure 3.6.9.22

(b) *See figure 3.6.9.23*

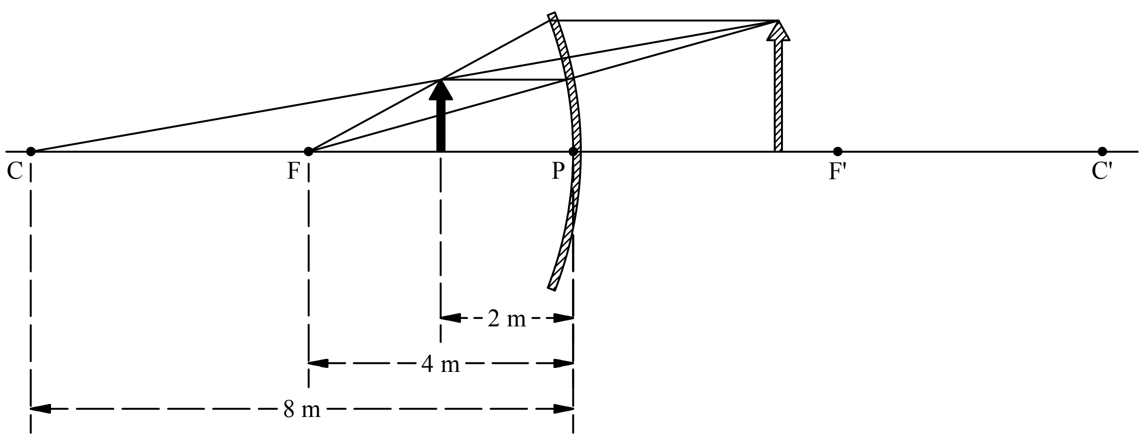

Figure 3.6.9.23

(c) *See figure 3.6.9.24*

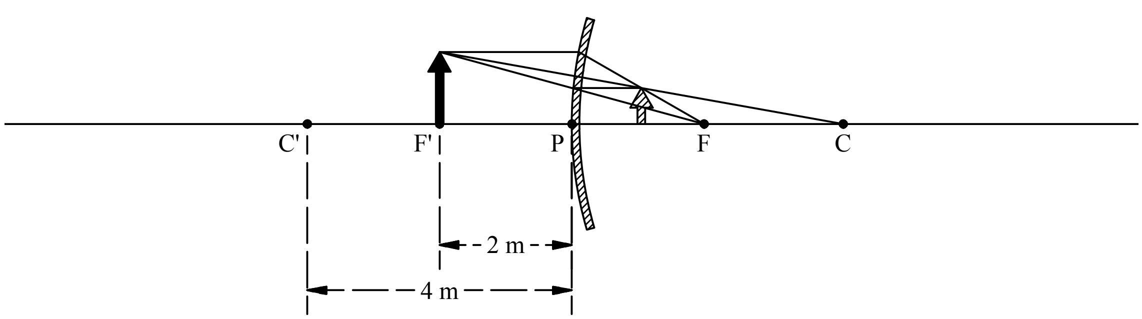

Figure 3.6.9.24

2. Figure 3.6.9.25 shows an object AB as well as a chart of centre C placed in front of a plane mirror.

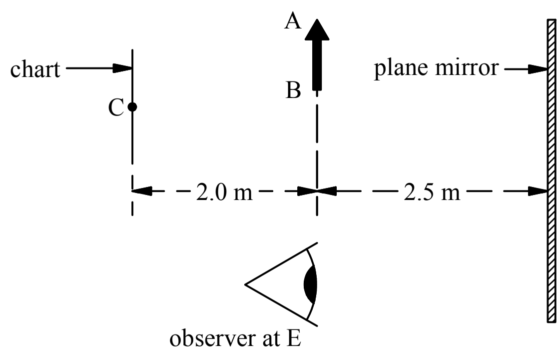

Figure 3.6.9.25

(a) Draw a ray diagram to show how the observer sees the image of AB as well as that of C in the mirror. Include relevant labels. **(4 mks)**

(b) Calculate the distance between the object AB and the image of C in the mirror. **(3 mks)**

(c) A friend bends the mirror slightly such that it forms part of a sphere whose radius of curvature is 2 m. The observer continues to see a virtual, upright image of both AB and C. Is the friend bending the mirror in a concave or convex manner? Explain. **(3 mks)**

. . . .

Solution

(a) *See figure 3.6.9.26*

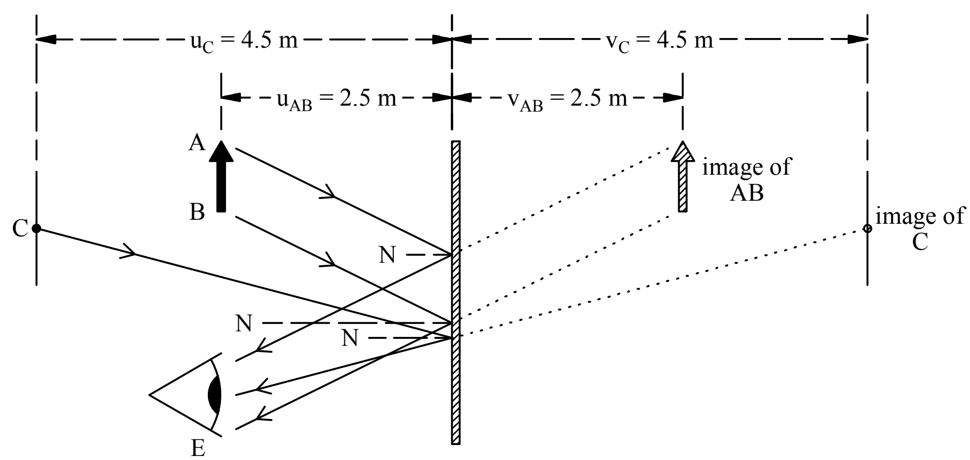

Figure 3.6.9.26

(b) *The distances concerned are shown in figure 3.6.9.26.*

$$\text{summing distances: } d_{\text{object } AB\rightarrow\text{image } C} = u_{AB} + v_C$$

$$\text{substituting known values: } d_{\text{object } AB\rightarrow\text{image } C} = 2.5\ m + 4.5\ m$$

$$\text{final answer: } \boxed{d_{\text{object } AB\rightarrow\text{image } C} = 7\ m}$$

(c) The mirror is being bent in a convex manner. This is because both AB and C form virtual, upright images while being located further from the mirror than its radius of curvature. If the mirror were bent in a concave manner, both AB and C at these positions would form real, inverted images.

3.6.10 Dispersion and Colours

Objectives

By the end of the lesson, students should be able to

1. define dispersion.
2. explain the dispersion of white light.
3. name and explain physical phenomena caused by dispersion of white light.
4. describe and explain how a pure spectrum can be obtained from white light.
5. discuss colours and their combination.

White Light and Visible Colours

- **White light** is visible light composed of all visible colours combined.
- While white light is composed of an infinite quantity of visible colours, seven are often distinguished.

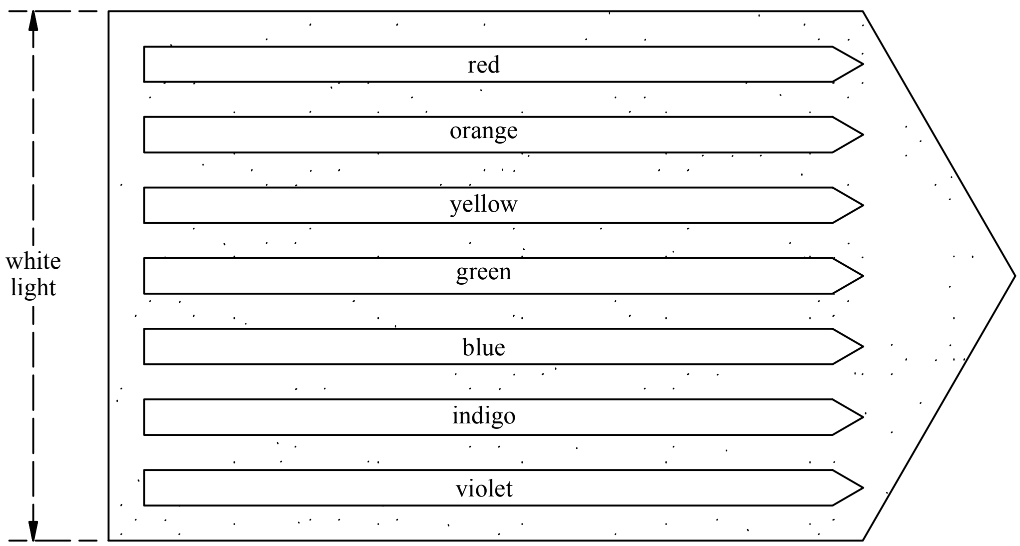

Figure 3.6.10.1

- NB: **r**espectable **o**perators **y**ell "**g**o" **b**efore **i**gniting **v**apours

Refraction of White Light

- When visible light is incident at an angle on a transparent media boundary, the angle of refraction depends
  - primarily on the ratio of the absolute refractive indices of both media involved;
  - secondly on the wavelength/frequency (colour) of each wave that composes the white light.
- That is, during refraction of white light
  - the entire beam is refracted significantly as a whole;
  - the deviation of any one colour varies slightly from colour to colour.
- Colours close to red are deviated the least while those close to violet are deviated the most.

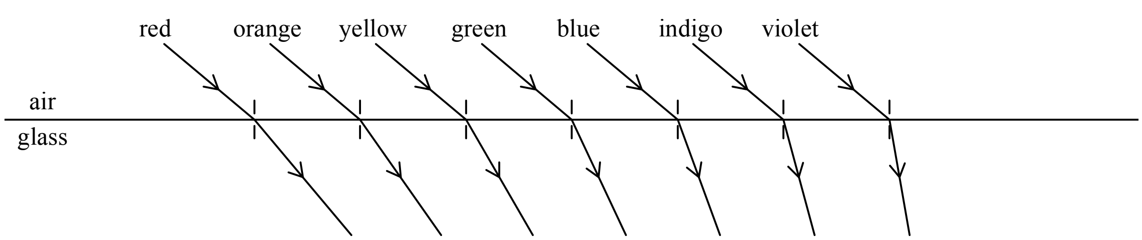

Figure 3.6.10.2

Dispersion of White Light

- **Dispersion of white light** is the separation of white light light into its component colors.
- Dispersion occurs because the different colours of white light travel at different speeds in a refracting medium.

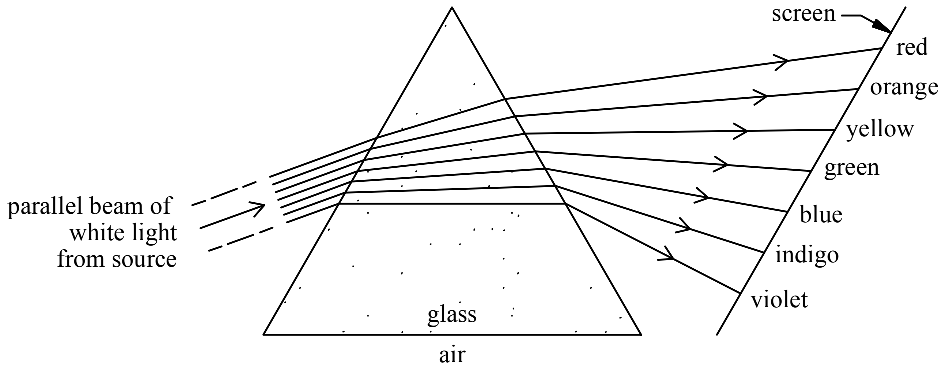

Figure 3.6.10.3

- As white light enters an equilateral triangular glass prism at a non-90° angle of incidence,
  - the more red colours are refracted the least;
  - the more violet colours are refracted the most;
  - light emerges from the prism as a light spectrum.
- A **light spectrum** is a continuum or range of varying colours.
- NB: The plural of "spectrum" is "spectra".
- Dispersion only occurs when light enters a refracting medium at a non-normal or non-90° angle of incidence.
- Examples of light dispersion include
  - the shimmering of light in diamonds
  - rainbows as light is dispersed through rain droplets in the sky
- An **impure light spectrum** is one in which colours overlap.
- An impure light spectrum is produced when only an equilateral triangular prism is used for dispersion.
- A **pure light spectrum** is one in which colours are clearly separated and distinct from each other.
- A pure light spectrum can be produced with
  - a source of white light;
  - two converging (biconvex) lenses;
  - one equilateral triangular glass prism;
  - a blank screen.

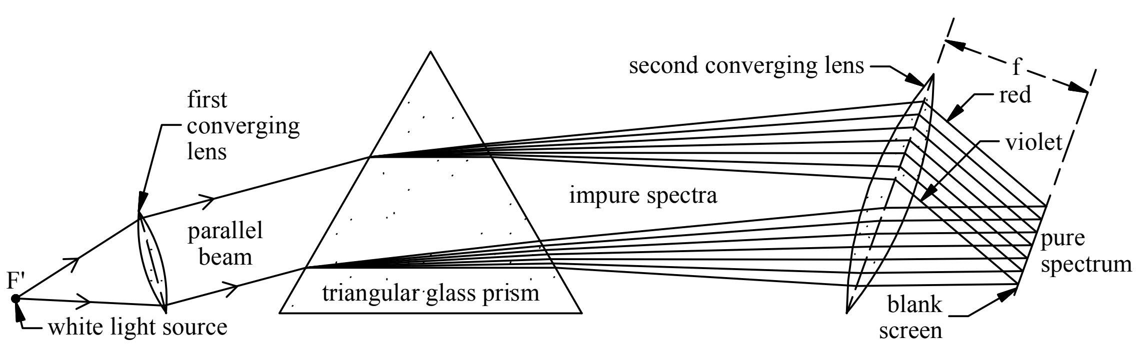

Figure 3.6.10.4

- In order to produce a pure spectrum, white light
  - starts from the source located at the first lens' principal focus;
  - emerges from the first lens as a parallel beam;
  - enters the prism and is dispersed when refracted;
  - emerges from the prism as an impure spectra;
  - is converged by the second converging lens;
  - appears as a pure spectrum on the blank screen located at the second lens' principal focus

Combining Colours of Light

- A **primary colour of light** is a light colour that cannot be made by mixing together any other light colours.
- Primary light colours include red, green and blue.
- The human eye can only truly sense colours of light similar to red, green and blue.
- All other light colours are perceived by the eye as some combination of primary light colours.

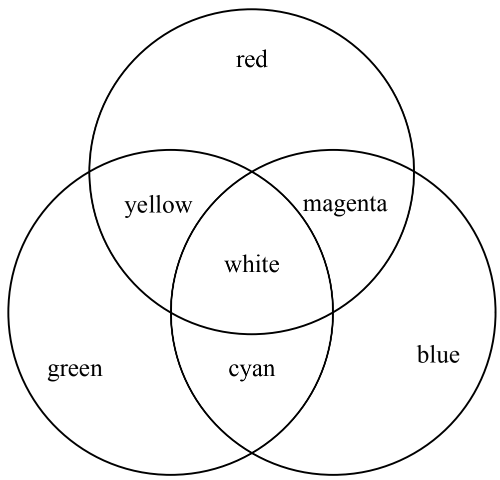

Figure 3.6.10.5

- A **secondary colour of light** is any light colour made by the combination of two primary colours.
- Secondary light colours include yellow, cyan and magenta.

$$\text{red} + \text{green} = \text{yellow}$$
$$\text{red} + \text{blue} = \text{magenta}$$
$$\text{green} + \text{blue} = \text{cyan}$$

- NB: "Peacock blue" and "turquoise" are other names for cyan.
- The mixing of all primary colours of light creates the perception of white light to the eye.

$$\text{red} + \text{green} + \text{blue} = \text{white}$$

- The perception of white light can also be created by mixing any two complimentary light colours.
- **Complimentary colours of light** are two light colours that combine to make the perception of white light.
- A complimentary light colour pair always includes one primary light colour and one secondary light colour.

$$\text{red} + \text{cyan} = \text{white}$$
$$\text{green} + \text{magenta} = \text{white}$$
$$\text{blue} + \text{yellow} = \text{white}$$

Appearance of Colours by Subtraction of Light

- The human eye sees an object's colour only as the the light colour that it reflects and doesnt absorb.
  - The eye sees a red object in white light after it absorbs green and blue and reflects red.

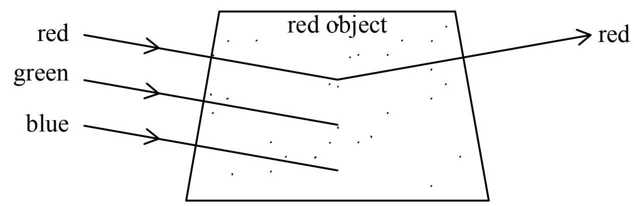

Figure 3.6.10.6

  - The eye sees a green object in white light after it absorbs red and blue and reflects green.

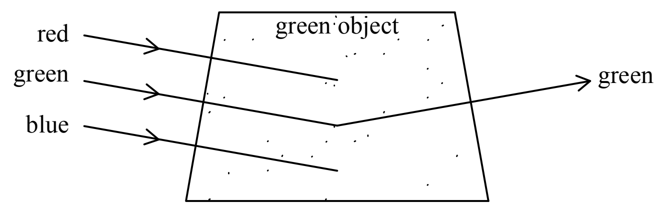

Figure 3.6.10.7

  - The eye sees a yellow object in white light after it absorbs blue and reflects a mixture of red and green.

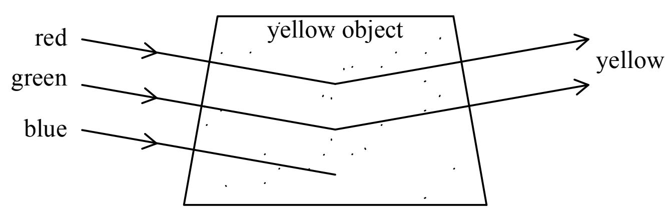

Figure 3.6.10.8

  - A black object in white light absorbs all colours and reflects no light.

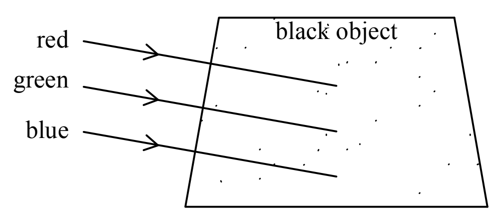

Figure 3.6.10.9

  - A white object in white light reflects all colours.

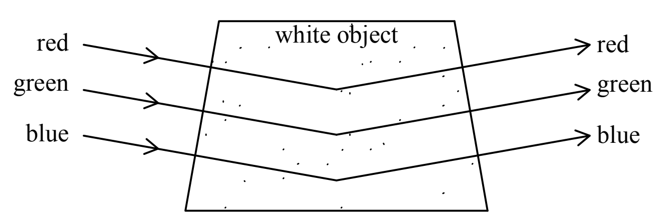

Figure 3.6.10.10

Colour Light Filters

- **Colour light filters** are sheets of plastic or glass which only allow light of certain colours to pass through.
- Colour light filters are opaque to some colours and transparent to others.
- A colour light filter is named according to the colour to which its transparent.

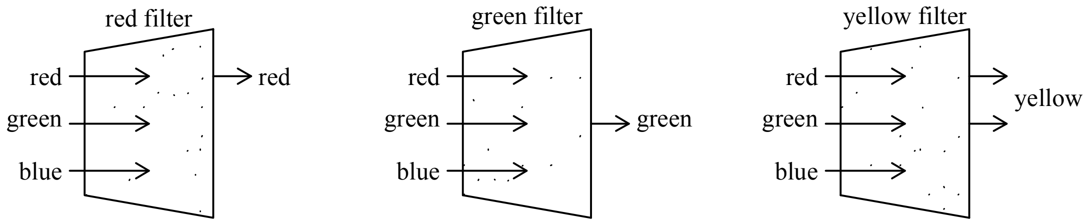

Figure 3.6.10.11

- NB: While neither terms are commonly used, a true “black filter” would be a fully opaque object while a true “white filter” would be a fully transparent object.
- The colour of white light viewed through multiple filters appears only as the colour that can pass through each.
- White light viewed through a yellow filter first and a cyan filter second is seen as green.

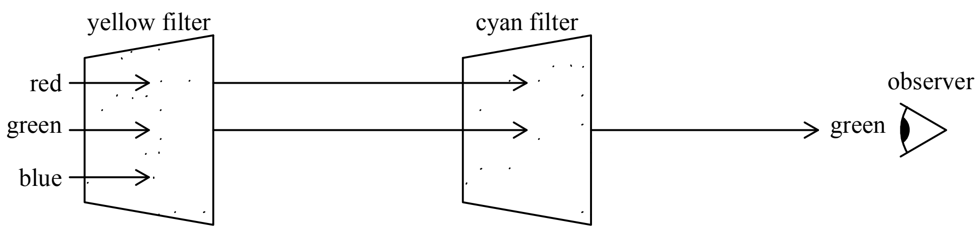

Figure 3.6.10.12

- In reverse, white light viewed through a cyan filter first and a yellow filter second is also seen as green.

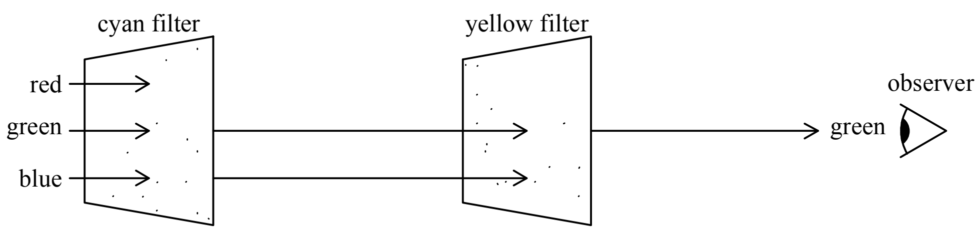

Figure 3.6.10.13

- Any light capable of passing through a cyan filter first is unable to also pass through a red filter.

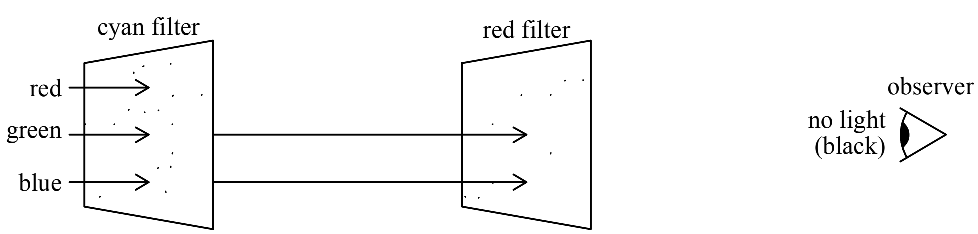

Figure 3.6.10.14

- Therefore, if two filters are of complimentary colours, white light will appear black when viewed through both.

- If white light that has been filtered down to a particular colour or mix of colours happens to strike an object, the object will absorb all colours except its own and reflect any remaining colours.
- If red and green light passing through a yellow filter strikes a green object, only green light is finally reflected.
  - Thus, a green object appears green in yellow (red and green) light.

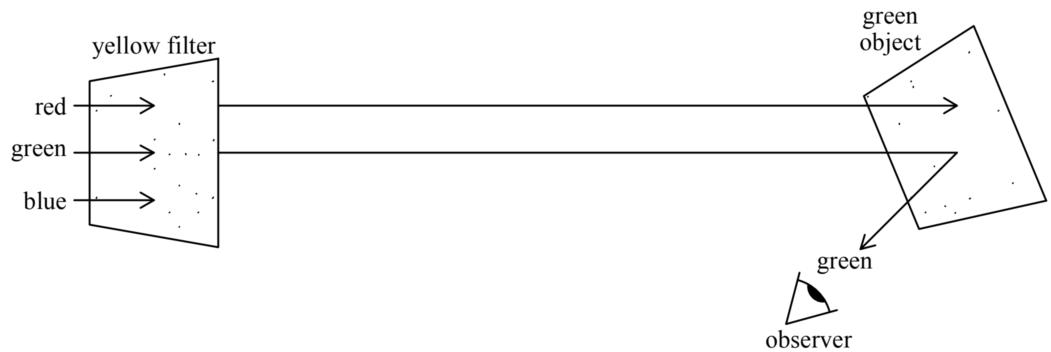

Figure 3.6.10.15

- If red and green light passing through a yellow filter strikes a cyan object, it is again only green that is reflected.
  - Thus, a cyan object appears green in yellow (red and green) light.

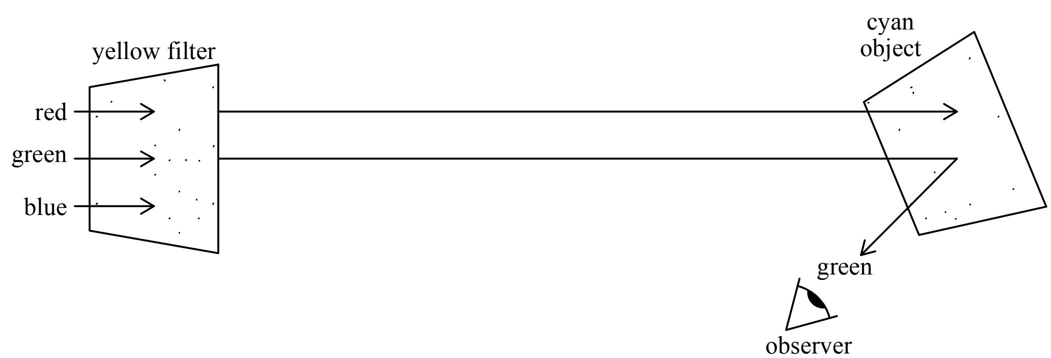

Figure 3.6.10.16

- If red and green light passing through a yellow filter strikes a blue object, there is no blue light to reflect.
  - Thus, a blue object appears black in yellow (red and green) light.

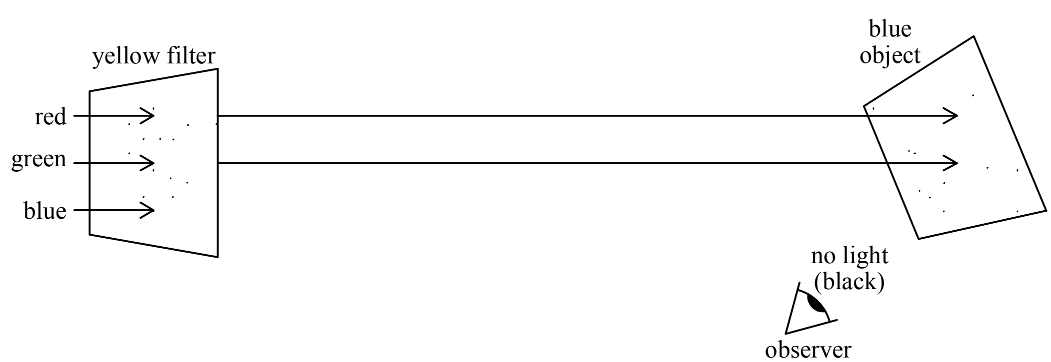

Figure 3.6.10.17

- Therefore, if a filter and an object are of complimentary colours, white light will appear black when passing through one and reflecting off the other, or vice versa.

GCE Paper 1 Questions

1. White light is not dispersed when passing through the prisms of prism binoculars because

   A dispersion and lateral vision don't occur together.
   B the light enters the prisms normally.
   C virtual images cannot be coloured.
   D two sets of prisms at right angles are used.

2. Which behaviour of light is responsible for the formation of rainbows?

   A dispersion
   B diffraction
   C refraction
   D reflection

3. A pure spectrum of white light is made up of

   A white light
   B overlapping colours
   C red and violet light
   D distinct colours

4. A white shirt is placed in both blue and yellow light. It appears

   A bluish-yellow.
   B black.
   C white.
   D cyan.

5. Which of the following groups of colours are entirely primary?

   A red, green and yellow
   B green, blue and cyan
   C red, blue and magenta
   D red, green and blue

6. Which of the following groups of colours are entirely secondary?

   A yellow, cyan, magenta
   B blue, cyan, magenta
   C red, blue, magenta
   D blue, green, red

7. Which of the following is <u>not</u> true about mixing coloured light?

   A Red and green make yellow.
   B Yellow and blue make green.
   C Red and cyan make white.
   D Green and magenta make white.

8. A red tie with white dots is observed in a red light. It appears to be

   A red
   B red with white dots
   C black
   D white

9. A cyan tie with black dots is observed in a red light. It appears to be

   A red
   B red with white dots
   C black
   D white

10. A blue dress may appear almost black in artificial yellow light from an ordinary electric lamp because

    A coloured cloths produce colours by subtraction.
    B blue cloth is a bad reflector and a good absorber of light.
    C the artificial light contains yellow which is a complementary colour to blue.
    D the artificial light itself contains very little blue.

GCE Paper 1 Solutions

1. B	2. A	3. D	4. C	5. D	6. A	7. B	8. A	9. C	10. D

GCE Paper 2 Questions

1. Figure 3.6.10.18 shows a ray of white light striking one face of a triangular prism of an unknown material whose absolute refractive index is 1.400 for red light and 1.873 for violet light.

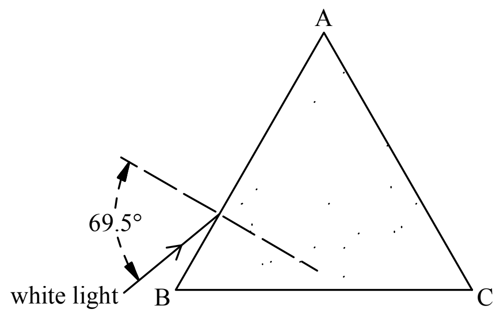

Figure 3.6.10.18

(a) Copy and complete the diagram with rays to show how the prism produces a spectrum of the white light. Label the red and violet colors in the spectrum. **(5 mks)**

(b) The spectrum produced above is said to be impure. Explain why. **(3 mks)**

. . . .

Solution

(a) *The angle of incidence is* 60° *for both red and violet. The slightly different refraction angles that cause the dispersion depend on the slightly different absolute refractive indices.*

$$\text{given equation for refraction of light incident from air: } n_2 = \frac{\sin(i)}{\sin(r)}$$

$$\text{turning refraction angle into subject: } r = \sin^{-1}\left(\frac{\sin(i)}{n_2}\right)$$

$$\text{substituting known values for red light: } r_{\text{red}} = \sin^{-1}\left(\frac{\sin(69.5°)}{1.400}\right)$$

$$\text{refraction angle of red light: } r_{\text{red}} \approx 42°$$

$$\text{substituting known values for violet light: } r_{\text{violet}} = \sin^{-1}\left(\frac{\sin(69.5°)}{1.873}\right)$$

$$\text{refraction angle of violet light: } r_{\text{violet}} \approx 30°$$

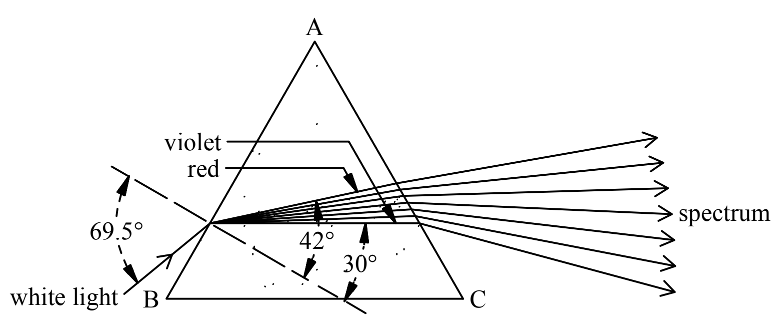

Figure 3.6.10.19

(b) The spectrum produced is impure because its colours are overlapping and indistinct from one another.

2. Figure 3.6.10.20 shows a student observing white light through two filters. While the first filter is known to be magenta, the second is of an unknown colour.

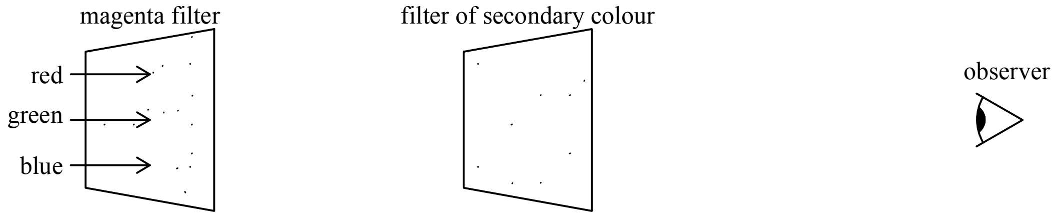

Figure 3.6.10.20

Copy and complete the diagram with the second filter's colour indicated if the student sees
(a) red light knowing the second filter is a secondary colour. **(2 mks)**
(b) blue light knowing the second filter is a secondary colour. **(2 mks)**
(c) only black knowing the second filter is a primary colour. **(2 mks)**

Solution

(a) *See figure 3.6.10.21*

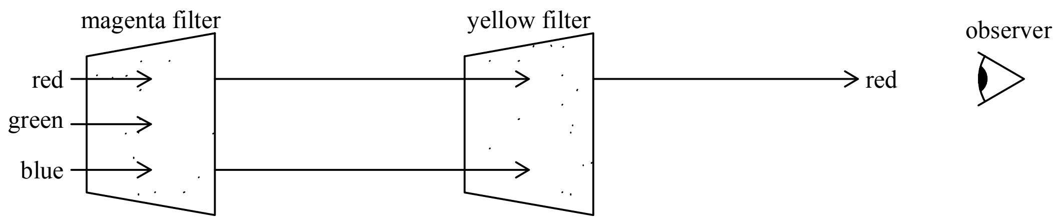

Figure 3.6.10.21

(b) *See figure 3.6.10.22*

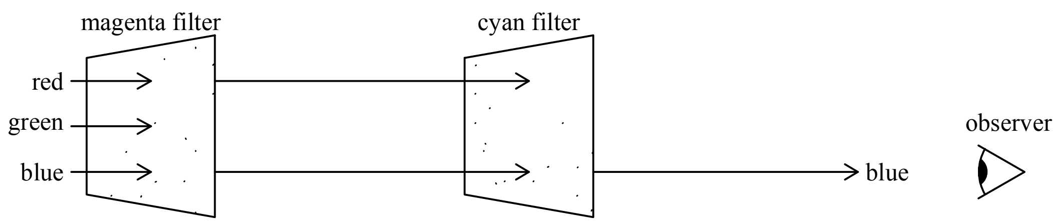

Figure 3.6.10.22

(c) *See figure 3.6.10.23*

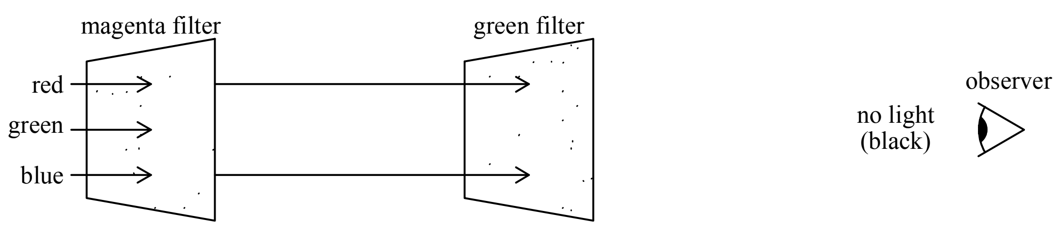

Figure 3.6.10.23